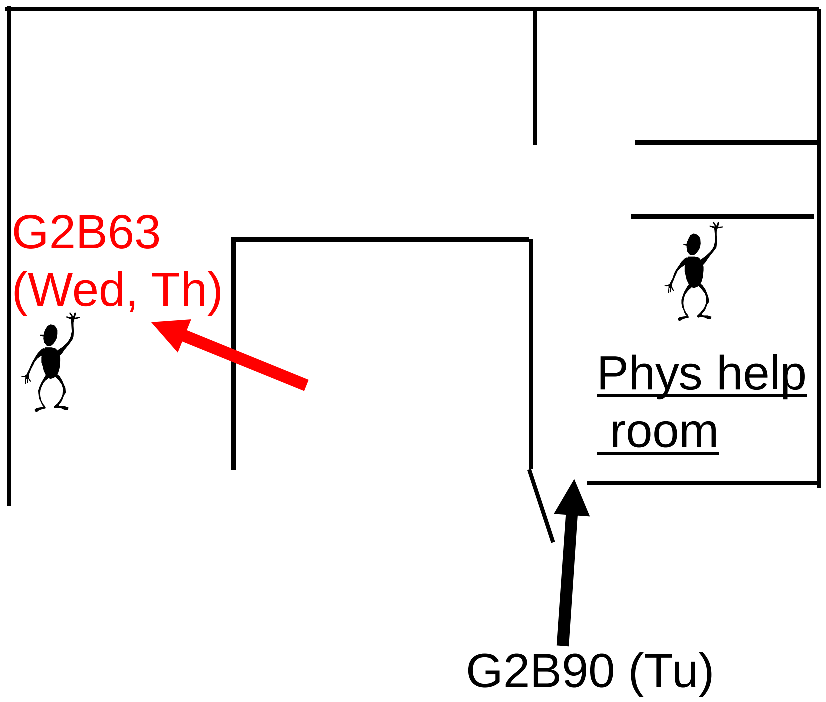
# Problem solving sessions

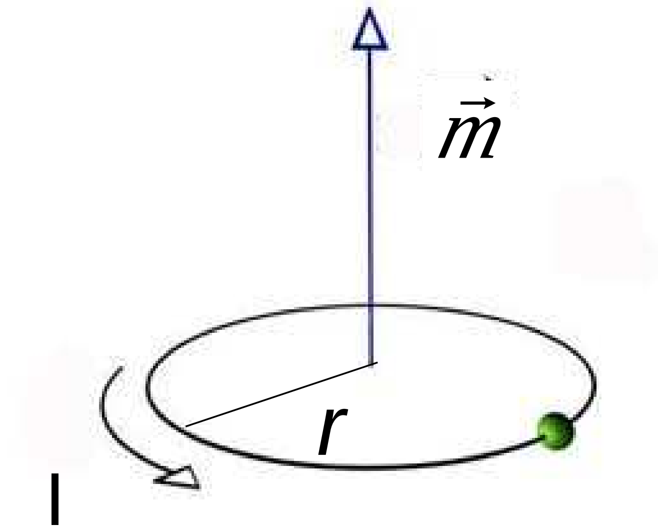



Tues: in G2B90

Wed and Thurs: in G2B63

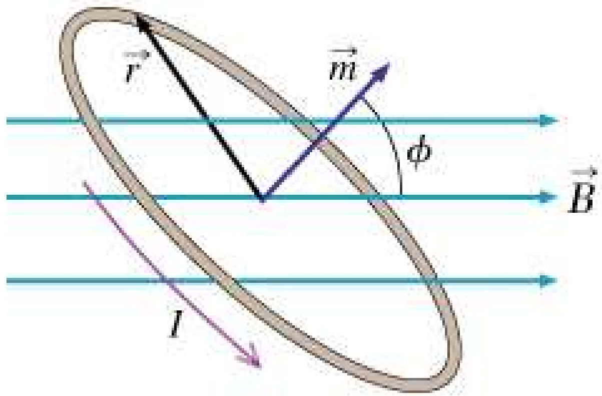
# Magnetic Moment of a Current Loop

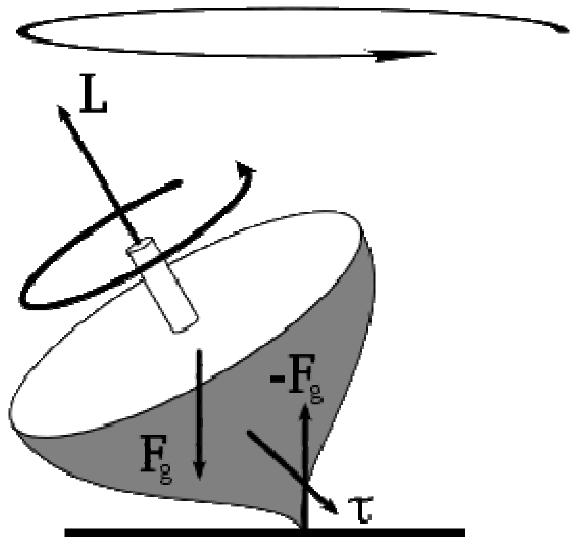
$$\vec{m} = I \cdot \vec{a}$$




$$\vec{a} = \pi r^2 \cdot \hat{a}$$

$\hat{a}$ points in the direction perpendicular to the plane of the current loop, in the direction given by the ***right-hand rule***.

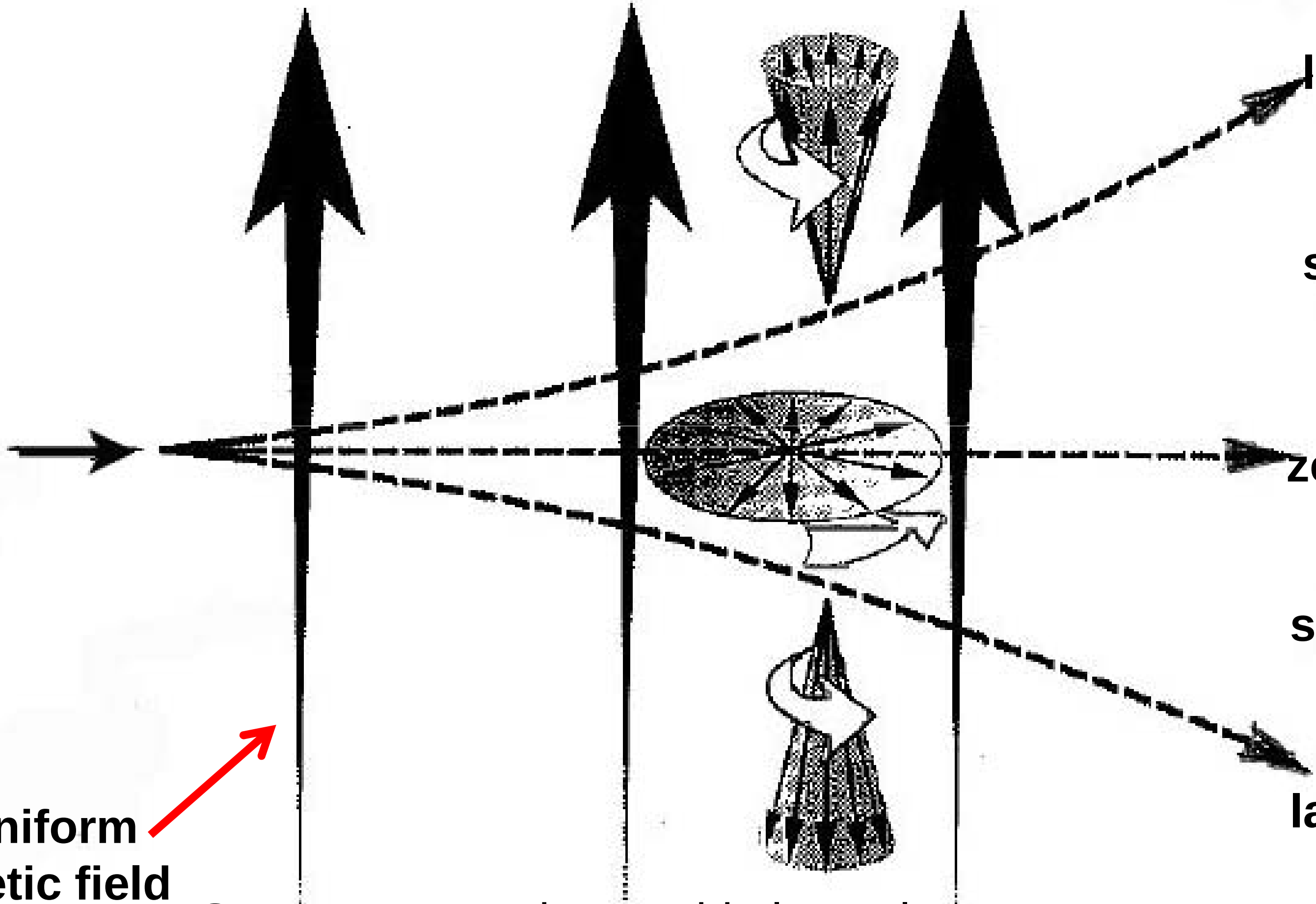



The torque does not cause the magnetic moment vector of the current loop to flip directions.

Since $\Phi$ = constant, the projection of the magnetic moment vector onto the direction of the B-field does not change while the current loop is interacting with the B-field.

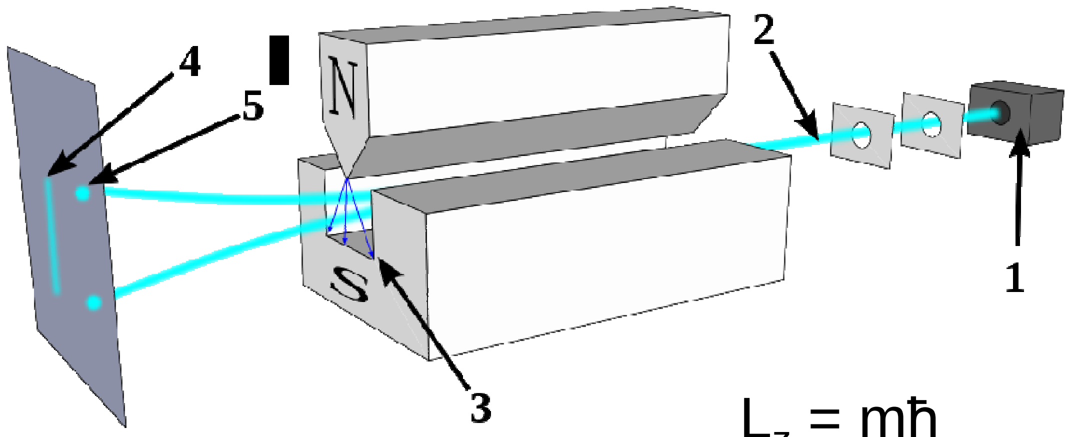
# Magnetic Moment in a Non-Uniform Magnetic Field




Suppose atoms have orbital angular momentum with $L^2 = l(l+1)\hbar^2$ with $l = 2$ and $L_z$ is not specified. In how many directions can the atoms be deflected?

(A) 0 (B) 2 (C) 3 (D) 5 (E) other

# Stern-Gerlach experiment




Silver (or hydrogen) atoms

$L_z = m\hbar$

with $m = -l, -(l+1), \ldots, l$

Can the observation (5) in the Stern-Gerlach experiment be due to the quantization of the orbital angular momentum? (A) Yes (B) No (C) Cannot decide

# Stern-Gerlach experiment

Electron has an inherent magnetic moment due to an inherent angular momentum
(property of electron, such as mass and charge).

Inherent angular momentum is called **spin**.

Spin is pure quantum phenomenon, no classical analogue

Magnetic moment of the electron is the Bohr magneton $m_B = \frac{e\hbar}{2m_e}$

Projection of spin: $S_z = m_s \hbar$ with $m_s = \pm \frac{1}{2}$

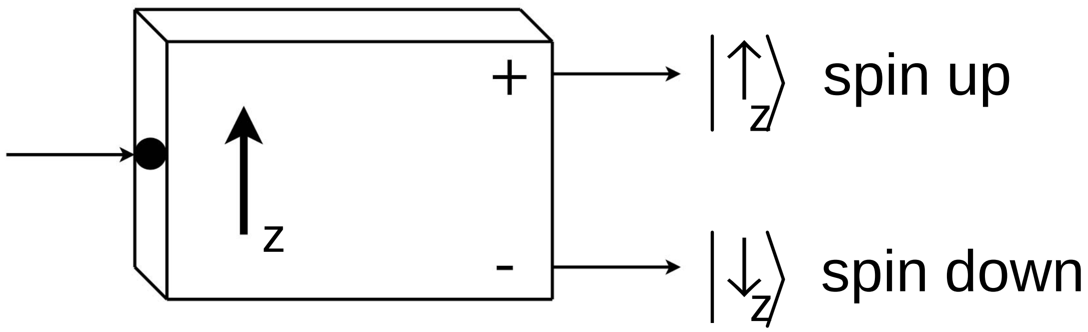
Projections (components) of spin are **complementary variables**

# Spin of atoms

Some atoms have more than two possible projections of the magnetic moment vector along a given axis, but we will only deal **with “two-state” systems** when talking about atomic spin.

When we say a particle is measured as “***spin up***” (or ***down***) along a given axis, we are saying that we measured the projection of the magnetic moment along that axis (+1/2 or -1/2), and observed it exiting the ***plus-channel*** (***minus-channel***) of a Stern-Gerlach type apparatus.

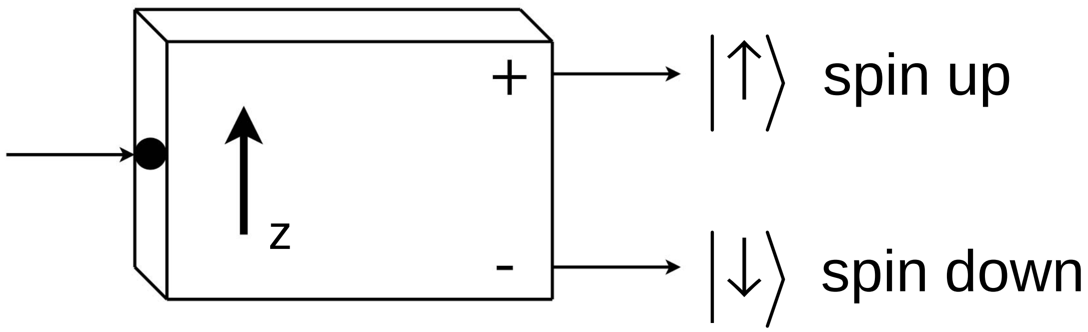
# A simplified Stern-Gerlach analyzer




Atoms are injected on left hand side as either

- Ensemble of atoms with randomly oriented spin (magnetic moment) or
- Ensemble of atom in a “prepared spin (magnetic moment) state”

# A simplified Stern-Gerlach analyzer




Suppose we inject atoms prepared in $|\uparrow_z>$. What are the probabilities that the atoms will leave through the two exit channels?

(A) ↑: 50% ↓: 50%

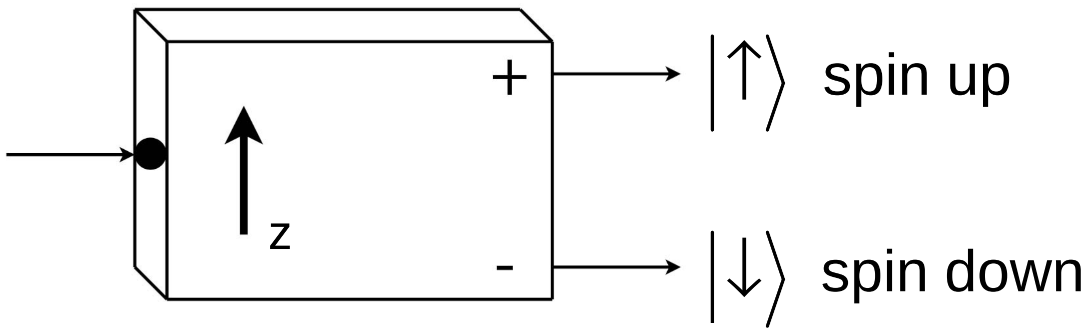
(B) ↑: 100% ↓: 0%

(C) ↑: 0% ↓: 100%

(D) Other

(E) Not enough information to decide

# A simplified Stern-Gerlach analyzer




Suppose we inject atoms prepared in $|\uparrow_x>$. What are the probabilities that the atoms will leave through the two exit channels?

(A) ↑: 50% ↓: 50%

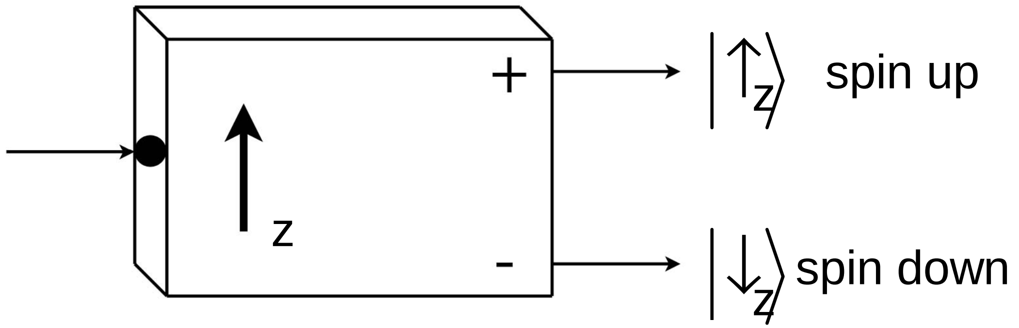
(B) ↑: 100% ↓: 0%

(C) ↑: 0% ↓: 100%

(D) Other

(E) Not enough information to decide

For atoms entering with spin up at angle θ (with respect to + axis)




For Θ = 0, 100% = cos$^2$(0$^0$) of atoms exit from "+ channel"

For Θ = 90$^0$, 50% = cos$^2$(45$^0$) of atoms exit from "+ channel"

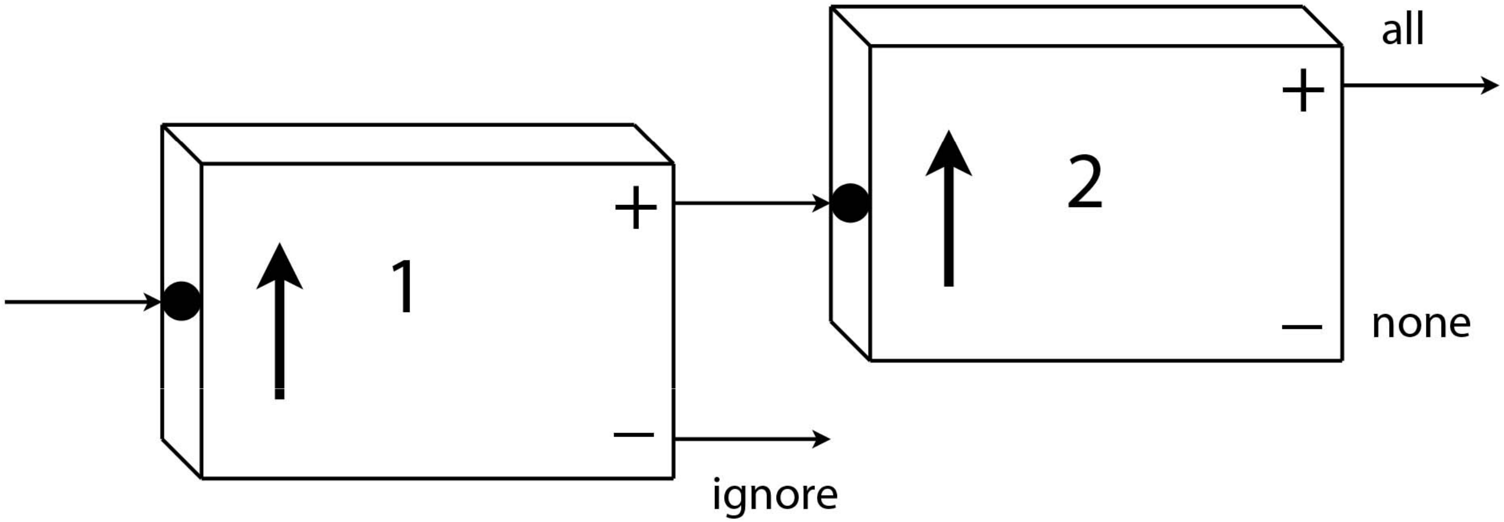
For Θ = 180$^0$, 0% = cos$^2$(90$^0$) of atoms exit from "+ channel"

For arbitrary Θ: atoms exit + channel with probability: $P\left[|\uparrow\rangle\right] = \cos^2\left(\frac{\theta}{2}\right)$

What is the probability that atoms (for arbitrary Θ) exit "– channel"?

$$P\left[|\downarrow\rangle\right] = \sin^2\left(\frac{\theta}{2}\right)$$

# Repeated spin measurements:

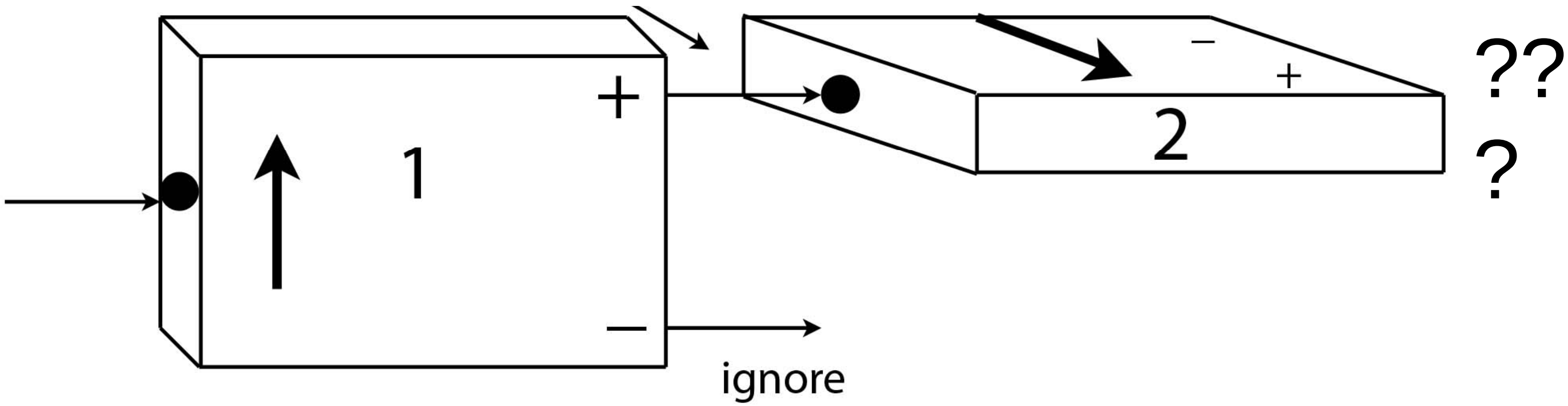



Ignore the atoms exiting from the ***minus-channel*** of **Analyzer 1**, and feed the atoms exiting from the ***plus-channel*** into **Analyzer 2**.

All of the atoms also leave from the ***plus-channel*** of **Analyzer 2**.

# Analyzer 2 is now oriented horizontally (+x):

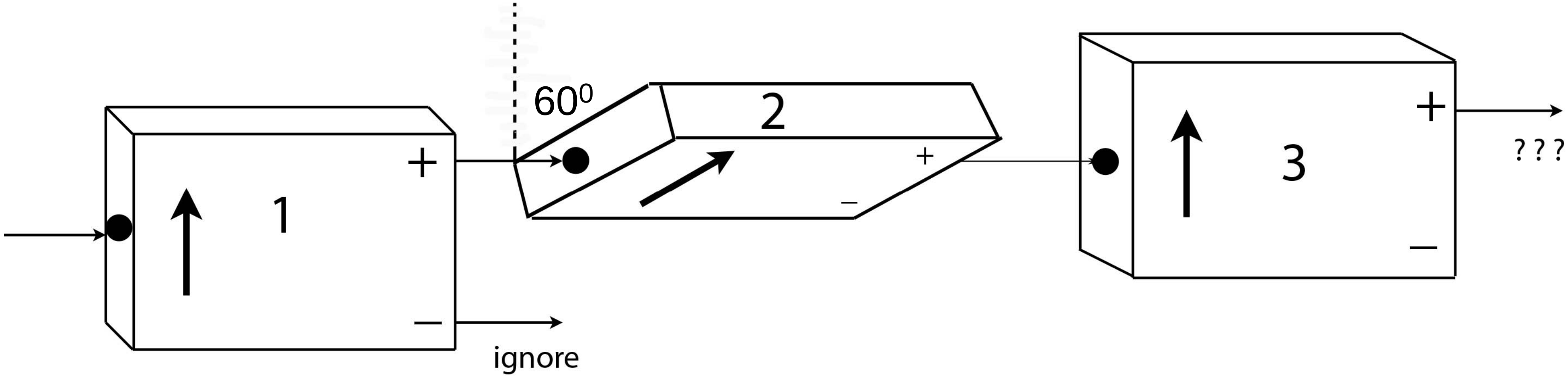
$$\left| m_Z = +m_B \right\rangle$$




***Ignore*** the atoms exiting from the ***minus-channel*** of **Analyzer 1**, and feed the atoms exiting from the ***plus-channel*** into **Analyzer 2**.

What happens when these atoms enter **Analyzer 2**?

A) They all exit from the plus-channel.
B) They all exit from the minus-channel.
C) Half leave from the plus-channel, half from the minus-channel.
D) Nothing, the atoms' magnetic moments have zero projection along the +x-direction




Instead of horizontal, suppose **Analyzer 2** makes an angle of $60^0$ from the vertical. **Analyzers 1** & **3** both are in +z direction.

What is the ***probability*** for an atom leaving the ***plus-channel*** of **Analyzer 2** to exit from the ***plus-channel*** of **Analyzer 3**?

A) 0% B) 25% C) 50% D) 75% E) 100%

Hint: Remember that $P\left[\left|\uparrow_\theta\right\rangle\right] = \cos^2\left(\frac{\theta}{2}\right)$