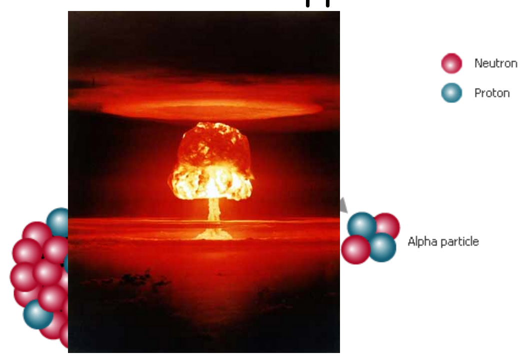
Golly… tunneling seemed pretty abstract… I wonder what those ideas could be applied to? Hmmm…

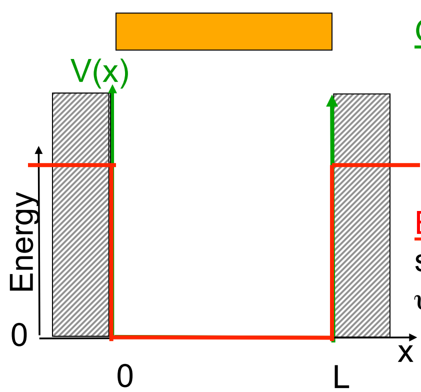


I know not with what weapons World War III will be fought, but World War IV will be fought with sticks and stones.

- A. Einstein

Day 31:
Questions?
Reminders: Schrödinger Wave Eq'n
Potential Wells & Tunneling

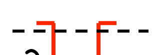
Reminders:
HW
Next up: STMs & Nukes

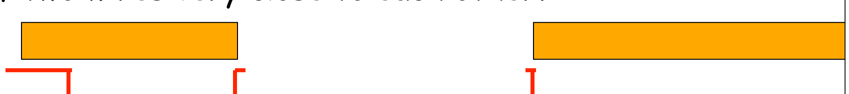


Good Approximation:
Electrons never got out of wire
ψ(x<0 or x>L) =0.
(OK when Energy << work function)

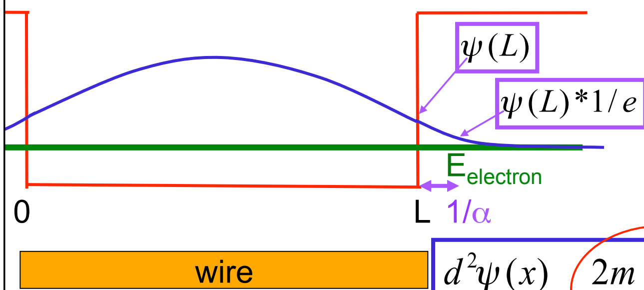
Exact Potential Energy curve (V):
small chance electrons get out of wire
ψ(x<0 or x>L)~0, but not exactly 0!

What happens if electron Energy bigger?
What if two wires very close to each other?





Then whether ψ leaks out a little or not, is very important.
How much coupling to other wire?



$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi(x) = \alpha^2\psi(x)$$

$$\alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}$$

How far does wave extend into this "classically forbidden" region?

$$\psi(x) = Be^{-\alpha x}$$

α big -> quick decay
α small -> slow decay

Measure of penetration depth = 1/α = η (Knight book)
➔ ψ decreases by factor of 1/e

For V-E = 4.7eV, 1/α ..9x10[-11] meters (very small ~ an atom!!!)

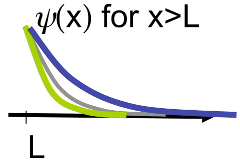
Thinking more about what α means

Putting these together!

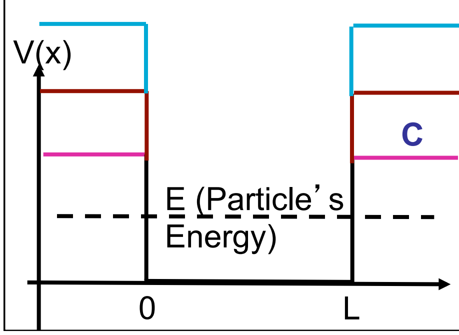
$$\alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}$$

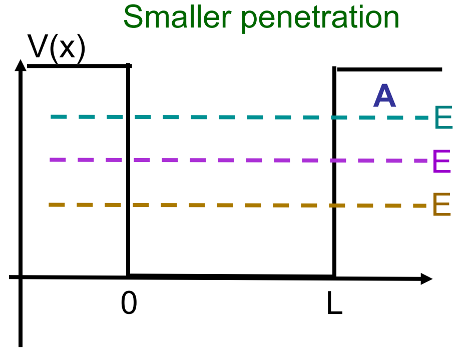
$$\psi(x) = Be^{-\alpha x}$$

Smaller difference between V & E, smaller α
Slower decay
Larger penetration

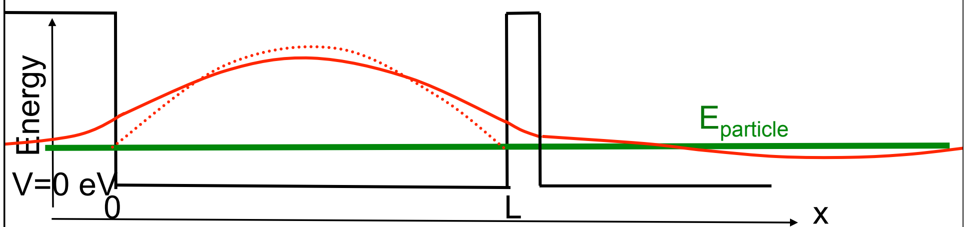


And bigger diff btwn V and E,
Larger α
Faster decay
Smaller penetration





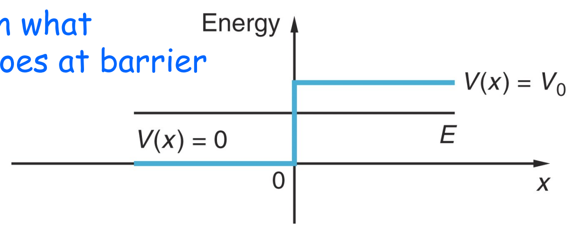
So the thinner, the shorter the barrier, the easier it is to tunnel …

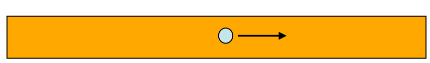


And particle can escape…

Application: Alpha-Decay, Scanning tunneling microscope

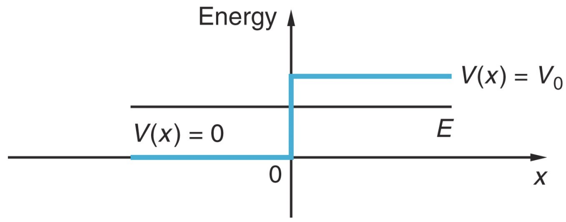
warm up on what electron does at barrier then apply





If the total energy E of the electron is LESS than the work function of the metal, V_0, when the electron reaches the end of the wire, (and no other wire is near by) it will…

A. stop.
B. be reflected back.
C. exit the wire and keep moving to the right.
D. either be reflected or transmitted with some probability.
E. I have *no* idea

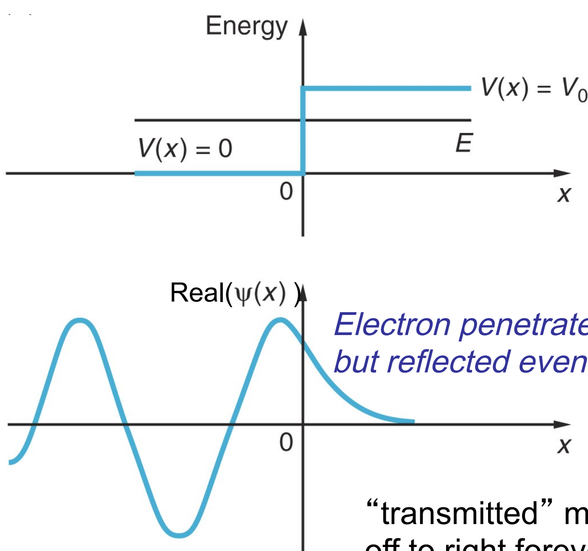


If the total energy E of the electron is LESS than the work function of the metal, V_0, when the electron reaches the end of the wire, it will…

Quantum physics is not so weird that electron can keep going forever in region where V>E. Remember that ψ decays exponentially in this region!

A. stop.
B. be reflected back.
C. exit the wire and keep moving to the right.
D. either be reflected or transmitted with some probability.
E. I have *no* idea

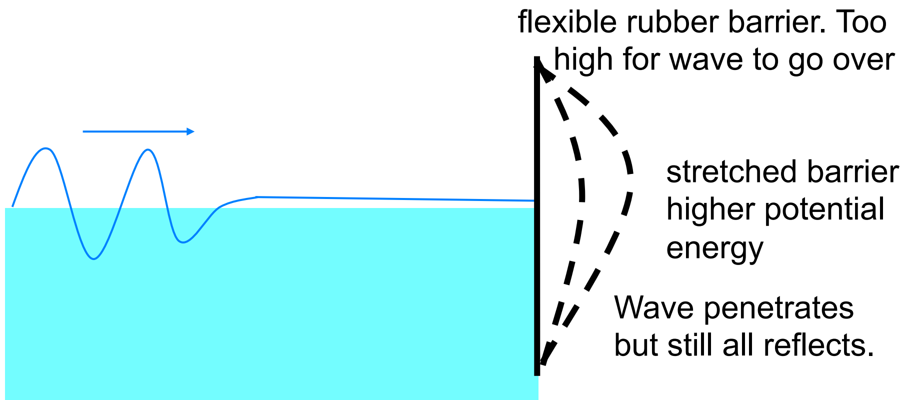
Once you have amplitudes,can draw wave function:



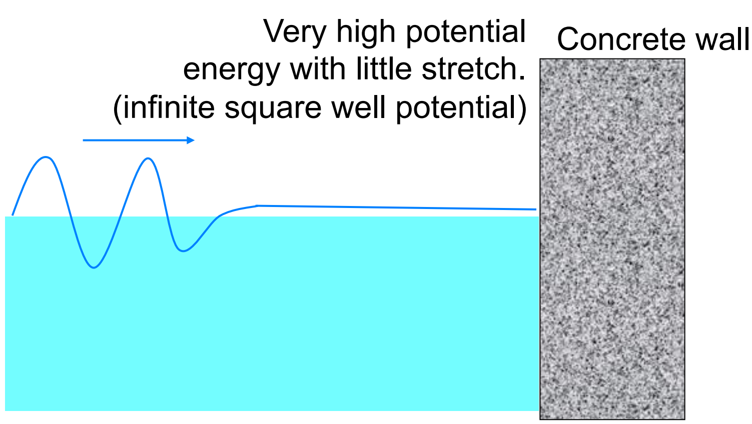
Electron penetrates into barrier, but reflected eventually.

"transmitted" means continues off to right forever. Wave function not go down to zero.

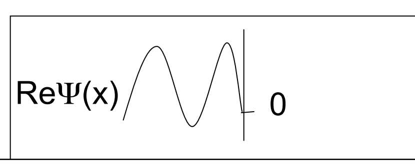
water wave ~ analogy for reflection and tunneling



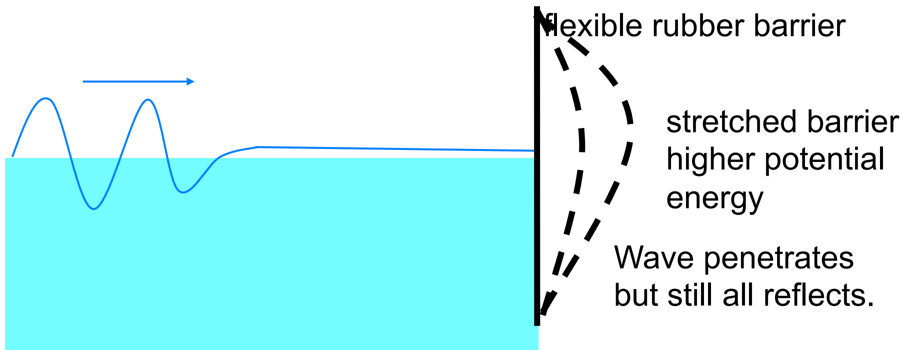
water wave ~ analogy for reflection and tunneling



Wave penetrates, but MUCH less.



water wave ~ analogy for reflection and tunneling

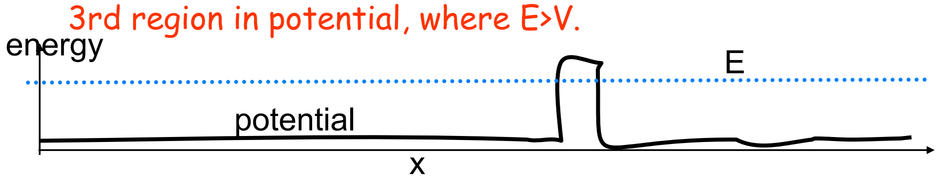


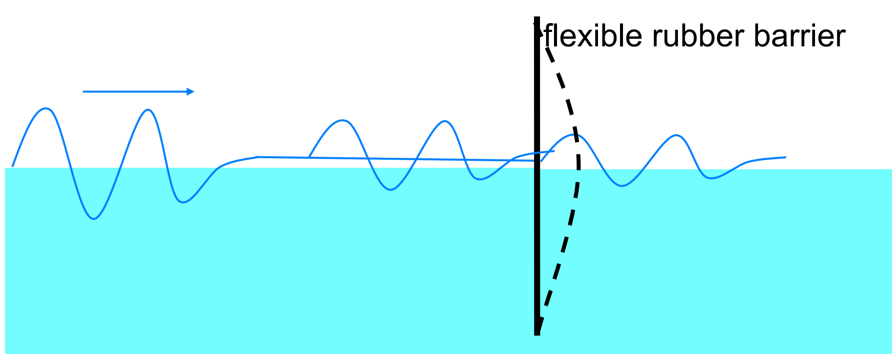
What is required for wave to be transmitted through rubber barrier?

a. make it really stretchy,
b. make it very stiff,
c. have water on other side,
d. none of the above

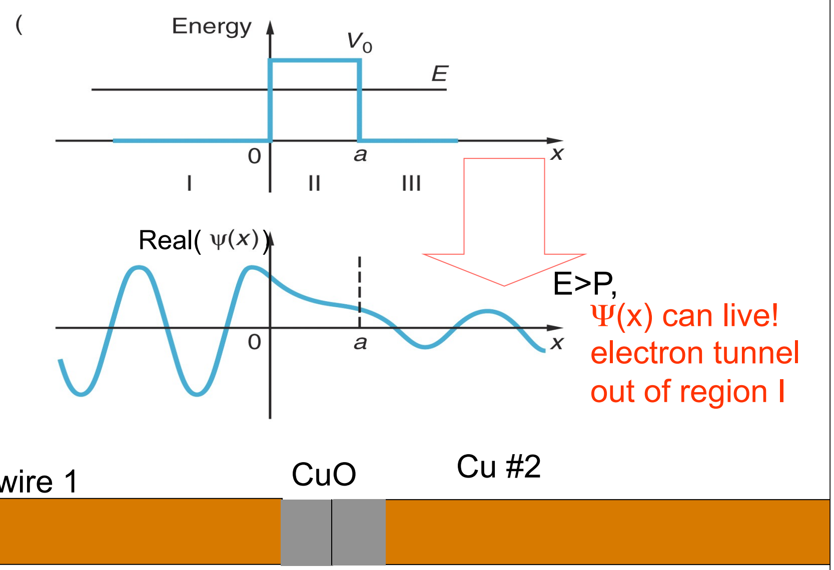
water wave analogy-- for tunneling need water on other side!

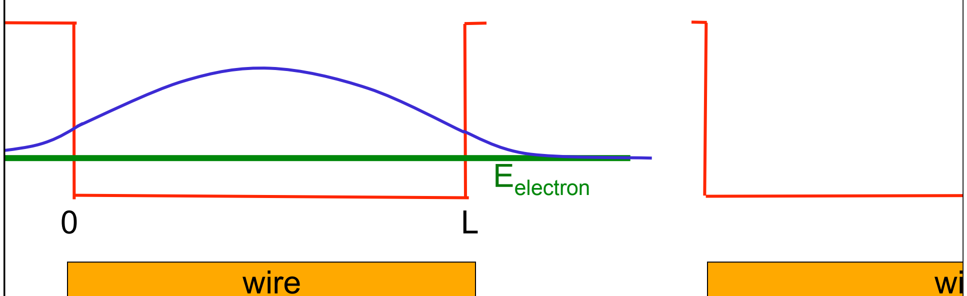
3rd region in potential, where E>V.





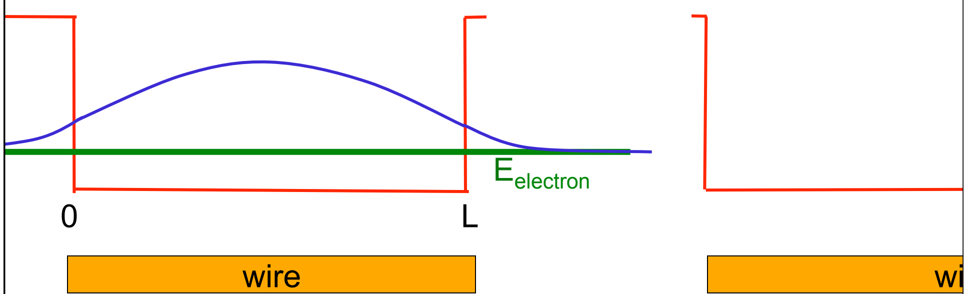
Can have transmission only if third region where solution is not real exponential! (electron tunneling through oxide layer between wires)





If very very long wire gets closer and closer to this very short, what will eventually happen?

a. electron is "shared" between wires, with fraction in each constant over time
b. the electron will flow away through wire 2
c. electron will jump back and forth between wire 1 and wire 2
d. electron stays in wire 1.
e. something else happens.



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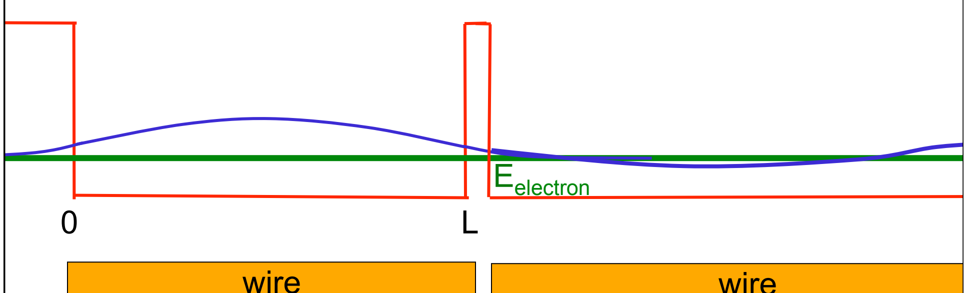

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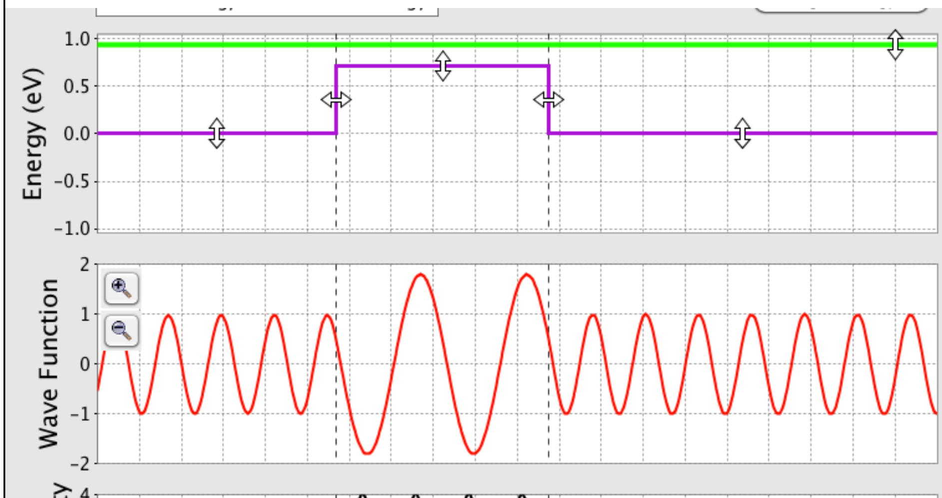
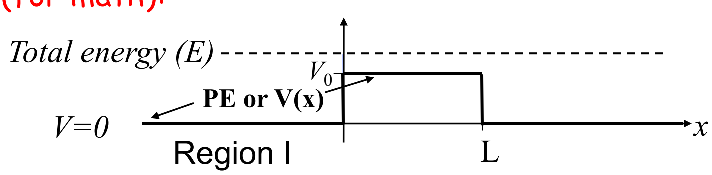
Tutorial (maybe)



Find functional form… guess and make sure it works.
(On tutorial ….

Easy way (for math):



The general solution for total energy (E) > potential energy (V):

$$\Psi(x,t) = (Ae^{ikx} + Be^{-ikx})e^{-iEt/\hbar}$$

Schrodinger Eqn: $$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Plugging into the Schrodinger Equation gives:

$$-\frac{\hbar^2}{2m}(-k^2)\Psi(x,t) + 0 = i\hbar\left(\frac{-iE}{\hbar}\right)\Psi(x,t)$$

This simplifies to: $\hbar^2k^2/2m = E$ OK