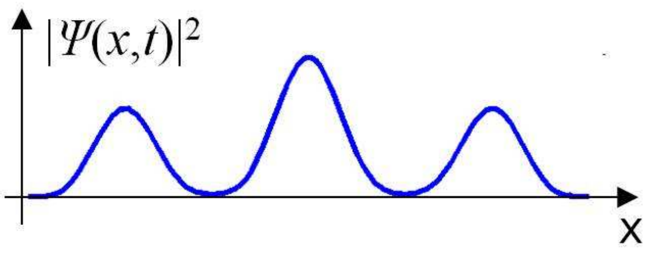
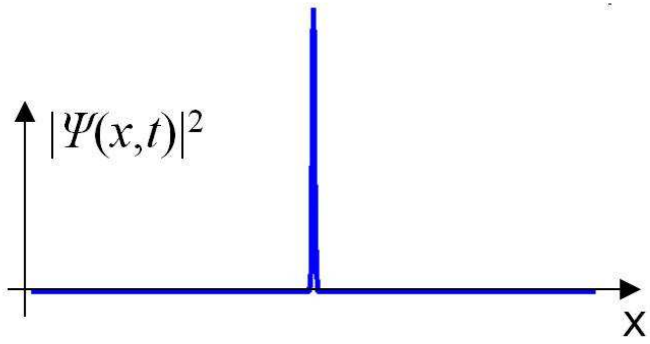
Below there are three probability densities $|\Psi(x,t)|^2$ for a particle. The wavefunction $\Psi(x,t)$ is

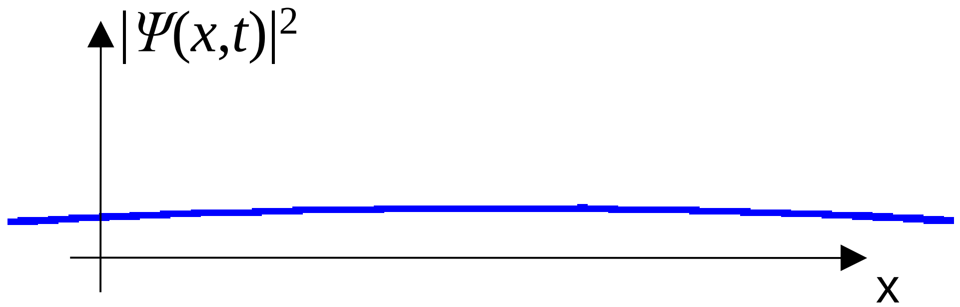
(A) the same

(B) different

in each of these cases.







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Info Session: **Monday, February 29**
UMC 235 | 5-6 PM

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APPLY: Monday, February 29 - Monday, March 14
at **LAcentral.colorado.edu**

$\Psi(x,t) = \exp[i(kx-\omega t)]$ is a solution of

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

since

LHS:

$$-\frac{\hbar^2}{2m}(ik)^2 \exp[i(kx-\omega t)] = \frac{\hbar^2 k^2}{2m}\exp[i(kx-\omega t)]$$

$$= \frac{p^2}{2m}\exp[i(kx-\omega t)] = E\exp[i(kx-\omega t)]$$

RHS:

$$i\hbar(-i\omega)\exp[i(kx-\omega t)] = \hbar\omega\exp[i(kx-\omega t)]$$

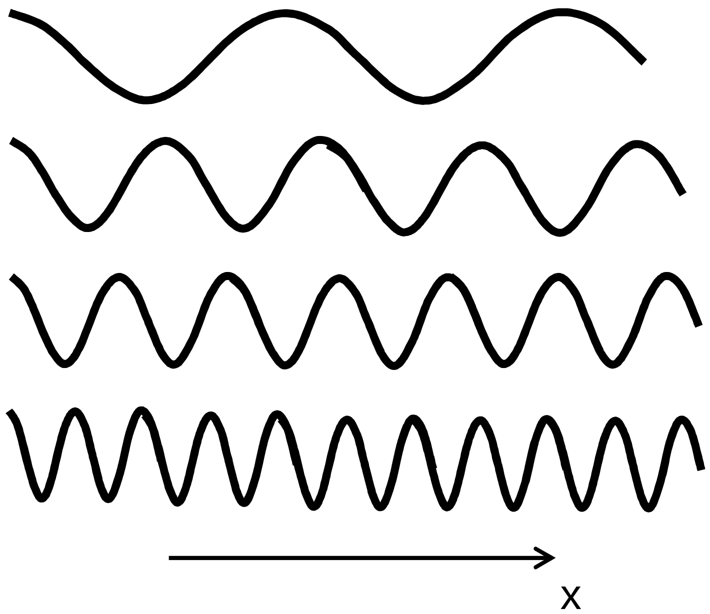
$$= hf\exp[i(kx-\omega t)] = E\exp[i(kx-\omega t)]$$

$$\Psi_1(x,t) = \exp\left[i\left(k_1 x - \omega_1 t\right)\right]$$

$$\Psi_2(x,t) = \exp\left[i\left(k_2 x - \omega_2 t\right)\right]$$

$$\Psi_3(x,t) = \exp\left[i\left(k_3 x - \omega_3 t\right)\right]$$

$$\Psi_4(x,t) = \exp\left[i\left(k_4 x - \omega_4 t\right)\right]$$



(A)

(B)

(C)

(D)

Which of these solutions represents a free particle with the *largest* (kinetic) energy?

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{\hbar^2 k^2}{2m}$$

Shortest wavelength = largest wavenumber = larger energy

Superposition principle

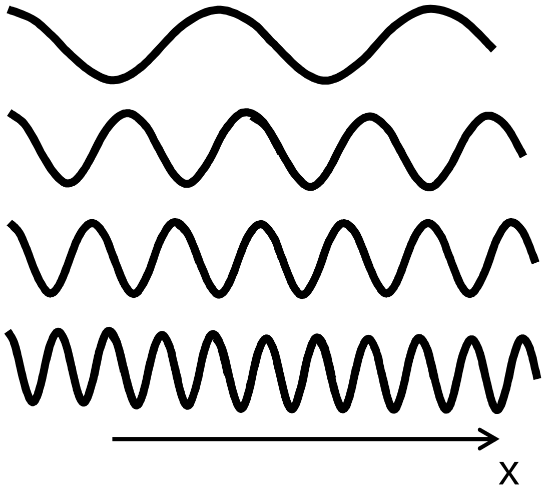
If each of these wave function is a solution,
then the sum is a solution as well

$$\Psi_1(x,t) = \exp\left[i\left(k_1 x - \omega_1 t\right)\right]$$

$$\Psi_2(x,t) = \exp\left[i\left(k_2 x - \omega_2 t\right)\right]$$

$$\Psi_3(x,t) = \exp\left[i\left(k_3 x - \omega_3 t\right)\right]$$

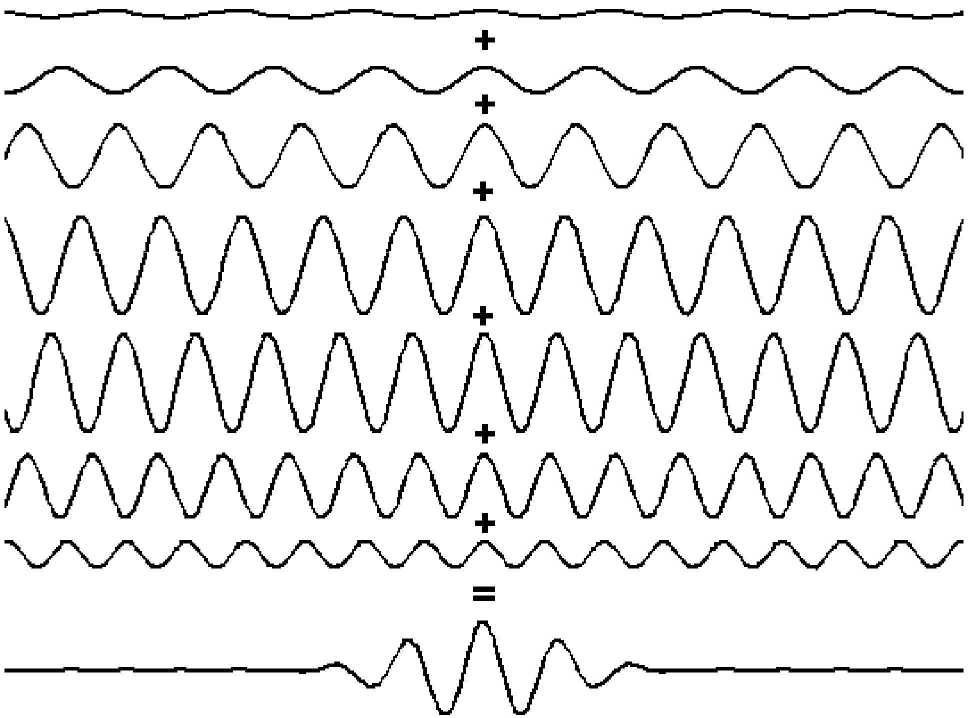
$$\Psi_4(x,t) = \exp\left[i\left(k_4 x - \omega_4 t\right)\right]$$



http://phet.colorado.edu/en/simulation/fourier

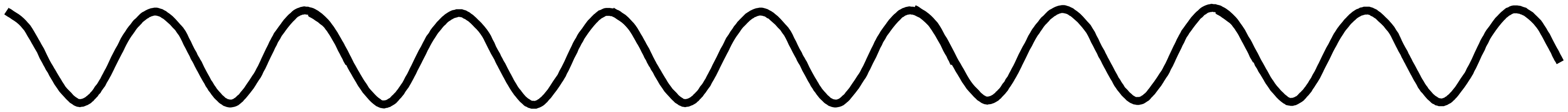
Superposition principle

$$\Psi(x,t) = \sum_n A_n \exp\left[i\left(k_n x - \omega_n t\right)\right]$$

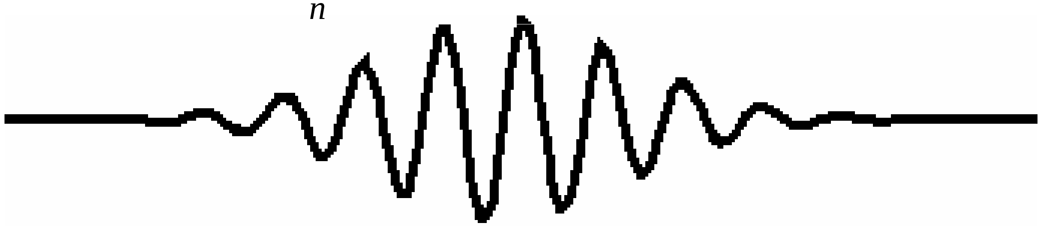


Plane waves vs. wave packets

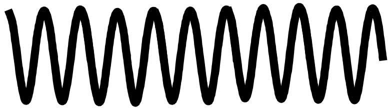
$$\Psi(x,t) = A\exp\left[i\left(kx-\omega t\right)\right]$$



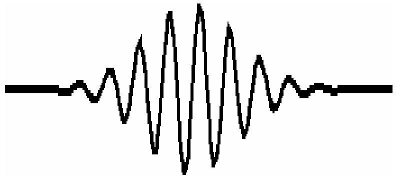
$$\Psi(x,t) = \sum_n A_n \exp\left[i\left(k_n x-\omega_n t\right)\right]$$



Plane waves vs. wave packets



$$\Psi(x,t) = A\exp\left[i\left(kx-\omega t\right)\right]$$



$$\Psi(x,t) = \sum_n A_n \exp\left[i\left(k_n x-\omega_n t\right)\right]$$

For which wave is the position (x) and momentum (p) most well-defined?

(A) x: plane wave, p: plane wave

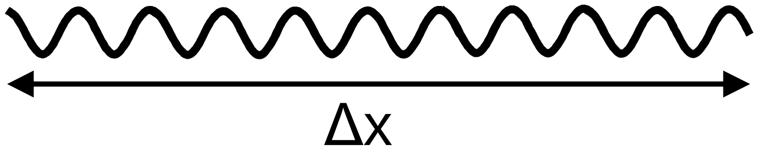
(B) x: plane wave, p: wave packet

(C) x: wave packet, p: plane wave

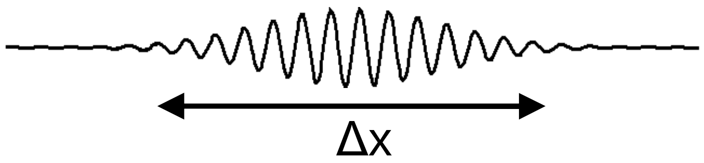
(D) x: wave packet, p: wave packet

(E) Equally well defined for both

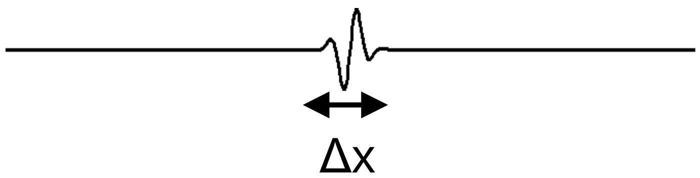
Uncertainty principle



small Δp – only one wavelength

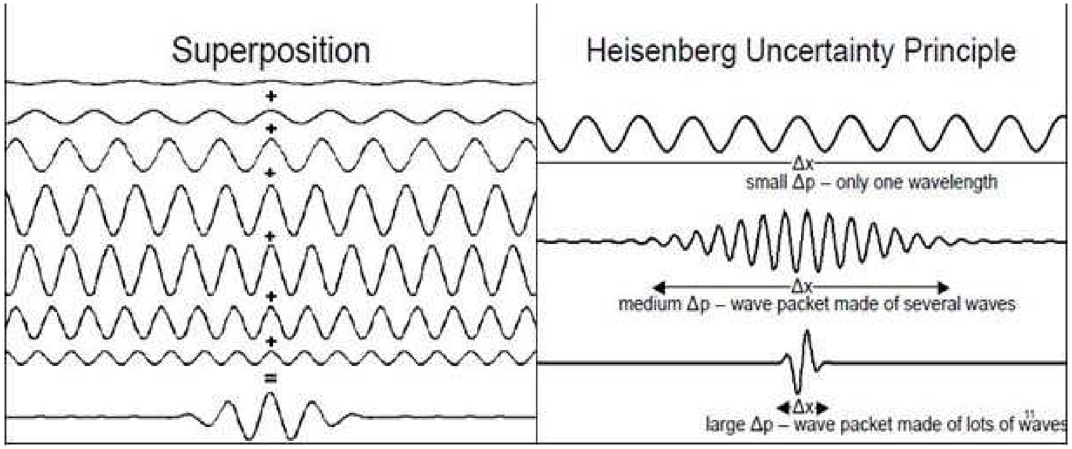


medium Δp – wave packet made of several waves



large Δp – wave packet made of lots of waves

Uncertainty principle



- Wave packets are constructed from a series of plane waves
  → superposition principle
- The more spatially localized the wave packet, the
  less uncertainty in position.
- With less uncertainty in position comes a greater
  uncertainty in momentum.

Uncertainty principle

In math: $$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

In words:
The position and momentum of a particle cannot **both** be determined with complete precision.

The more precisely one is determined, the less precisely the other is determined.

Uncertainty principle $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$

Two students plan to first measure the position of a particle and then some time later the momentum of the particle. Before they do so, they discuss the outcomes of the measurements. Consider their statements:

Student 1: The uncertainty principle tells us that we can neither measure the position nor the momentum of the particle with absolute precision in these two measurements.

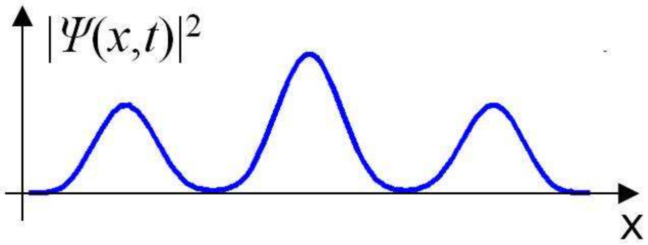
Student 2: No, it tells us that we can make one of the two measurements with absolute precision, but not the other one.

Who is correct?

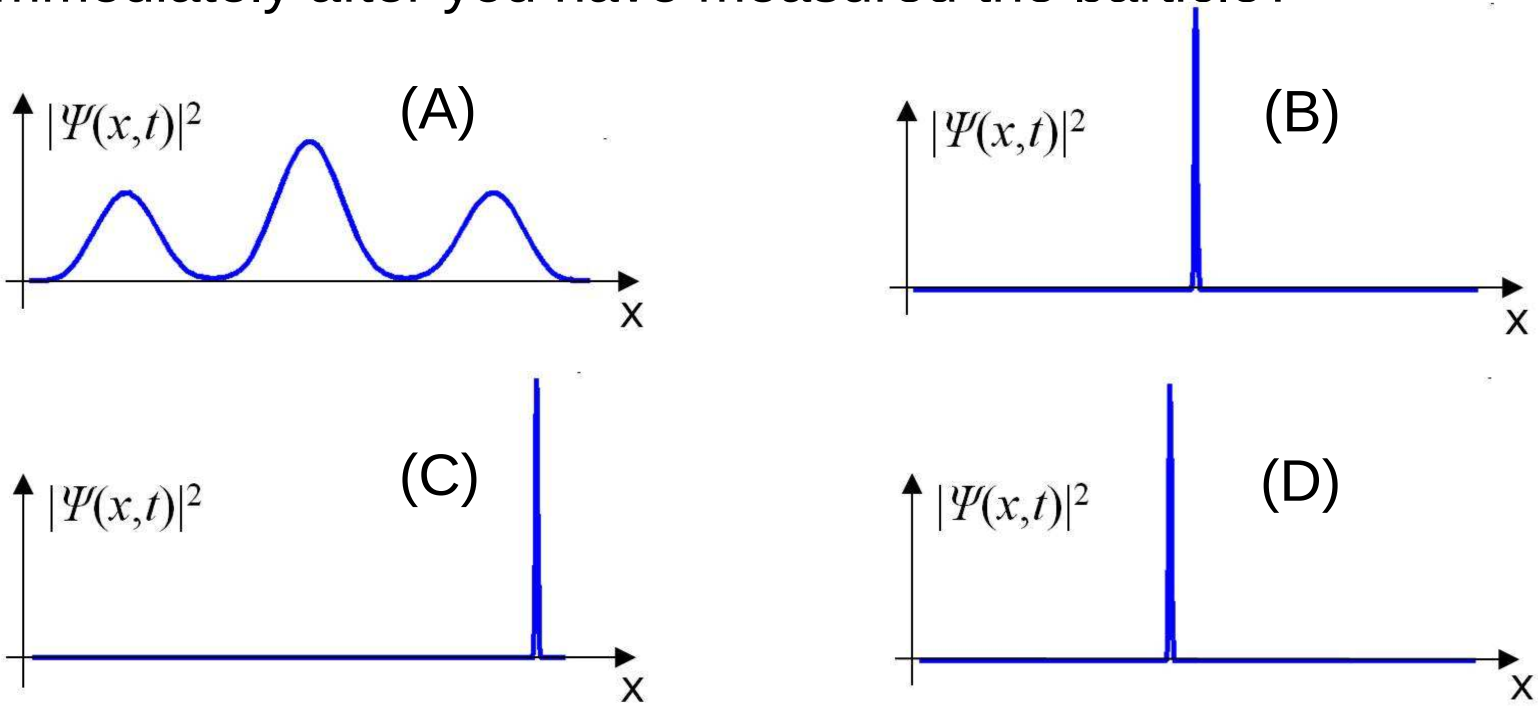
(A) Student 1 (B) Student 2 (C) None of them

Uncertainty principle is about simultaneous measurement of position and momentum of a particle, not about the measurement of one of the observables.

Assume that the probability density of a particle is given by

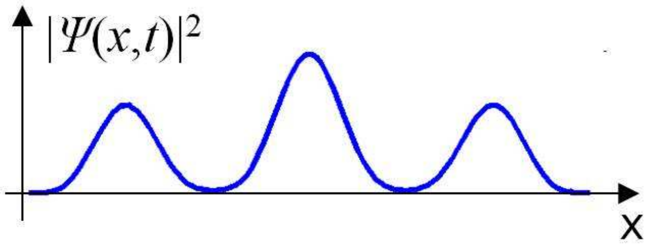


You measure the position of the particle. Which of the following graphs displays best the probability density immediately after you have measured the particle?

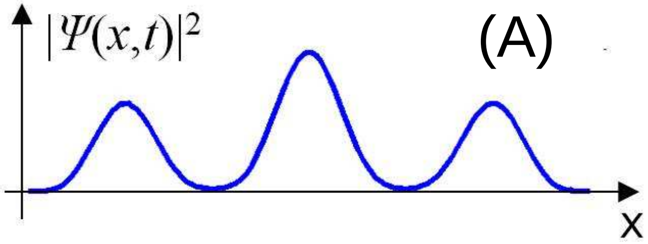


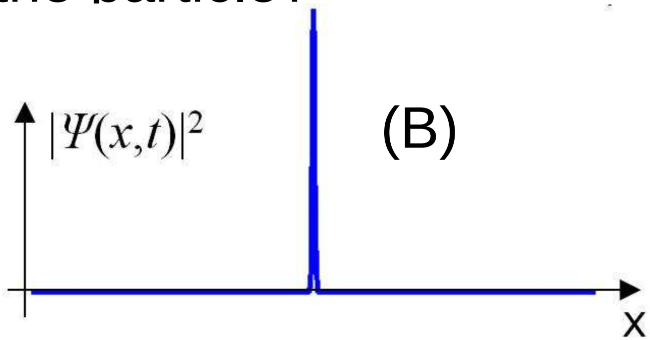
(E) Could be B, C or D, depending on where you have measured the particle

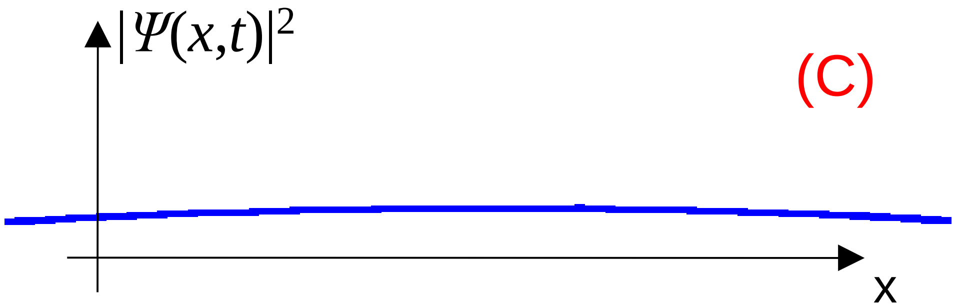
Assume that the probability density of a particle is given by



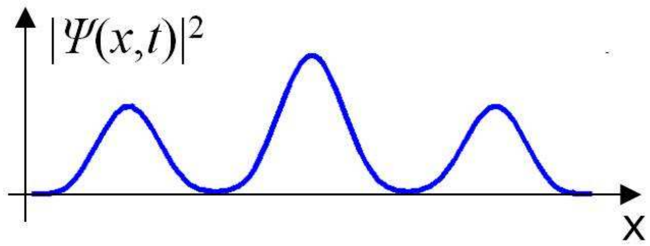
You measure the position of the particle and after a short time the momentum of the particle. Which of the following graphs displays best the probability density immediately after you have measured the momentum of the particle?



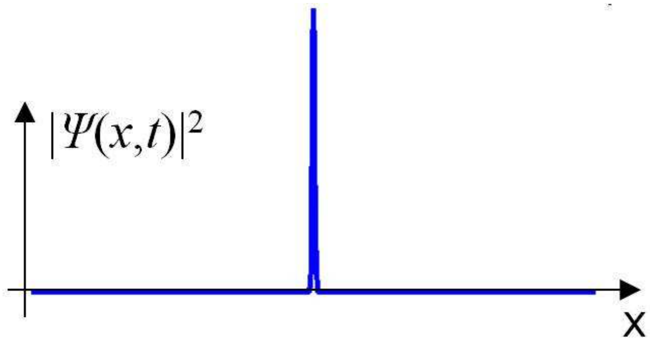




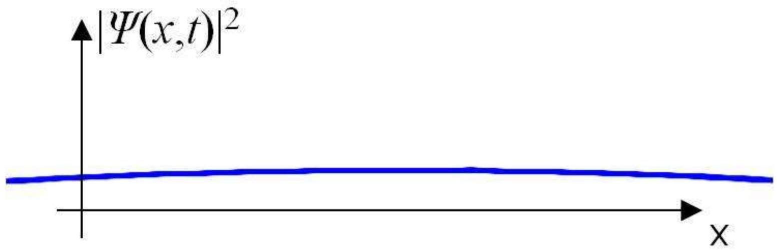
Before measurement:



After position measurement:



After momentum measurement:



Remember: Wavefunction is different for each of these probability densities. Act of measurement changes the wavefunction. Schrödinger equation describes everything before and after the measurement, but not the measurement itself (discontinuous process).