Complementary variables

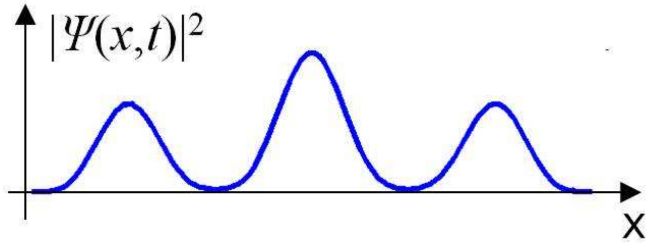
The

- position and momentum of a particle
- components of the angular momentum (L_x, L_y, L_z)
- components of the spin (S_x, S_y, S_z)

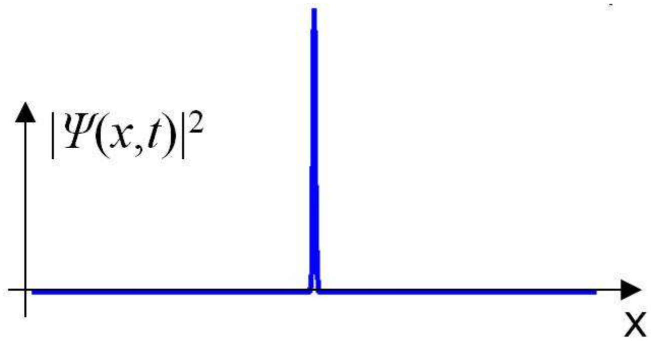
of a particle cannot be determined with complete precision simultaneously.

The more precisely one is determined, the less precisely the other(s) is (are) determined.

Before measurement:

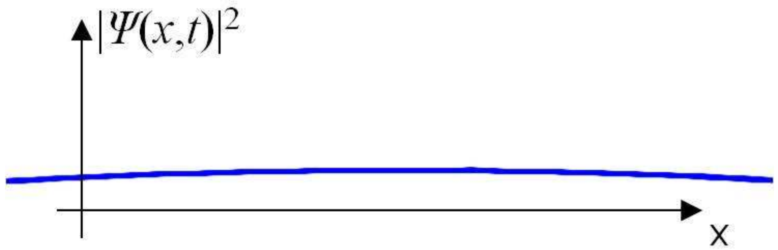


After position measurement:



small Δx
large Δp

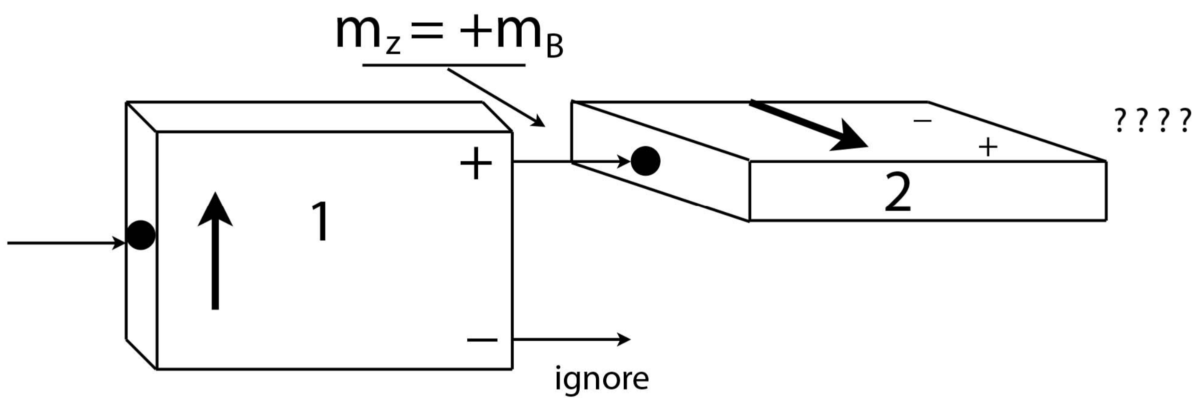
After momentum measurement:

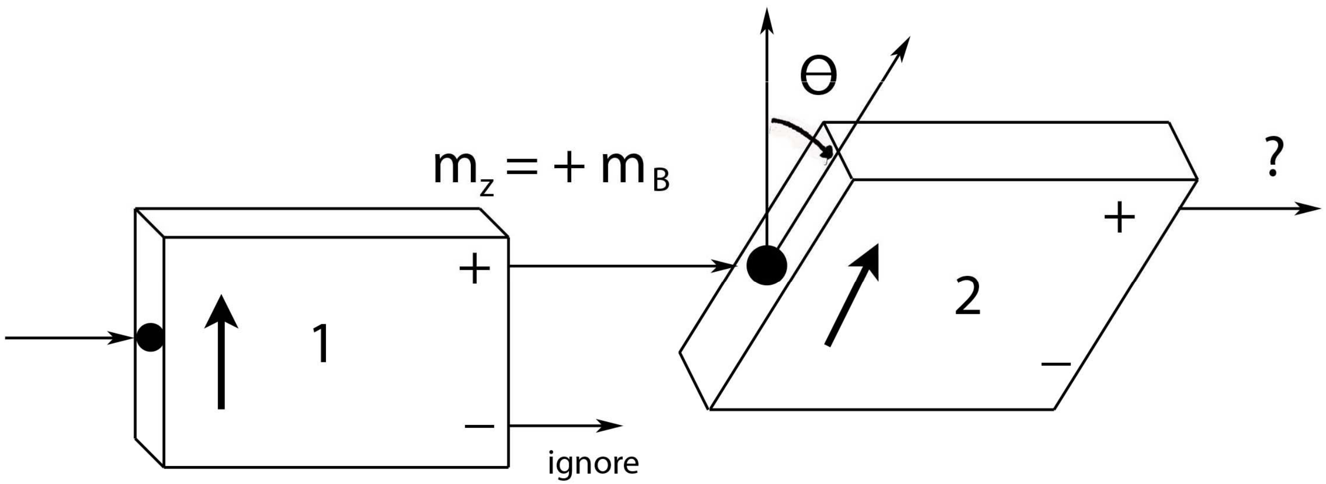


large Δx
small Δp

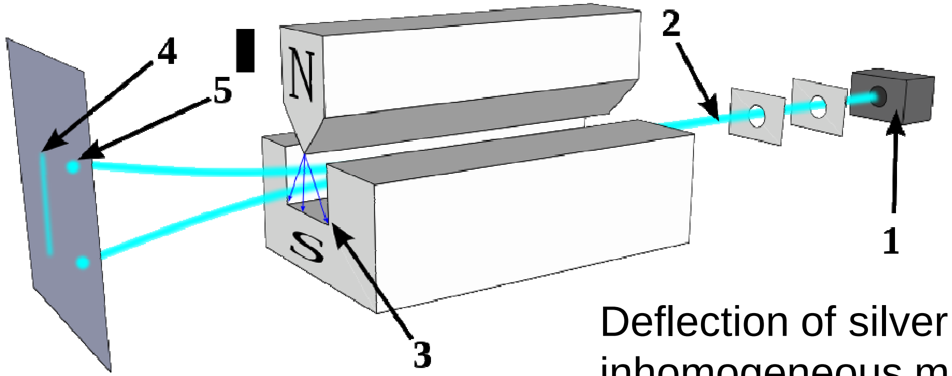
Remember: Wavefunction is different for each of these probability densities. Act of measurement changes the wavefunction. Schrödinger equation describes everything before and after the measurement, but not the measurement itself (discontinuous process).

Spin measurements





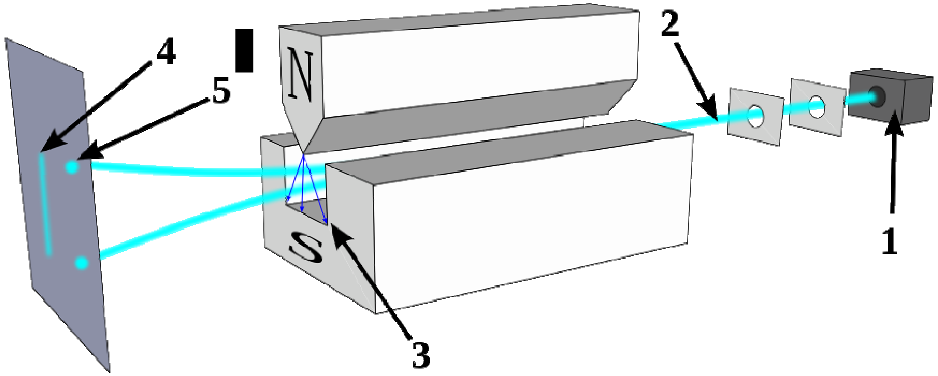
Stern-Gerlach experiment (1925)



Deflection of silver atoms in inhomogeneous magnetic field (picture: Wikipedia)

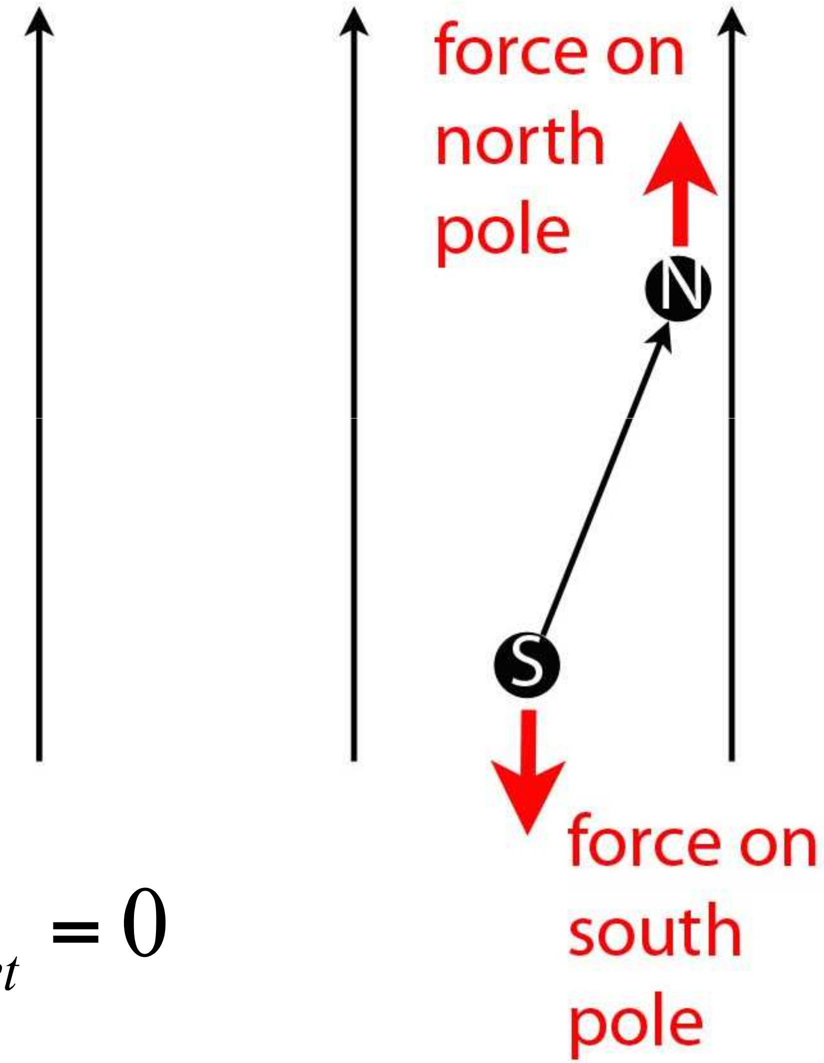
Observation: Discrete spectrum (5) and ***not*** classically expected continuous spectrum (4)

Stern-Gerlach experiment (1925)



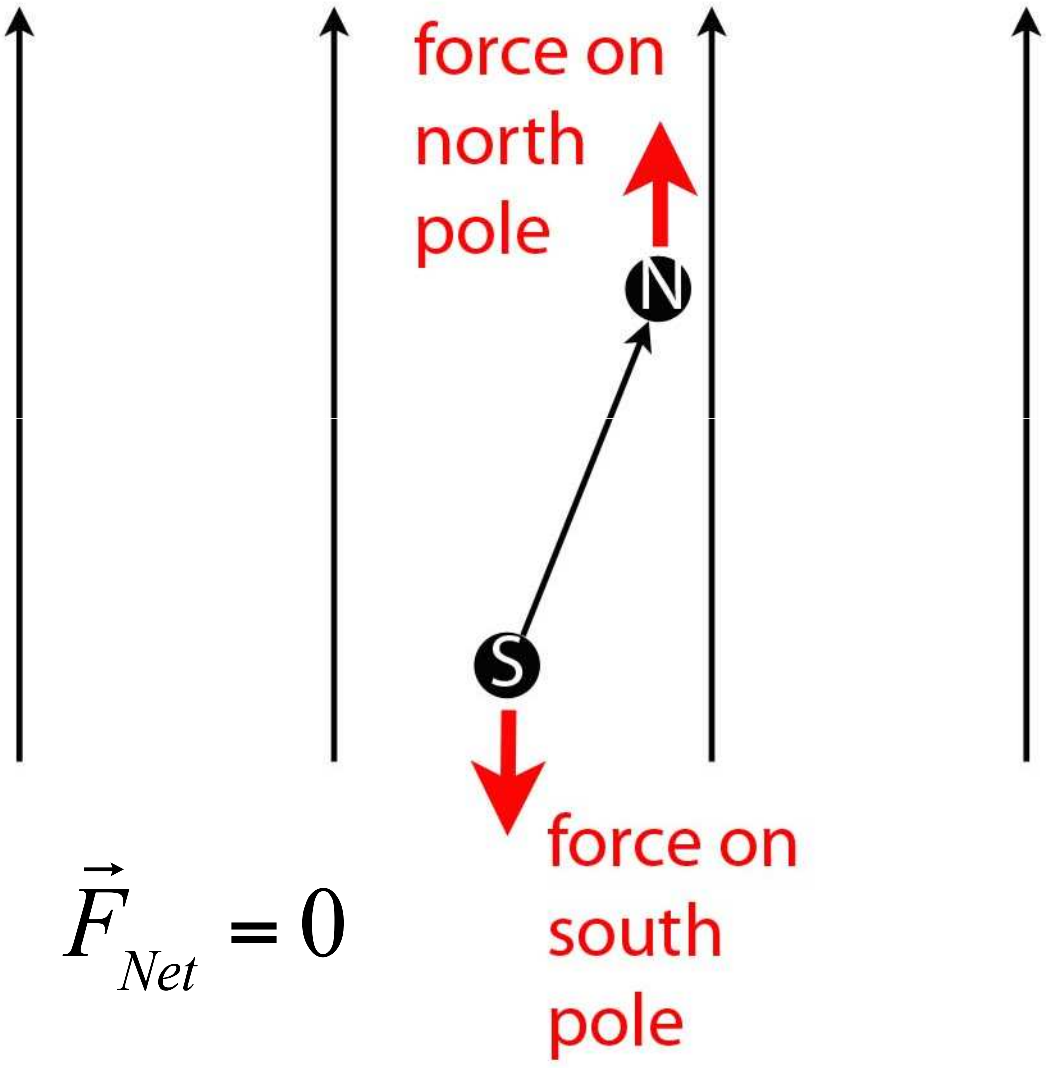
- Atoms must have a quantized (projection/component of) magnetic moment
- Magnetic moment is due to orbital angular momentum and spin

Compass Needle in a Uniform Magnetic Field



$$\vec{F}_{Net} = 0$$

Compass Needle in a Uniform Magnetic Field



$$\vec{F}_{Net} = 0$$

What happens to the compass needle?

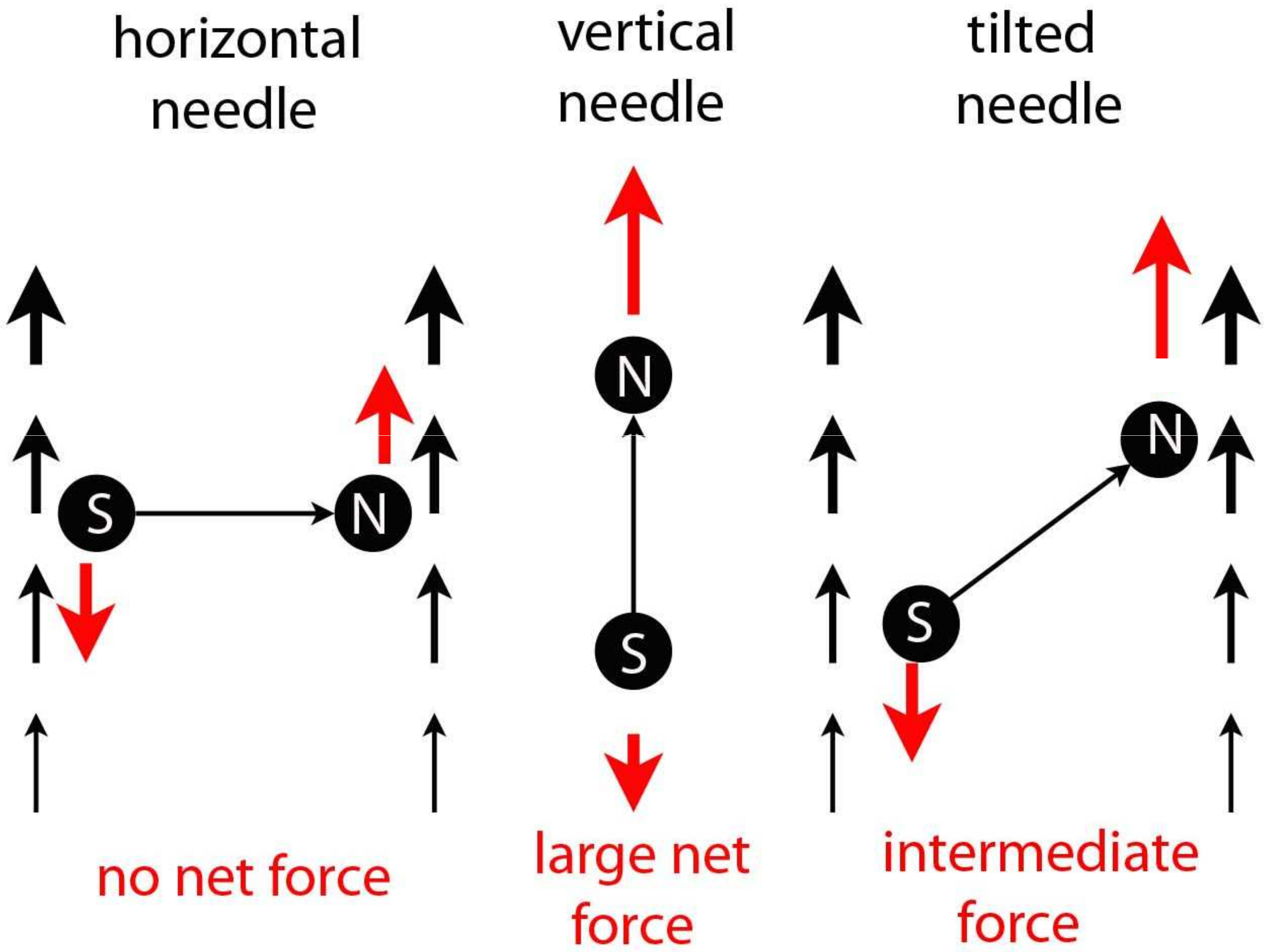
(A) Nothing, $\mathbf{F}_{net} = 0$

(B) Oscillates about the center-of-mass

(C) Whole compass needle accelerates either up or down.

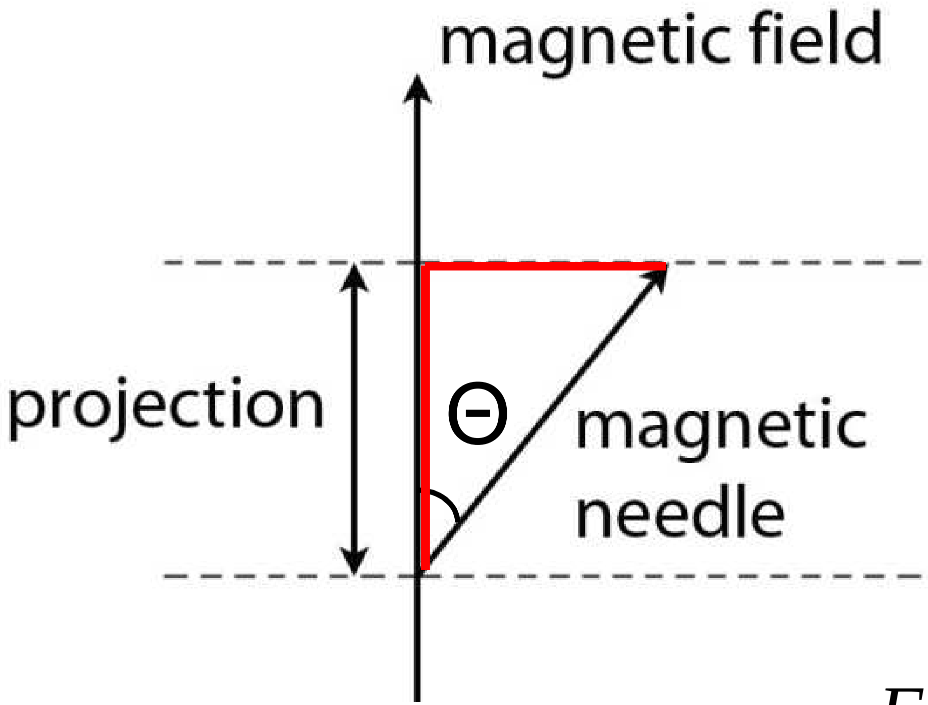
(D) B and C

Compass Needle in a <u>Non-Uniform</u> Magnetic Field



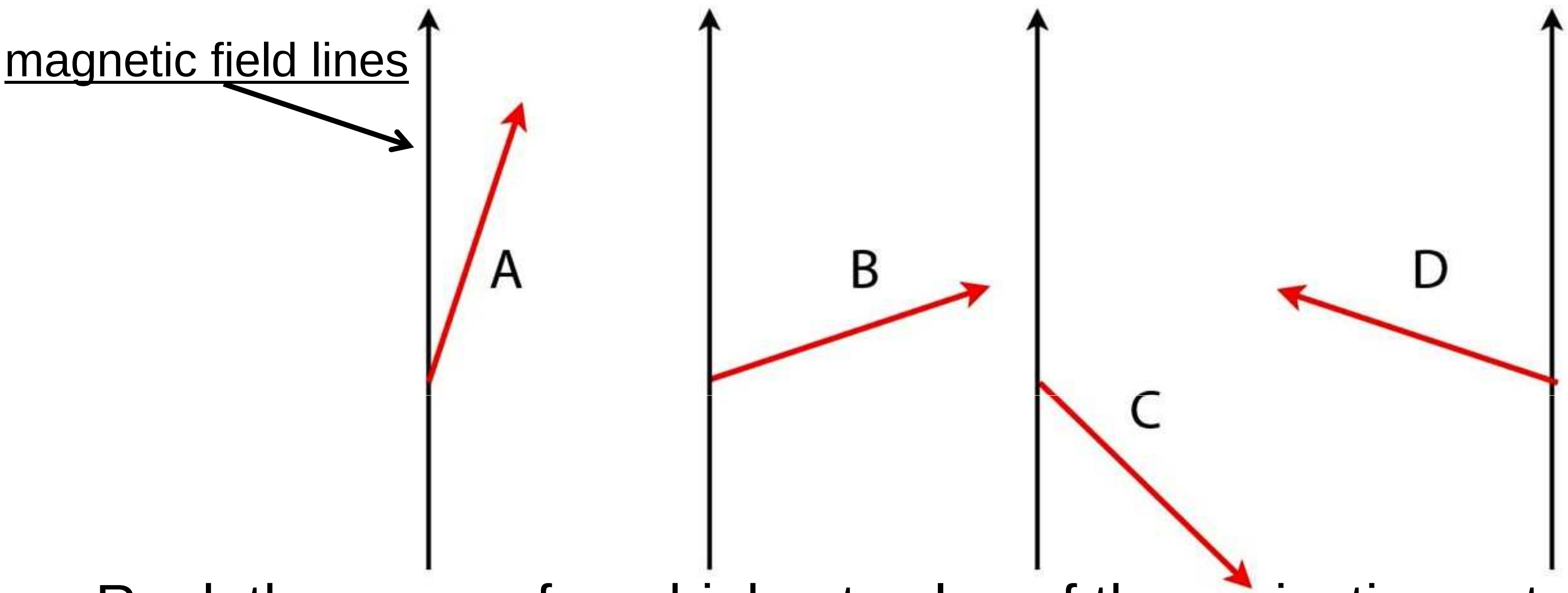
Compass Needle in a Non-Uniform Magnetic Field

It turns out the net force is proportional to the ***projection*** of the magnetic needle onto the magnetic field.



$$F_{net} \propto \cos(\Theta)$$

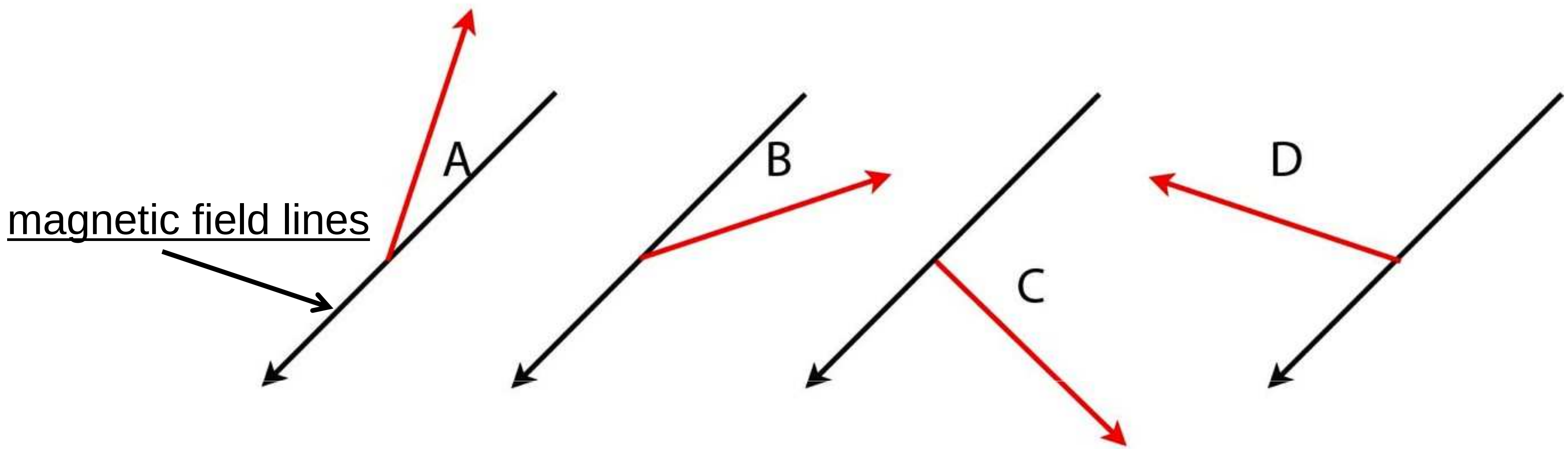
Below are four magnetic arrows (**A**, **B**, **C**, and **D**), shown relative to magnetic field lines pointing vertically upward.
All four arrows have the same length.



Rank the arrows from highest value of the projection onto the vertical axis to the lowest (sign matters).

A) A > B > C > D
B) A > B > D > C
C) A > B = D > C
D) A > C > B = D
E) None of the above.

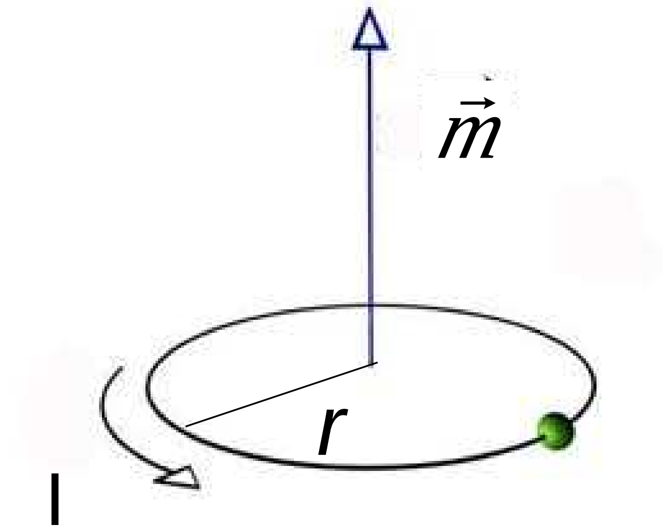
Below are four magnetic arrows (**A**, **B**, **C**, and **D**), shown relative to magnetic field lines pointing vertically upward.
All four arrows have the same length.



Rank the arrows from highest value of the projection onto the vertical axis to the lowest (sign matters).

A) A > B = D > C
B) D > C > B = A
C) C > D = B > A
D) A = B > C > D
E) None of the above.

Magnetic Moment of a Current Loop



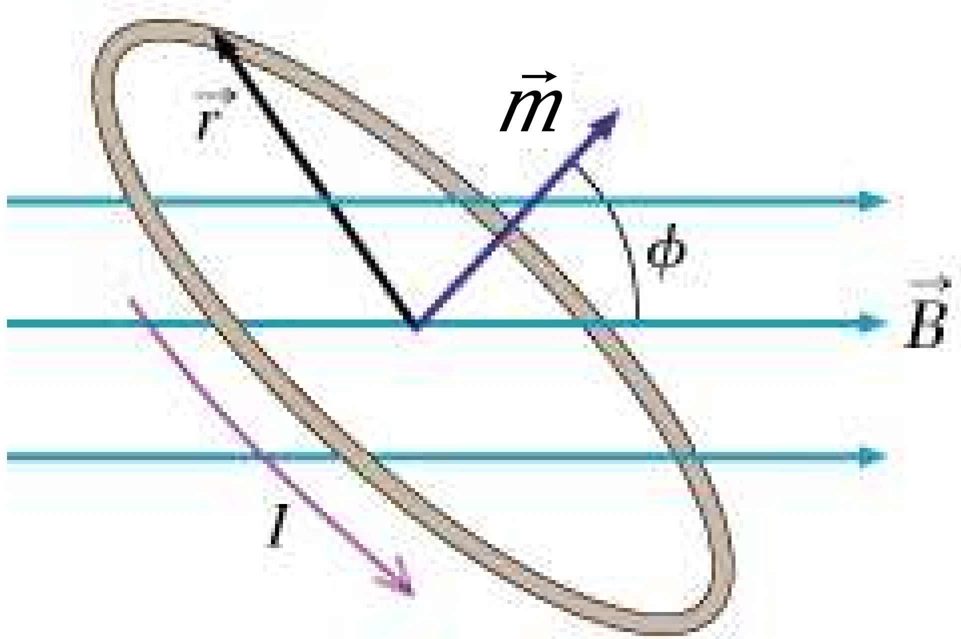
$$\vec{m} = I \cdot \vec{a}$$

$$\vec{a} = \pi r^2 \cdot \hat{a}$$

$\hat{a}$ points in the direction perpendicular to the plane of the current loop, in the direction given by the ***right-hand rule***.

Torque on a Current Loop in a Magnetic Field

$$\vec{\tau} = \vec{m} \times \vec{B} \qquad \left|\vec{\tau}\right| = \left|\vec{m}\right| \cdot \left|\vec{B}\right| \sin(\phi)$$

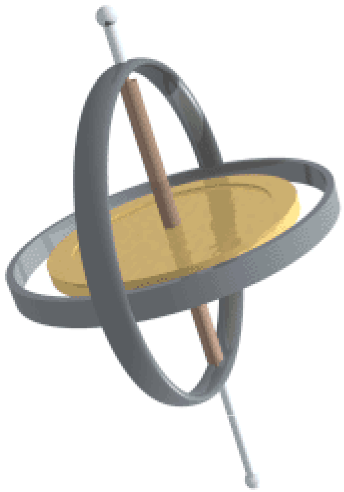


The magnetic moment vector ***precesses*** about the magnetic field lines, but the angle φ remains constant.

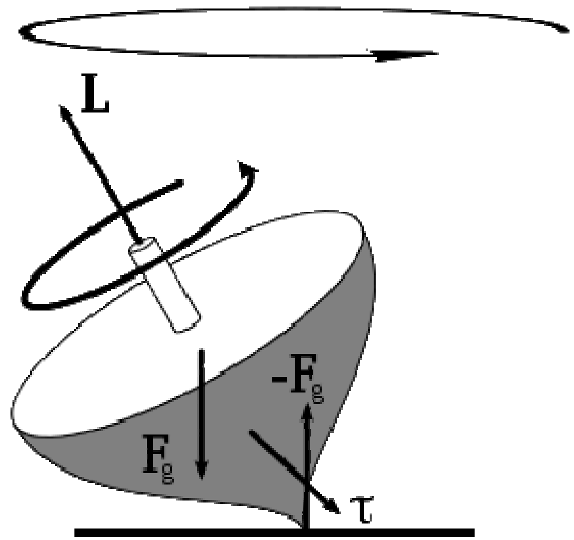
What effect does this torque have on the current loop?

$$\boxed{\phi = \text{constant}}$$

Precession of a Gyroscope in a Gravitational Field:



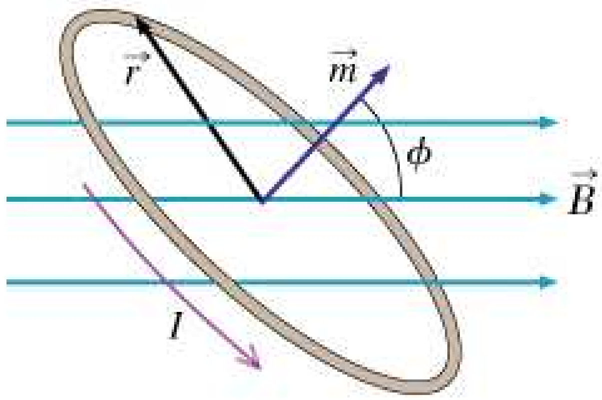
$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

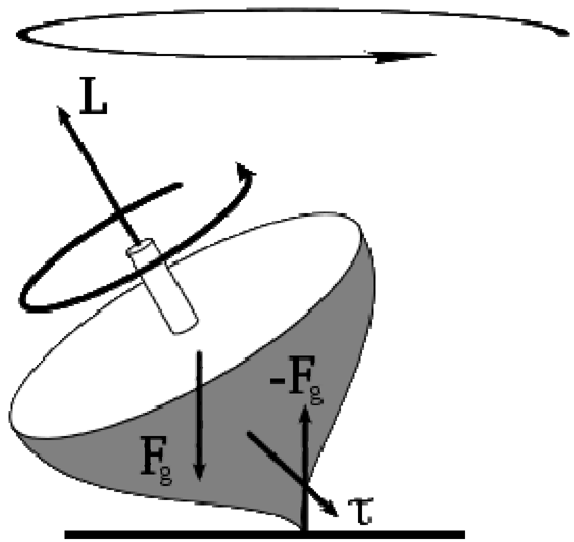


The gyroscope can only spin about the symmetry axis

The torque acts at ***right angles*** to the angular momentum vector, so it changes the ***direction*** of the angular momentum vector, but ***not its magnitude***.

What corresponds to what?



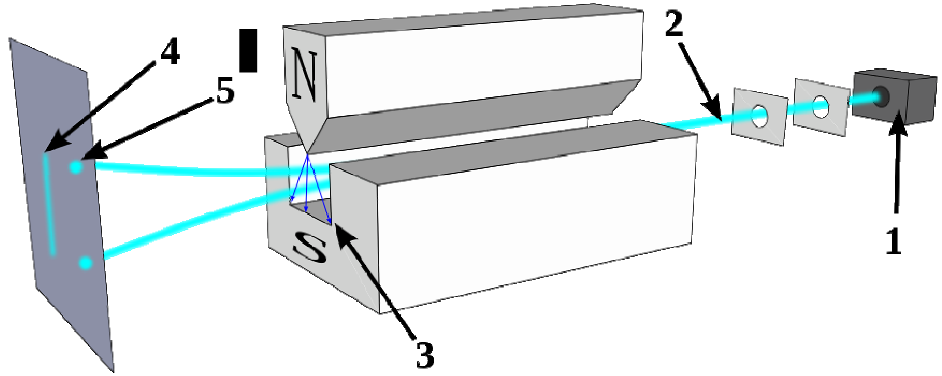


Take-Home Message:

A) The torque does not cause the magnetic moment vector of the current loop to flip directions.

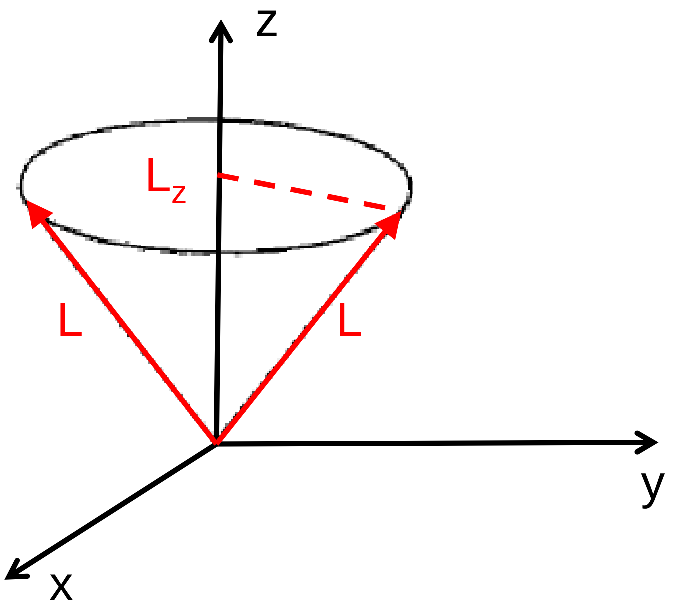
B) Since φ = constant, the projection of the magnetic moment vector onto the direction of the B-field does not change while the current loop is interacting with the B-field.

Stern-Gerlach experiment (1925)



- Atoms must have a quantized (projection/component of) magnetic moment
- Magnetic moment is due to orbital angular momentum and spin

Quantization



Quantization of orbital angular momentum:

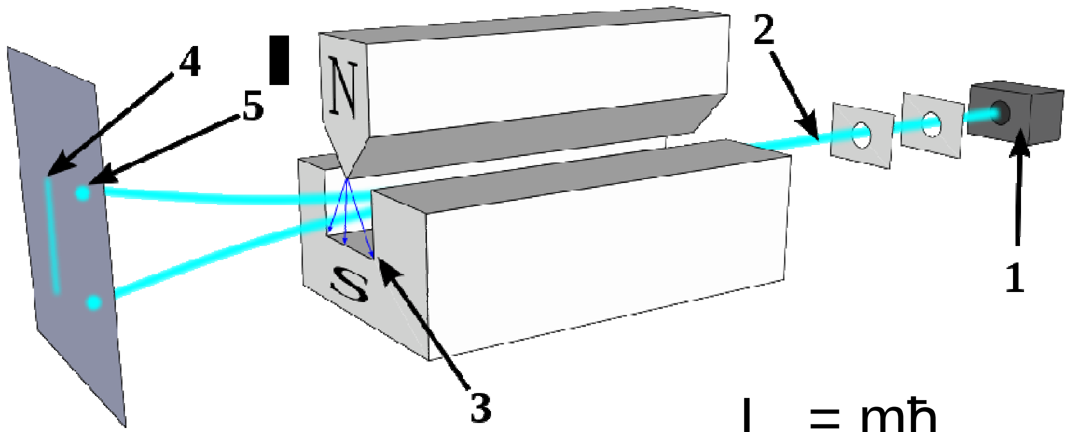
$L^2 = l(l+1)\,\hbar^2$
with $l = 0, 1, 2, 3, \ldots$

$L_z = m\hbar$
with $m = -l, -(l+1), \ldots, l$

How many values of L_z do we have?

(A)0 (B) 2 (C) 2l (D) 2l+1
(E) something else

Stern-Gerlach experiment



$L_z = m\hbar$
with $m = -l, -(l+1), \ldots, l$

Can the observation (5) in the Stern-Gerlach experiment be due to the quantization of the orbital angular momentum? (A) Yes (B) No (C) Cannot decide