4.8. Spin

The Stern-Gerlach experiment can be explained by the fact that an electron itself has an inherent magnetic moment due to an inherent angular momentum, which is called spin

Remarks:

- Spin is a pure quantum phenomenon. There is no classical analogue to it.
- The magnitude of the magnetic moment of the electron is the Bohr magneton:

$$m_B = \frac{e\hbar}{2 m_e} = 9.27 \times 10^{-24} \frac{J}{T}$$

- Projection of spin onto an axis is quantized

$$S_z = m_s \hbar \quad \text{with} \quad m_s = \pm \frac{1}{2}$$

$\rightarrow$ Two values

- Projections (Components) of spin are complementary variables

$\Rightarrow$ They cannot be measured simultaneously.

Spin of atoms

In general, projection of magnetic moment vector onto a given axis can have more than two possible values. This occurs since in general the magnetic moment of atoms is due to both orbital angular momentum and spin (angular momentum).

In order to further study the measurement of complementary variables (here: projections of spin), we will consider <u>two-state</u> systems only, i.e. as in the original Stern-Gerlach experiment.

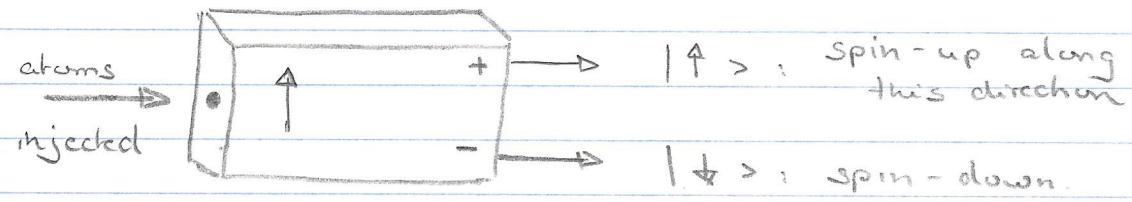
<u>Notation</u>: An atom is either measured as

spin-up : $|\uparrow\rangle$ or $|+\tfrac{1}{2}\rangle$ or

spin-down : $|\downarrow\rangle$ or $|-\tfrac{1}{2}\rangle$

along a given axis.

For the measurements we will consider a simplified Stern-Gerlach analyzer



For the injection of atoms we can consider two options

- Ensemble of atoms with randomly oriented spin (magnetic moment)

  ⇒ Atoms will exit with equal probability of $\frac{1}{2}$ the plus and minus channels

- Ensemble of atoms in a "prepared spin (magnetic moment" state

Examples:

(i) Atoms are prepared in spin-up state along the z-direction and are analyzed with a spin-up (along z) Stern-Gerlach analyzer

Result: Probability to exit + channel: 100%

Probability to exit - channel: 0%

since atoms are prepared in a state along the axis that is analyzed.

(ii) Atoms are prepared in spin-up (spin-down) state along x-direction (or y-direction) and are analyzed with a spin-up or spin-down (along z) analyzer

Result: Probability to exit + channel: 50%

Probability to exit - channel: 50%

since atoms are prepared in a state complementary to the one analyzed.

(iii) Atoms are prepared in a spin-up state along a direction at an angle of Θ to the axis analyzed.

Result: Probability to exit + channel : $\cos^2\left(\frac{\Theta}{2}\right)$

Probability to exit - channel : $\sin^2\left(\frac{\Theta}{2}\right)$

Note : $\cos^2\left(\frac{\Theta}{2}\right) + \sin^2\left(\frac{\Theta}{2}\right) = 1$

Remarks about repeated spin measurements :

- Total probability for atom to pass successive Stern-Gerlach analyzers along a certain path is given by the product of the individual probabilities

- Expectation value (Average value) of measurement is

$$\langle x \rangle = \sum_{i=1}^{2} P(x_i)\, x_i$$

$P(x_i)$: probability for individual result.

x_i : individual possible results