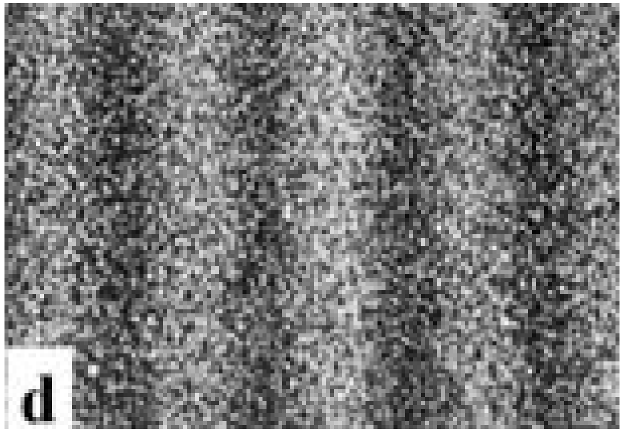
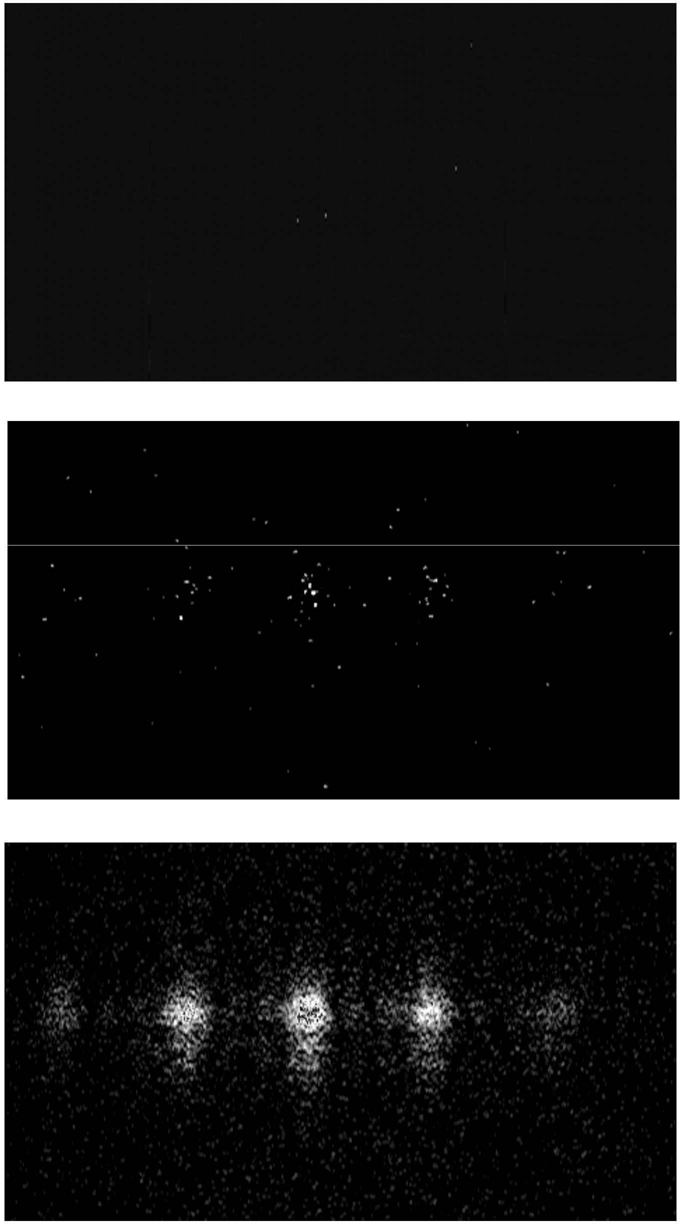


Summary so far:

- Individual light flashes, never two flashes at same time
  → particle feature
- Light flashes appear randomly with pattern
  → wave feature, probabilistic, must be property of electron

Double slit experiment with single photons



Same experiment with single photons:

Individual flashes of light

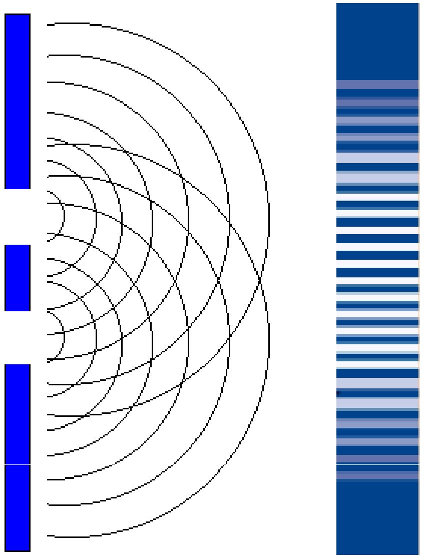
Build up to double slit interference pattern

Pattern is a measure of the probability where the next photon will hit the screen

How can we describe the pattern?

What does this mean concerning our probabilistic interpretation?

Double slit interference

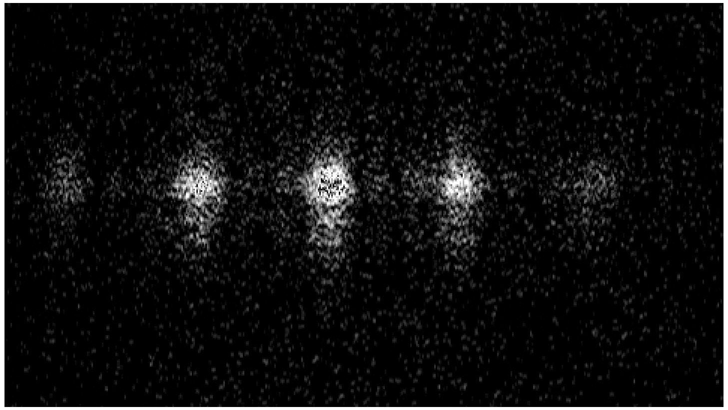


$$\int_{\Delta x} E^2 dx = \int_{\Delta x} (E_1 + E_2)^2 dx$$

: Measure for intensity in Δx

: Measure for brightness in Δx

Double slit experiment with single photons



$$\int_{\Delta x} E^2 dx = \int_{\Delta x} (E_1 + E_2)^2 dx : \text{Intensity in } \Delta\text{x}$$

$$: \text{Brightness in } \Delta\text{x}$$

$$: \text{Probability to find photon in } \Delta\text{x}$$

Wave function – Probability

$$\int_{\Delta x} E^2 dx = \text{Probability to find photon in } \Delta\text{x}$$

$$E^2 = \left(E(x)\right)^2 = \text{Probability to find photon at x}$$

$$= \text{Probability density}$$

$$\text{E(x)} = \text{Probability amplitude} = \text{Wave function}$$

Wave function – Probability

According to our probabilistic interpretation:

$$\text{What is } \int_{all\ x} E^2 dx = ?$$

(A) 0 (B) 1

(C) Some value between 0 and 1

(D) Some positive value, but cannot be determined

Wave function – Probability

According to our probabilistic interpretation, what can you say about $(E(x))^2$?

(A) Could be any value between -∞ and +∞

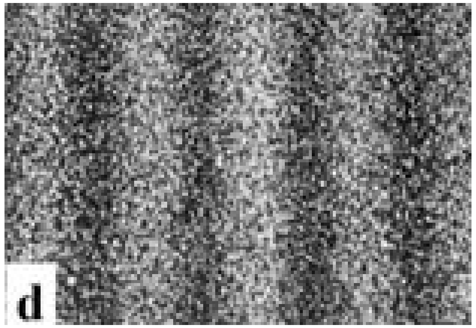
(B) Could be any value but not ±∞

(C) Is either 0 or positive

(D) Must be between 0 and 1

(E) Something else

Double slit experiment with single electrons



wavefunction

$$\int_{\Delta x} |\psi|^2 \, dx = \text{Brightness in interval } \Delta \text{x}$$

$$= \text{Probability to find electron in } \Delta \text{x}$$

particle nature of electron

Wave function – Probability

$$\int_{\Delta x} |\psi|^2 \, dx = \text{Probability to find electron in } \Delta \text{x}$$

$$|\psi|^2 = |\psi(x)|^2 = \text{Probability density}$$

$$\psi(x) = \text{Probability density amplitude}$$

$$= \text{Wave function}$$

Role of wavefunction in quantum mechanics

EM Waves (light/photons)

Amplitude of E-field: E(x)

Probability density of finding a photon: E^2

Wave equation from Maxwell equations

Matter Waves (electrons, atoms, etc.)

Amplitude of matter field: $\Psi(x)$

Probability density of finding a particle: $|\Psi|^2$

Wave equation = Schrödinger equation

Matter waves

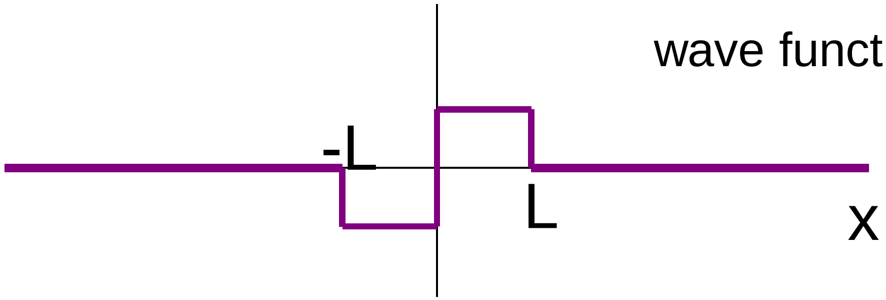
Describe a particle with a wave function $\Psi(x,y,z;t)$

Wave function contains information about the ***probability*** to find a particle at x, y and z & t

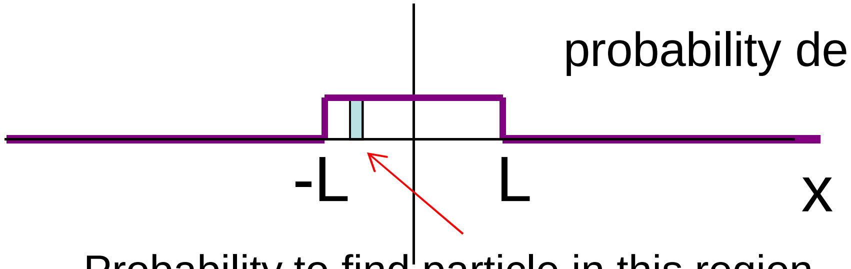
Wave function does **not** describe the path of the particle

In general: $\Psi(x,y,z;t)$ We simplify: $\Psi(x;t)$

Matter waves



wave function



probability density

Probability to find particle in this region

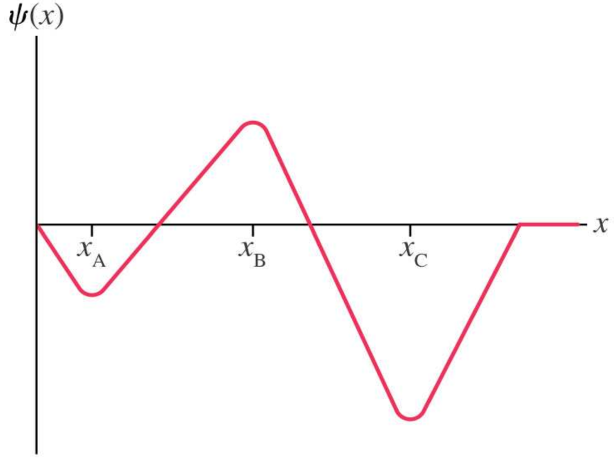
Below is a wave function for a neutron. At what value of x is the neutron most likely to be found?

(A) X_A

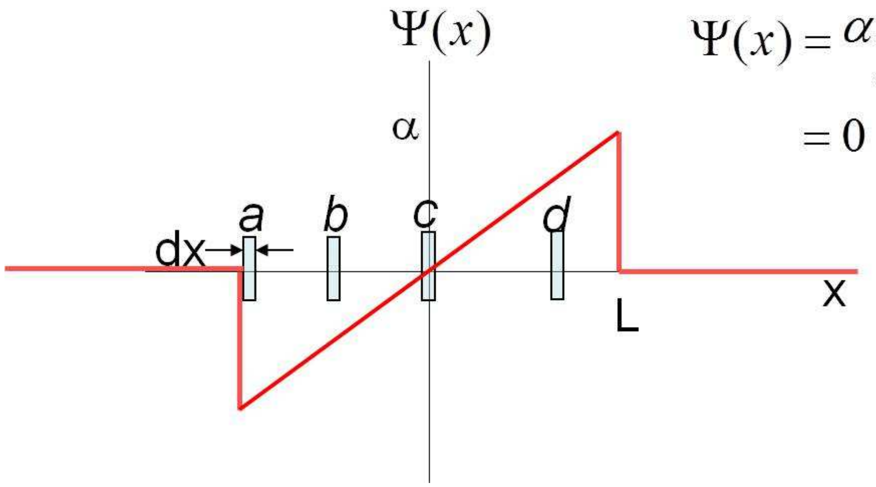
(B) X_B

(C) X_C

(D) There is no most likely place



An electron is described by the following wave function



$$\Psi(x) = \alpha x / L \text{ from } x = -L \text{ to } x = +L$$

$$= 0 \text{ elsewhere}$$

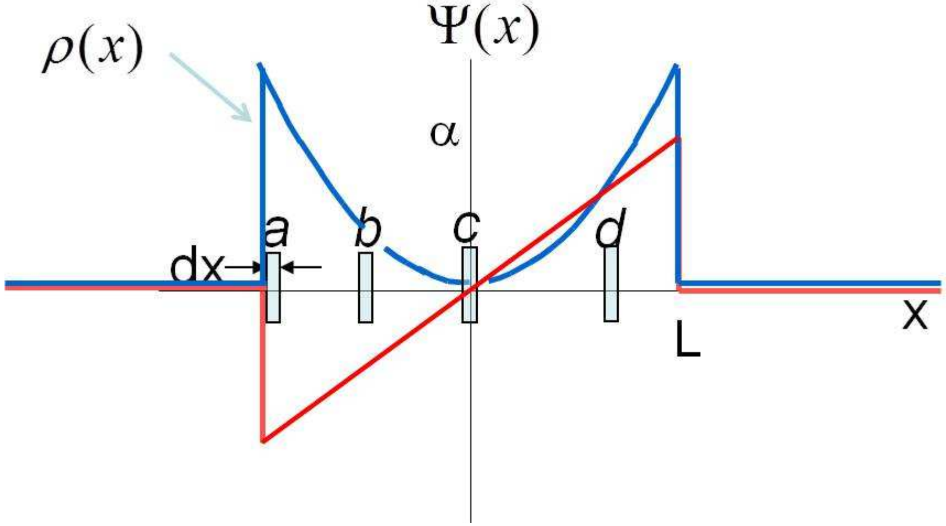
How do the probabilities of finding the electron near (within dx) of a,b,c, and d compare?

(A) d > c > b > a

(B) a = b = c = d

(C) d > b > a > c

(D) a > d > b > c

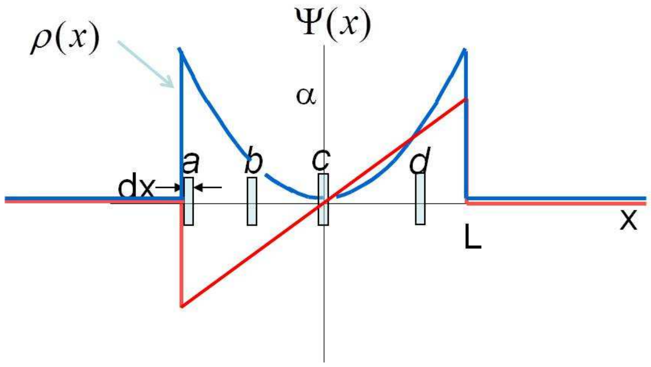


$$\Psi(x) = {\alpha x}/{L} \text{ from } x = -L \text{ to } x = +L$$

$$= 0 \text{ elsewhere}$$

$$\rho(x) = \left|\Psi(x)\right|^2 = \frac{\alpha^2 x^2}{L^2}$$

P(a)>P(d)>P(b)>P(c)



$$\Psi(x) = \alpha x / L \text{ from } x = -L \text{ to } x = +L$$

$$= 0 \text{ elsewhere} \qquad \rho(x) = \left|\Psi(x)\right|^2 = \frac{\alpha^2 x^2}{L^2}$$

Can you determine α in terms of L?

(A)Yes, of course (B) Yes, maybe

(C) No, not possible