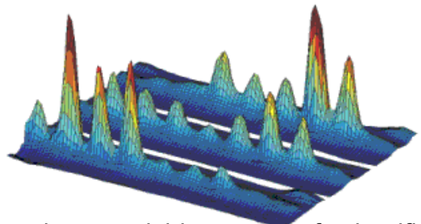
What happens when wires are so small that QM does determine their behavior? & can we take advantage of thi$?



We virtually ignore the astonishing range of scientific and practical applications that quantum mechanics undergirds: today an estimated 30 percent of the U.S. gross national product is based on inventions made possible by quantum mechanics, from semiconductors in computer chips to lasers in compact-disc players, magnetic resonance imaging in hospitals, and much more.

Max Tegmark and John Archibald Wheeler Sci.American, Feb.2001

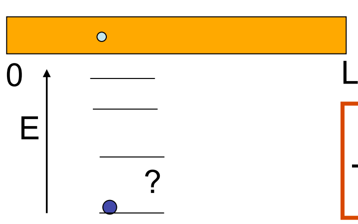
Phys 2130, Day 30:
Questions?
Review of Quantum Wells
& tunneling

Reminders:
Next up: Tunneling
HW Due Thurs

Nanotechnology: how small does a wire have to be before movement of electrons starts to depend on size and shape due to quantum effects?

Look at energy level spacing compared to thermal energy, kT= 1/40 eV at room temp.

Calculate energy levels for electron in wire of length L. Know spacing big for 1 atom, what is L when E is ~1/40 eV?



Use time independ. Schrod. eq.

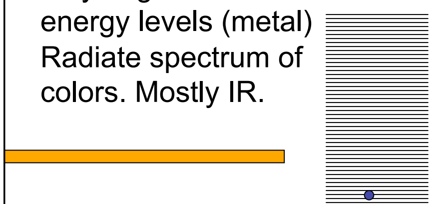
$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+V(x)\psi(x)=E\psi(x)$$

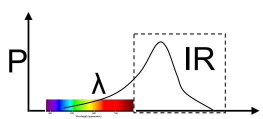
Figure out V(x), then figure out how to solve, what solutions mean physically.

Wire ([light bulb filament](#))

[Single atom](#) [(discharge lamps](#))

Hot electrons.
very large # close energy levels (metal)
Radiate spectrum of colors. Mostly IR.

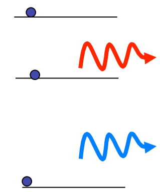


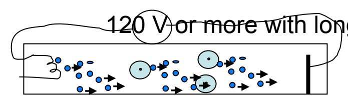


Can think of classically)

Electron jumps to lower levels.

Only specific wavelengths.





Need Quantum

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x,t)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Most physical situations, like H atom, no time dependence in V!

simplification #1 when V(x) only. *(works in 1D or 3D)*
(important, will use in all Shrod. Eq'n problems!!)

Ψ(x,t) separates into position part dependent part ψ(x) and time dependent part Φ(t) =exp(-iEt/ħ). Ψ(x,t)= ψ(x)Φ(t)

plug in, get equation for ψ(x)
You did this on your HW.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+V(x)\psi(x)=E\psi(x)$$

what is in book With V(x) for U(x)

"time independent Schrodinger equation"

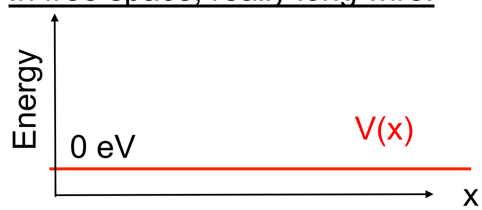
1. Figure out what V(x,t) is, for situation given.

V(x,t) = potential energy of the electron

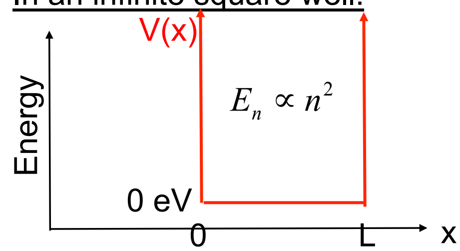
→ What is it as a function of position?

→ Is it changing with time? (Too complicated)

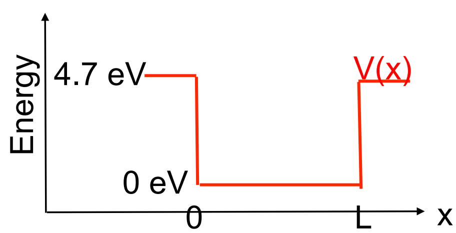
In free space, really long wire:



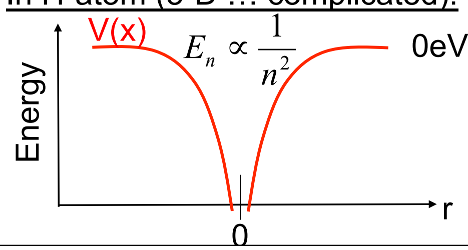
In an infinite square well:



In a wire:



In H-atom (3-D … complicated):

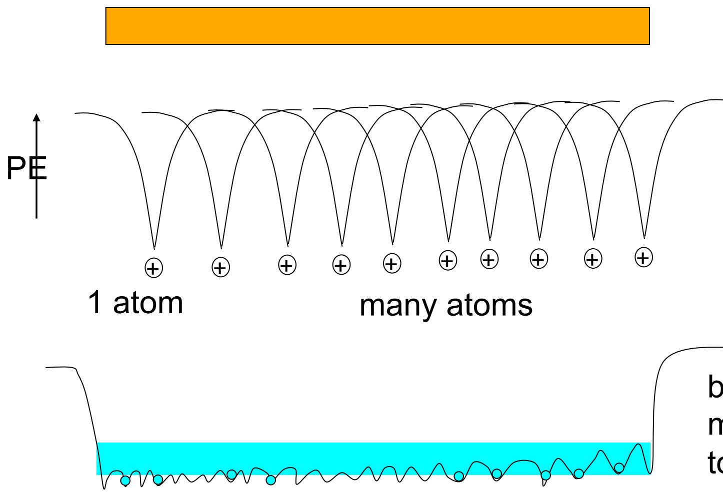


Where does the electron want to be?

⇒ potential energy vs position, V(x) & boundary conditions.

Electron wants to be at position where

a. V(x) is largest
b. V(x) is lowest
c. Kin. Energy > V(x)
d. Kin. E. < V(x)
e. where elec. wants to be does not depend on V(x)

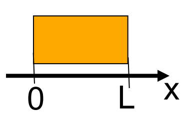


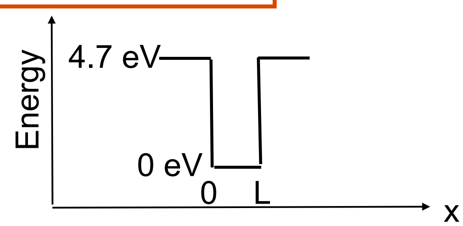
but lot of e's move around to lowest PE

repel other electrons = potential energy near that spot higher.

as more electrons fill in, potential energy for later ones gets flatter and flatter. For top ones, is VERY flat.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+V(x)\psi(x)=E\psi(x)$$





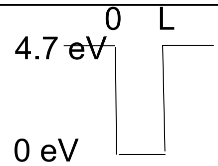
x<0, V(x) = 4.7 eV
x> L, V(x) = 4.7 eV
0<x<L, V(x) =0

How to solve?

1. mindless mathematician approach:

find Ψ in each region, make solutions match at boundaries, normalize.

Works, but bunch of math.

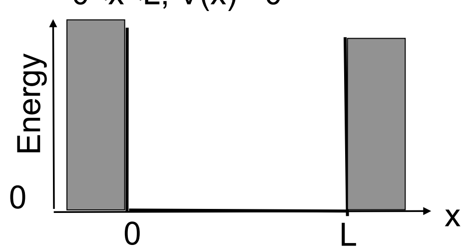


2. Clever physicist approach.

Reasoning to simplify how to solve.
Electron energy not much more than ~kT=0.025 eV.
Where is electron likely to be?

What is chance it will be outside of well?

x<0, V(x) ~ infinite
x> L, V(x) ~ infinite
0<x<L, V(x) =0



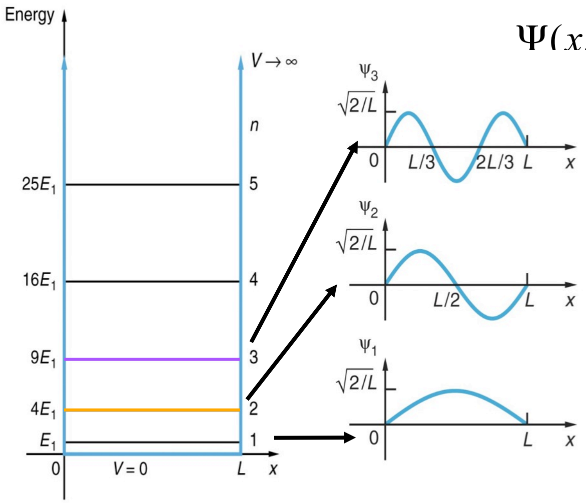
so clever physicist just has to solve

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\psi(x)$$

with boundary conditions,
$\psi(0)=\psi(L) =0$

solution a lot like microwave & guitar string

NOTE:
Book uses “rigid box” for “infinite square well”



$$\Psi(x,t) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)e^{-iEt/\hbar}$$

Quantized: $k=n\pi/L$

Quantized:

$$E = n^2\frac{\pi^2\hbar^2}{2mL^2} = n^2E_1$$

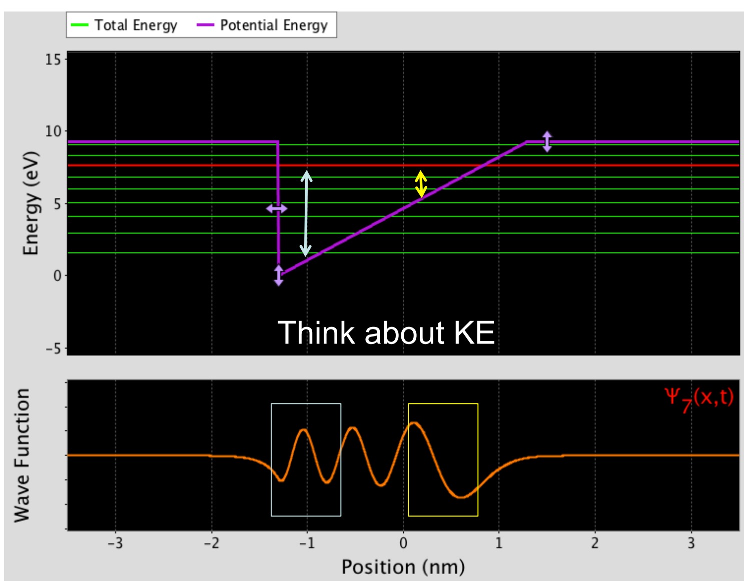
How does probability of finding electron close to L/2 if in n =3 excited state compared to probability for when n=2 excited state?

a. much more likely for n=3.

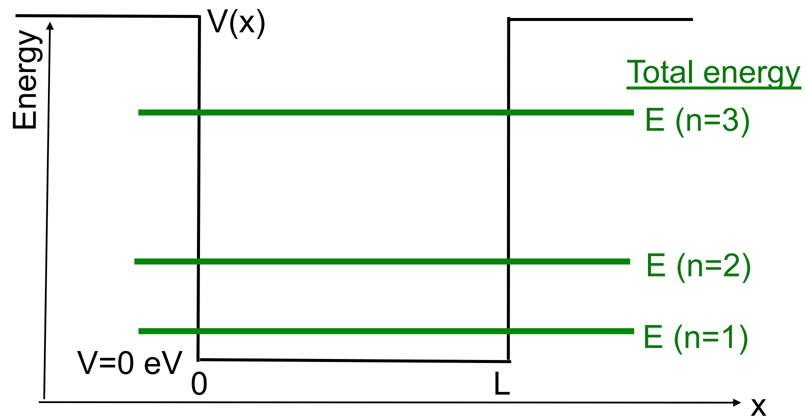
b. equal prob. for both n = 2 and 3.

c. much more likely for n=2

A quick word about asymmetric wells

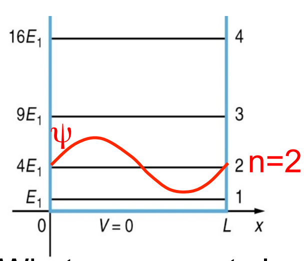


Careful about plotting representations…. Sometimes we're jerks



Careful… plotting 3 things on same graph:

Potential Energy V(x)
Total Energy E
Wave Function ψ(x)



$$\Psi(x,t) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})e^{-iEt/\hbar}$$

Quantized: k=nπ/L

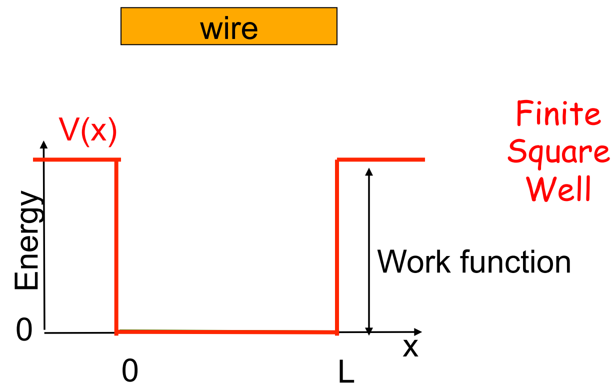
Quantized: $E = n^2 \frac{\pi^2\hbar^2}{2mL^2} = n^2 E_1$

What you expect classically:	What you get quantum mechanically:
Electron can have any energy	Electron can only have specific energies. (quantized)
Electron is localized	Electron is delocalized
Electron equally likely anywhere in wire	Electron likely measured different placed (depends on E!)

Need to solve for exact Potential Energy curve:

V(x): small chance electrons get out of wire

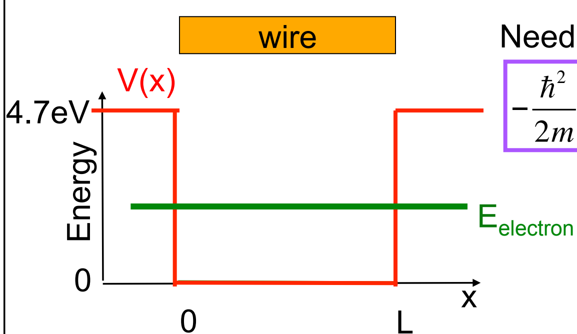
$\psi(x<0 \text{ or } x>L) \sim 0$, but not exactly 0!



Finite Square Well

Important for thinking about "Quantum tunneling":

- Radioactive decay
- Scanning tunneling microscope to study surfaces



Need to solve Schrodinger Eqn:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\,\psi(x) = E\psi(x)$$

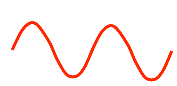
Region I | Region II | Region III

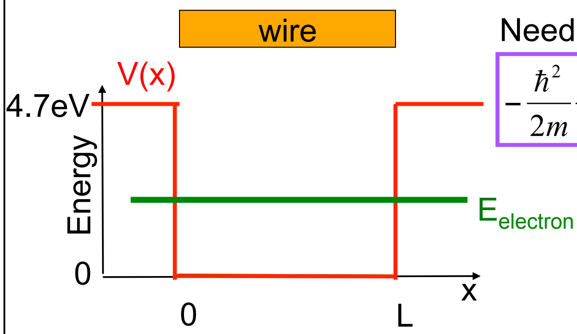
In Region II … total energy E > potential energy V

$$\frac{d^2\psi(x)}{dx^2} = \underbrace{\frac{2m}{\hbar^2}(V-E)}_{\text{Negative number}}\psi(x) = -k^2\psi(x)$$

k is real

When E>V: Solutions = sin(kx), cos(kx), e^{ikx}.
Always expect sinusoidal functions





Need to solve Schrodinger Eqn:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$$

Region I | Region II | Region III

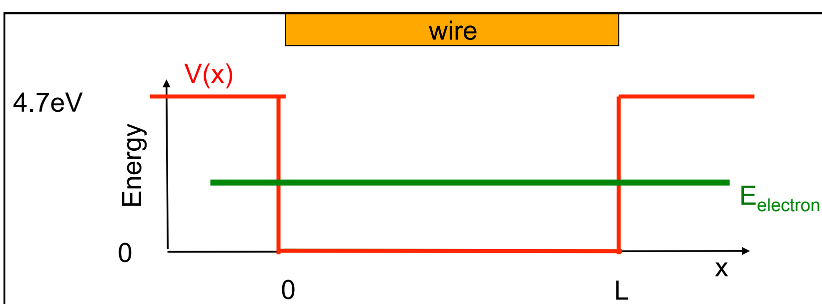
In Region III … total energy E < potential energy V

$$\frac{d^2\psi(x)}{dx^2}=\underbrace{\frac{2m}{\hbar^2}(V-E)}_{\text{Positive}}\psi(x) \quad =\alpha^2\psi(x)$$

α is real

What functional forms of $\psi(x)$ work?

a. $e^{i\alpha x}$ b. $\sin(\alpha x)$ c. $e^{\alpha x}$ d. more than one of these



Region I

$\psi_I(x)=Fe^{\alpha x}+Ge^{-\alpha x}$

Region II

$\psi_{II}(x)=C\sin(kx)+D\cos(kx)$

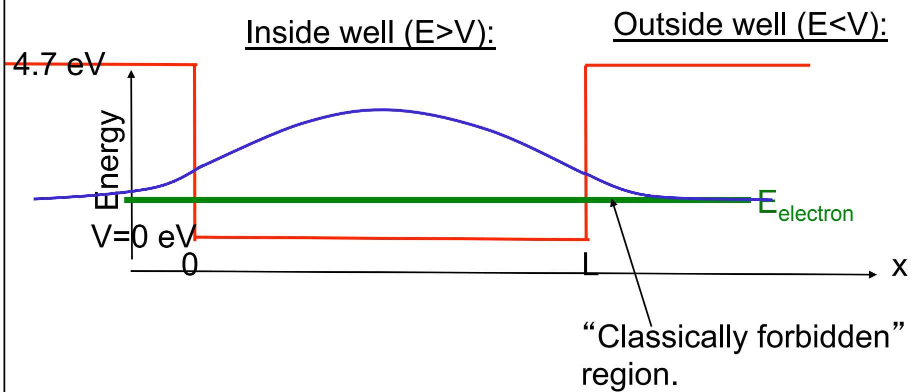
Region III

$\psi_{III}(x)=Ae^{\alpha x}+Be^{-\alpha x}$

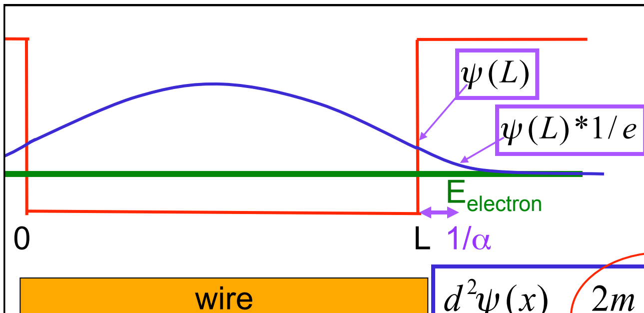
What will wave function in Region III look like?
What makes sense for constants A and B?

a. A must be 0 b. B must be 0 c. A and B must be equal
d. A=0 and B=0 e. A and B can be anything, need more info.

$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi(x)$$



Electron is delocalized … spread out.
Some small part of wave is where Total Energy is less than potential energy!



$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi(x) = \alpha^2\psi(x)$$

$$\alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}$$

How far does wave extend into this "classically forbidden" region?

$$\psi(x) = Be^{-\alpha x}$$

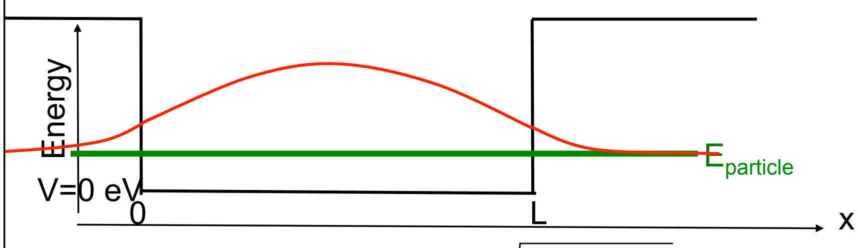
α big -> quick decay
α small -> slow decay

Measure of penetration depth = $1/\alpha = \eta$ (Knight book)
➔ ψ decreases by factor of 1/e

For V-E = 4.7eV, $1/\alpha$ ..9x10^{-11} meters (very small ~ an atom!!!)

$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi(x)$$

Inside well (E>V): Outside well (E<V):



$$\psi(x) = Be^{-\alpha x} \qquad \alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}$$

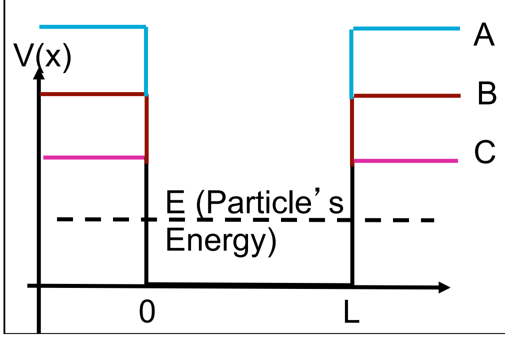
What changes could increase how far wave penetrates into classically forbidden region?
(And so affect chance of tunneling into adjacent wire)

Thinking about α and penetration distance

Under what circumstances would you have a largest penetration?
Order each of the following case from smallest to largest.

$$\psi(x) = Be^{-\alpha x} \qquad \alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}$$

To get largest penetration (tunneling), which Potential curve for a given energy level?



Thinking about α and penetration distance

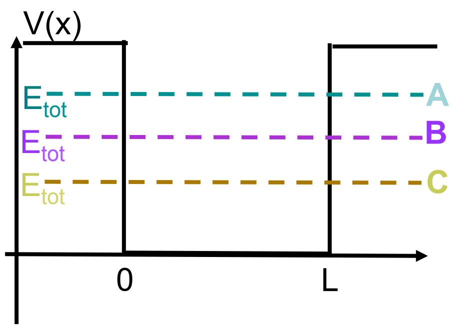
Under what circumstances would you have a largest penetration?

Order each of the following case from smallest to largest.

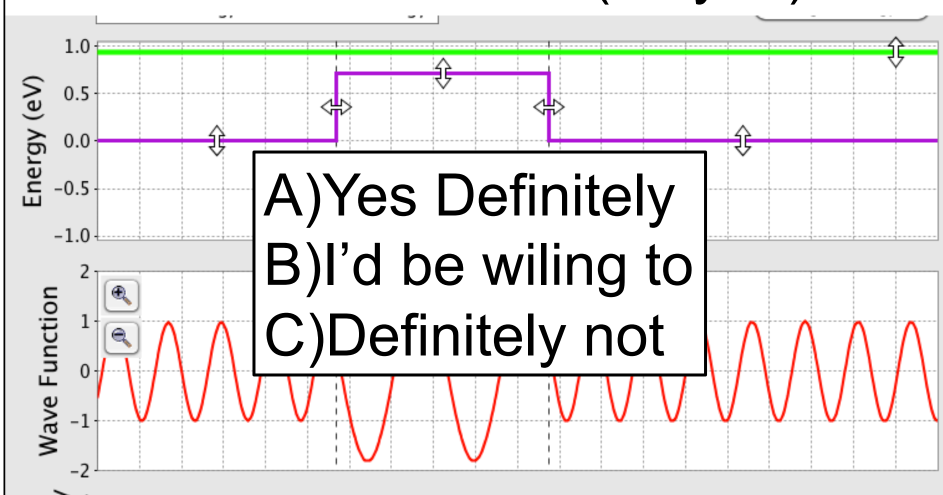
To get largest penetration (tunneling), which total energy level for a fixed potential curve?

$$\psi(x) = Be^{-\alpha x}$$

$$\alpha = \sqrt{\frac{2m}{\hbar^2}(V - E)}$$



Tutorial on Wed (maybe)



A)Yes Definitely

B)I'd be wiling to

C)Definitely not