Summary:

The Bohr atomic model can:

- quantitatively explain the Balmer formula via

$$\Delta E_{n \to m} = E_1 \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$: Energy difference between levels

$$\Rightarrow f_{n \to m} = \frac{E_1}{h} \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

$$\Rightarrow \lambda_{n \to m} = \frac{c}{f_{n \to m}} = \frac{ch}{E_1} \frac{1}{\frac{1}{n^2} - \frac{1}{m^2}}$$

Prefactor gives the constant in Balmer formula.

- explain also the energy spectra of other single-electron systems (He^+ ion, Li^{2+} ion, ...)

- predict approximately the size of hydrogen atom (Bohr radius)

The Bohr atomic model cannot

- explain why angular momentum is quantized
  why electrons do not radiate when in stationary orbitals

- explain between which orbitals the electrons jump

- be applied for more complex atoms, molecules ...

Overall the Bohr model is a mix of

classical physics and some arbitrary quantum rules

While it can explain certain things, it also shows somehow the limits of classical mechanics to explain microscopic phenomena.

4. From matter waves to Schrödinger's wave equation

4.1 Compton effect

So far, we have seen that Einstein proposed that light consists of energy quanta given by

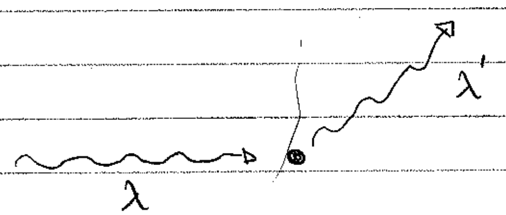
$$E = hf$$

On the other hand, from the theory of relativity momentum and energy of the light are related by

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

which relates particle features (E and p) and wave features (λ and f).

This relation was proven in the Compton effect where high energy X-ray light is irradiated on atoms. Compton observed that the light is scattered like a particle



The details of the results of the experiment can be explained via the above relation

$$p = h/\lambda$$

4.2 de Broglie wavelength

Thus, we see that light shows besides wave behavior also particle behavior. In other words: There is a wave-particle duality.

de Broglie (1923) took the wave-particle duality even further and postulated:

All (material) particles should also display a dual wave-particle behavior. Generalizing the relations for the photon momentum to any material particle with nonzero mass, the de Broglie relations relate the momentum of the particle to the wavelength λ and the wave number k of a matter wave as

$$\lambda = \frac{h}{p} \quad \text{and} \quad k = \frac{p}{\hbar} \quad \text{with} \quad \hbar = \frac{h}{2\pi}$$

Using: $E = \frac{1}{2} m v^2$ and $p = mv \Rightarrow p^2 = m^2 v^2$

$$\Rightarrow E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE}$$

$$\Rightarrow \lambda = \frac{h}{\sqrt{2mE}}$$

Thus:
- de Broglie wavelength is as shorter as larger is the energy of a given particle
- de Broglie wavelength is as shorter as larger is the mass of a particle at a given energy.