4.4. Matter waves and wavefunction

We are left with the task to introduce a new concept to quantitatively describe and interpret the results seen for the double-slit experiment. The key is that we observe the same kind of interference pattern for performing the experiment with

- single electrons
- other single microscopic "particles"
- single photon (i.e. light)

In the case of light the experiment can be interpreted as follows:

→ On the one hand: The light intensity at a certain point on the screen is determined by the superposition of the wave functions of the electric fields passing through the two slits

$$E^2(x,t) = \left(E_1(x,t) + E_2(x,t)\right)^2$$

This gives the characteristic interference pattern (wave nature of light)

→ On the other hand: The light intensity at a certain point on the screen is an information about the probability that a photon hits the screen (particle nature of light)

→ Thus, it is the wave function of the electric (and magnetic) field that gives information about the wave as well as the particle character of light:

$$\text{Probability} \sim \#\ \text{light flashes} \sim \#\ \text{of photons}$$
$$\sim \text{intensity of light} \sim (E_1 + E_2)^2$$

Back to the experiment with electrons. It shows the same dual wave-particle character.

→ Expectation: Electrons (massive particles) can be described by a <u>wave</u>f<u>unction</u> $\Psi(x,t)$. As we will see later, this wavefunction is in general a complex number.

And further on:

- In analogy to the interpretation of the light intensity, we interpret the (absolute) square of the wave function as a probability density. In other words; we expect that $|\Psi(x,t)|^2$ denotes the proba<u>bility</u> de<u>nsity</u> to find the electron at location x at time t.

- The probability to find the electron (at time t) in a certain interval Δx is then given by

$$P_{\Delta x} = \int_{\Delta x} |\Psi(x,t)|^2 \, dx$$

- Mathematical and physical restrictions:

$$|\Psi(x,t)|^2 \geq 0 \quad , \text{ of course}$$

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 \, dx = 1 \quad : \text{ particle has to be somewhere}$$

Note: We work in 1D, but you can of course also write down this concept of wavefunction in 3D as well: $\Psi(\vec{r},t)$

- We call $\psi(x,t)$ the probability (density) amplitude. The wavefunction contains the whole information about the particle. But, $\psi(x,t)$ itself has <u>no empirical</u> significance only $|\psi(x,t)|^2$ as being the probability density has it.

But, which equation determines $\psi(x,t)$?

Historical note:
As a Professor of Physics at Zürich University Erwin Schrödinger gave a colloquium talk on how de Broglie associated a wave with a particle. When he had finished his collegue Peter Debye remarked that he thought that this theory is childish because "to deal properly wi waves, one had to have a wave equation" (see Felix Bloch, Physics Today, December 1976, p. 23)

Just a few weeks later Erwin Schrödinger gave another talk in the colloquium which he started by saying: "My colleague Debye suggested that one should have a wave equation; well, I have found one!"