3.3 Bohr model

A particular line spectrum is the one observed for hydrogen atom. Balmer first noticed that the wavelengths of the lines observed in the visible part of the spectrum followed the following rule

$$\lambda_n = \frac{91.19\ nm}{\frac{1}{2^2} - \frac{1}{n^2}}$$

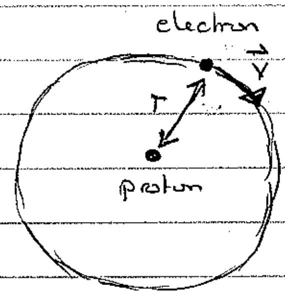
Balmer could neither explain the functional form of the equation nor the constant 91.19 nm.

However, later it was found that the relation can be extended for lines in other parts of the spectrum of hydrogen atom as well:

$$\lambda_{n,m} = \frac{91.19\ nm}{\frac{1}{n^2} - \frac{1}{m^2}}$$

Explanation of this formula will be one of the successes of the Bohr atomic model, which however turns out to be wrong. Nevertheless, it is an important step towards our understanding of quantum mechanics as the description of the microscopic world.

Let's return in our discussion of early atom models to the Rutherford model. What are the energies that an electron in an orbital motion can have, according to classical physics? Let's analyze for hydrogen atom, consisting of one positively charged proton and one negatively charged electron:



Potential energy: As a reference point we set the potential energy to be zero when the electron is infinitely far away from the proton.

The potential energy of the electron at distance r from the proton is then given by the work done to move the electron from ∞ to r

$$PE = -\int_{\infty}^{r} \vec{F} \cdot d\vec{r} = -\int_{\infty}^{r} k \frac{q_{elec}\, q_{prot}}{r^2} \hat{r} \cdot dr\, \hat{r}$$

Coulomb force

$$= -k(-e)(e) \int_{\infty}^{r} \frac{1}{r^2}\, dr = ke^2 \left[-\frac{1}{r} \right]_{\infty}^{r}$$

$$\Rightarrow \quad PE = -\frac{ke^2}{r}$$, Potential energy is negative

Kinetic energy: In order to derive the kinetic energy we use that the centripetal force is equal to the Coulomb force.

$$F_{\text{Centripetal}} = F_{\text{Coulomb}} \Rightarrow \frac{mv^2}{r} = \frac{ke^2}{r^2}.$$

$$\Rightarrow KE = \frac{1}{2} mv^2 = \frac{1}{2}\frac{ke^2}{r}$$: positive, of course

$$\Rightarrow \text{Total energy} = PE + KE = -\frac{ke^2}{r} + \frac{1}{2}\frac{ke^2}{r} = -\frac{1}{2}\frac{ke^2}{r}$$

total energy is negative

Given our knowledge about discharge lamps (discrete energy levels) and electrodynamics, we see the following problems:

- Why are there only discrete energy levels?

  According to the analysis above, the electron can have any energy.

- Why are atoms stable?

  Since the electron is accelerated in its orbital motion, according to electrodynamics the electron should radiate and loose energy

  $\Rightarrow$ Electron should fall into nucleus.