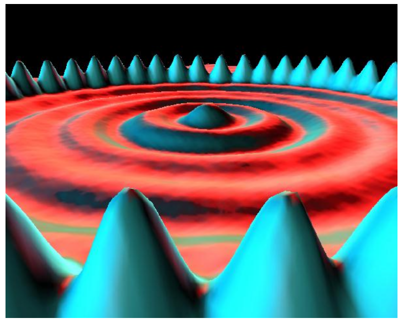
Why is this picture so cool and what does it have to do with John Travolta?



Here they have positioned 48 iron atoms into a circular ring in order to "corral" some surface state electrons and force them into "quantum" states of the circular structure. The ripples in the ring of atoms are the density distribution of a particular set of quantum states of the corral - IBM Research 3 Feb 2000

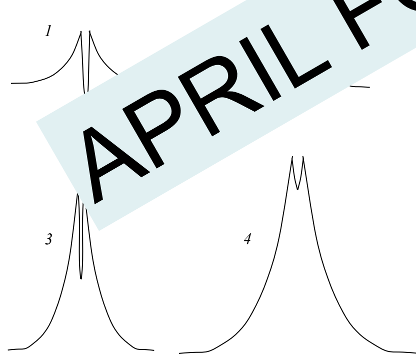
Day 32:
Questions?
STM

Reminders:
HW
Next up:
Alpha Decay and Nooks

POP QUIZ

HAVE YOU BEEN READING THE BOOK?

APRIL FOOLS

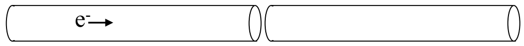


1. To the left are energy profiles for various atoms. Which decays most rapidly and which never decays?

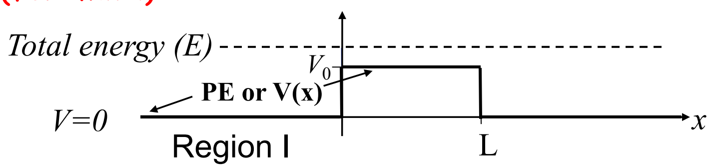
a. 3 most rapid, 4 never
b. 1 most rapid, 2 never
c. 2 most rapid, 1 never
d. 4 most rapid, 3 never
e. 4 most rapid, 1 never

2

2. Find functional form… guess and make sure it works.
(On tutorial ….



Easy way (for math):



The general solution for total energy (E) > potential energy (V):

$$\Psi(x,t) = (Ae^{ikx} + Be^{-ikx})e^{-iEt/\hbar}$$

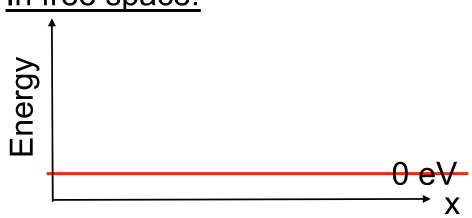
Schrodinger Eqn: $$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Plugging into the Schrodinger Equation gives:

$$-\frac{\hbar^2}{2m}(-k^2)\Psi(x,t) + 0 = i\hbar(\frac{-iE}{\hbar})\Psi(x,t)$$

This simplifies to: $\hbar^2 k^2 / 2m = E$ OK

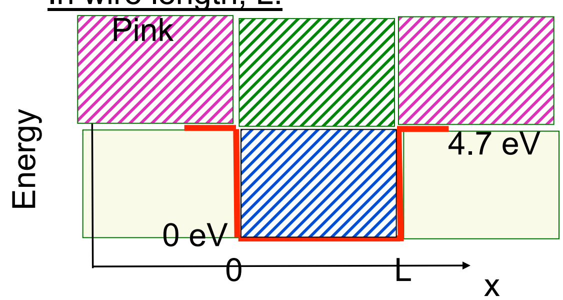
In free space:



For the case of an electron in free space (E> V=0), what energies are allowed?
A. Any energy is allowed
B. Only certain specific energies are allowed.

Answer is A.
Nothing to constrain allowed energy (wavelength) of wave function! Boundaries constrain wavelength …

In wire length, L:

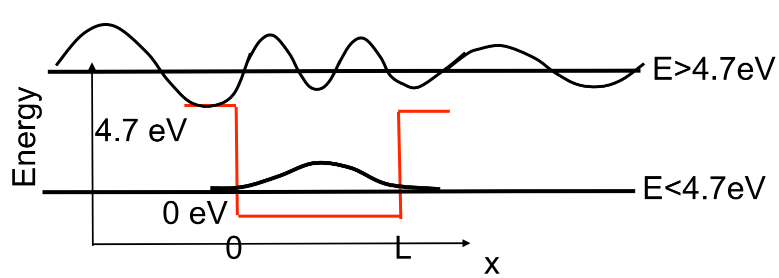


In which regimes is an electron allowed to have only specific fixed energies (quantized energy levels)? (In the other regimes… any energy is allowed)

A. Green only
B. Blue only
C. Pink only
D. Green and Blue
B. Green, Blue, and Pink

Answer is B.
Only in this regime (Energy is less than V_0) is wavelength of wave function constrained between two boundaries.

In wire length, L:

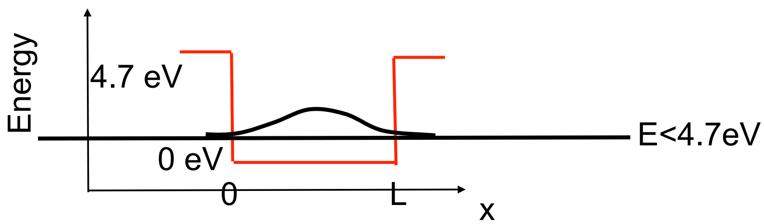


In which regimes is an electron allowed to have only specific fixed energies (quantized energy levels)? (In the other regimes… any energy is allowed)

A. Green only
B. Blue only
C. Pink only
D. Green and Blue
B. Green, Blue, and Pink

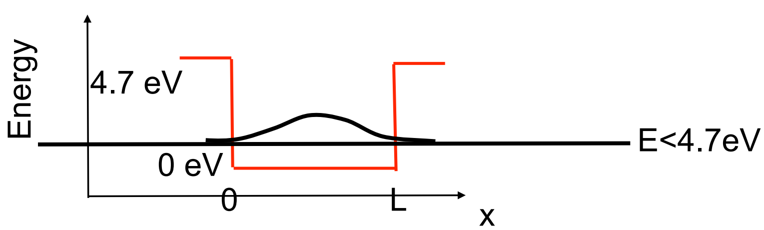
Answer is B.
Only in this regime (Energy is less than V_0) is wavelength of wave function constrained between two boundaries.

In wire length, L:



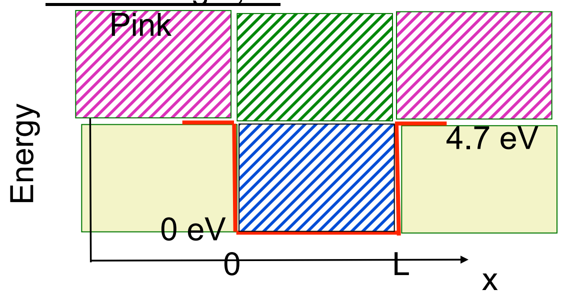
If only specific energy levels are allowed:
A. This outcome is only true in QM
B. This is true in QM and Classical instances (e.g. violin)
C. I have no idea

In wire length, L:



If only specific energy levels are allowed:
A. This outcome is only true in QM
B. This is true in QM and Classical instances (e.g. violin)
C. I have no idea

In wire length, L:



Which areas not possible classically?
A. Green
B. Blue
C. Pink
D. Yellow

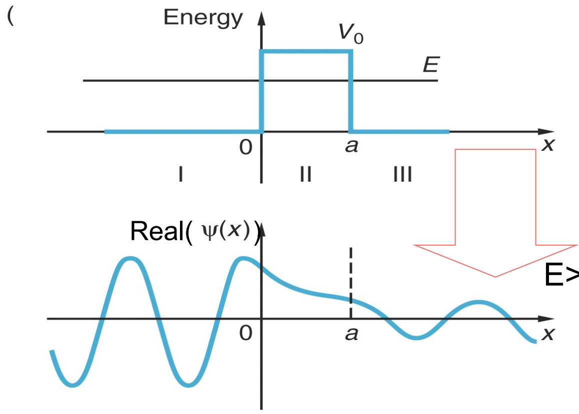
D: the classically forbidden region

Today:
other applications of tunneling in real world

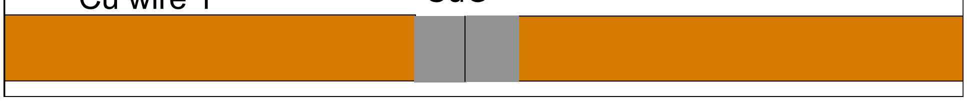
Scanning tunneling microscope (STM):
how QM tunneling lets us map individual atoms on surface

Interesting example may not have time to cover but may be in notes:
- Sparks and corona discharge (also known as field emission) electrons popping out of materials when voltage applied.
- Many places including plasma displays.

Can have transmission only if third region where solution is not real exponential! (electron tunneling through oxide layer between wires)



E>P,
$\Psi(x)$ can live!
electron tunnel
out of region I



Tunneling Probability as in HW

Prob. ~ $\Psi^2 \approx e^{-2\alpha D}$

from $\Psi \approx e^{-\alpha x}$

α = decay constant = $\dfrac{\sqrt{2m(V_0 - E)}}{\hbar}$

D = width of barrier

From homework, what is distance wave function of an electron of 1eV penetrates from copper into an air / vacuum?

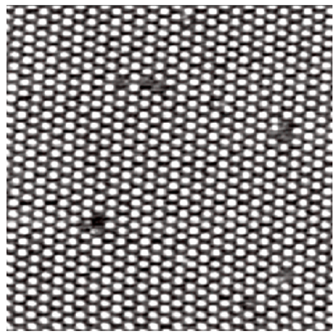
penetration depth = $1/\alpha = \hbar/[2m(V-E)]^{1/2}$

$$= \frac{10^{-34}\ \text{Js}}{[2 \times (9 \times 10^{-31}\ \text{kg}) \times (4.7-1)\ \text{eV} \times 1.6 \times 10^{-19}\ \text{J/eV}]^{1/2}}$$

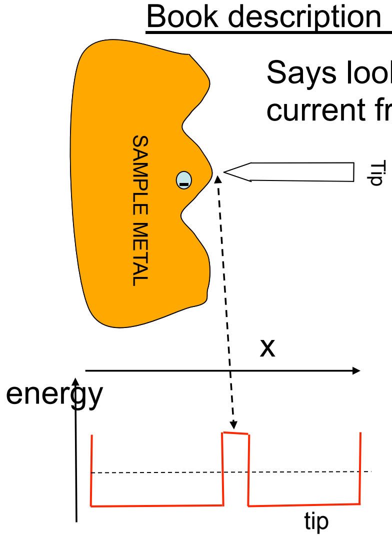
$= 1.01 \times 10^{-10}$ m = 2 Bohr radii ~ 1 atom diameter

implies that if have barrier of few eV, and change distance by one atom diameter will change tunneling current by large factor (1/e = 1/2.7)

Use tunneling to measure small changes in distance.
Nobel prize winning idea- invention of "scanning tunneling microscope (STM)". Measure atoms on surfaces.



Book description of STM wrong in earlier versions.



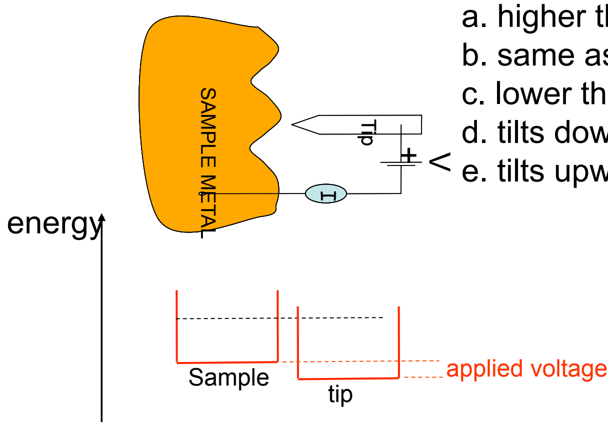
Says looks like this, and one looks at tunneling current from sample to tip to measure gap.

What is wrong with this?

Electron tunnels from sample to tip.
What would V(x) look like then?
a. same as before.
b. V in tip higher, V sample lower.
c. V in tip lower, V sample higher.
d. V same on each side as before but barrier higher.

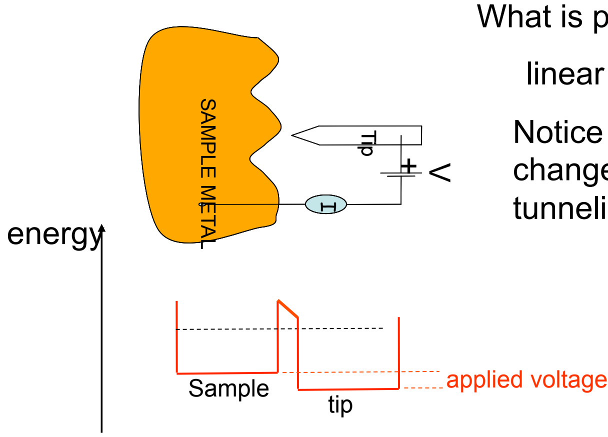
ans. b. electron piled on top (in energy) of many other electrons that contribute to V(x). Add electron, makes higher V(x), remove makes lower. So what does next electron want to do?

Correct picture of STM-- voltage applied between tip and sample. Holds potential difference constant, electron current.
Figure out what potential energy looks like in different regions so can calculate current, determine sensitivity to gap distance.



What does V tip look like?
a. higher than V sample
b. same as V sample
c. lower than V sample
d. tilts downward from left to right
e. tilts upward from left to right

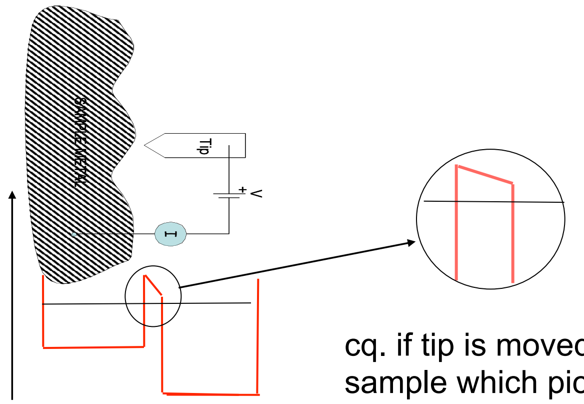
Correct picture of STM-- voltage applied between tip and sample.
Potential energy in different regions so can calculate current, determine sensitivity to gap distance.



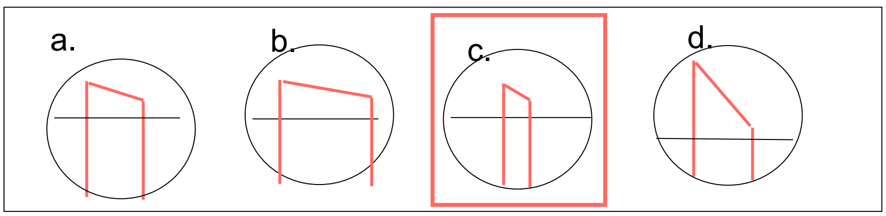
What is potential in air gap?

linear connection

Notice changing V will change barrier, and hence tunneling current.



cq. if tip is moved closer to sample which picture is correct?

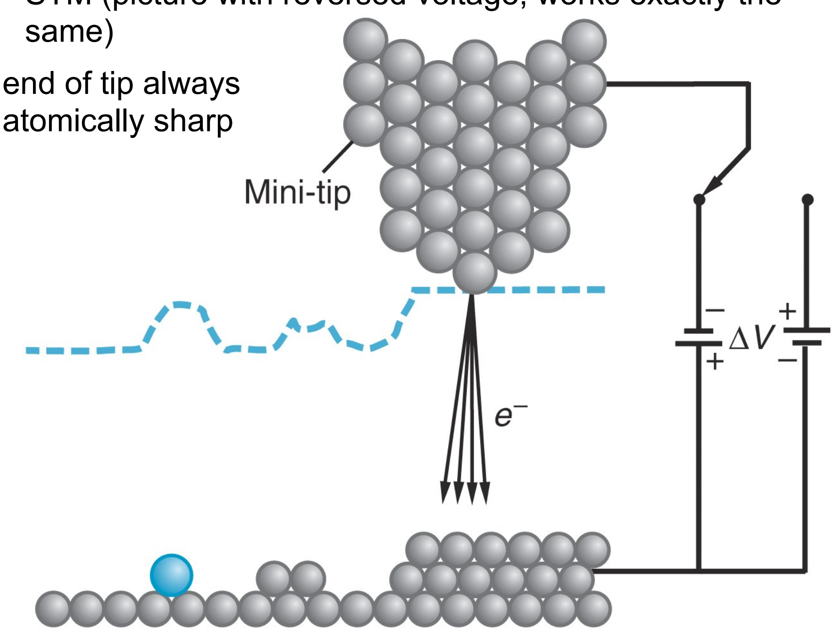


tunneling current will go: (a) up, (b) stay same, (c) go down

(a) go up. D is smaller, so $e^{-2\alpha D}$ is bigger (not as small), T bigger

STM (picture with reversed voltage, works exactly the same)

end of tip always atomically sharp

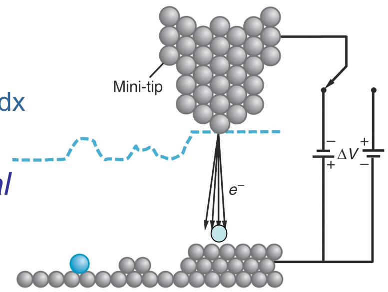


How sensitive to distance
Need to look at numbers.

$T \sim e^{-2\alpha D}$ from $P(x,t) = |\Psi(x,t)|^2 \, dx$

how big is α?

(= 1/(how long is exponential tail of wave function = how far can tunnel).



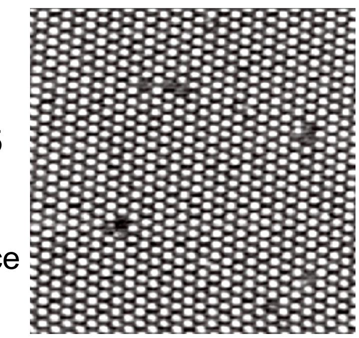
$$\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

calculate as in homework:
if V_0-E = 4 V, $\alpha = 1/(1 \times 10^{-10}$ m)

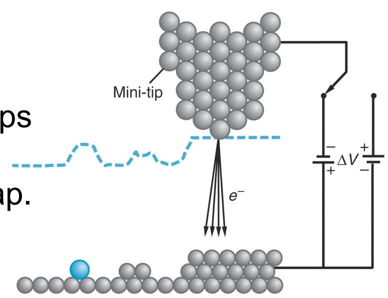
So if D = 3×10^{-10} m, T = e^{-6} = .0025
add 1 extra atom (d ~ 10^{-10} m),
how much does T change?
Hint atom makes tip closer to surface

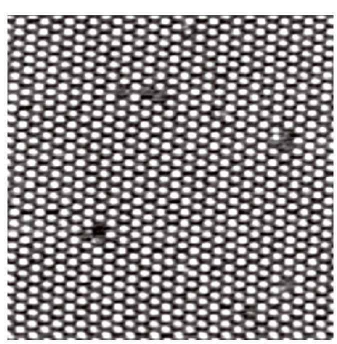
T = e^{-4} =.018

so one atom yields 7x the current!



actual STM moves tip across surface, adjusts distance to keep distance constant, keeps track of how much has to move in and out to make map.

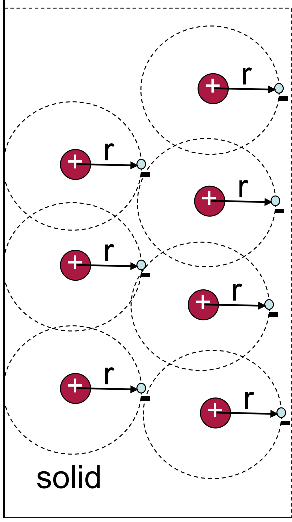




a more common manifestation of QM tunneling

1. understanding discharges- electrons popping out of surface when voltage applied.

What electric field needed to rip electron out of solid if no tunneling?

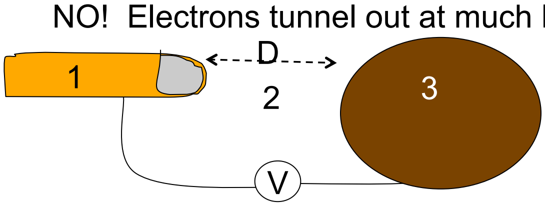


Applied E-field must exceed E-field of nucleus
Using Hydrogen estimate E_{nuc}.
E-field ~ kq/r^2
~ 13 V/ 0.053 nm
~ 3 x 10^{11} V/m
~ 3 x 10^{9} V/cm

so would need around 5 x 10^{9} V/cm J. Travoltage sim

Get few billion volts from rubbing feet on rug?

NO! Electrons tunnel out at much lower voltage.



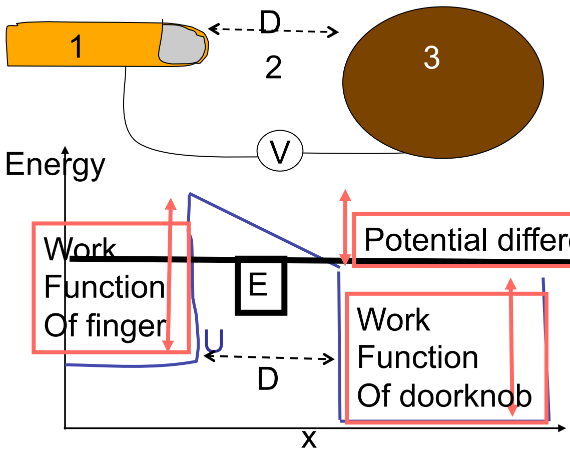
What is the minimum we need to know to figure out tunneling probability?

a. Only D
b. only V
c. V and D
d. V, D, and work functions of finger and doorknob
e. none of the above, or need additional information

ans. d. if have these, can get potentials, solve Schrod Eqn so would need around 5 x 10^9 V/cm J. Travoltage sim

Get few billion volts from rubbing feet on rug?

NO! Electrons tunnel out at much lower voltage.



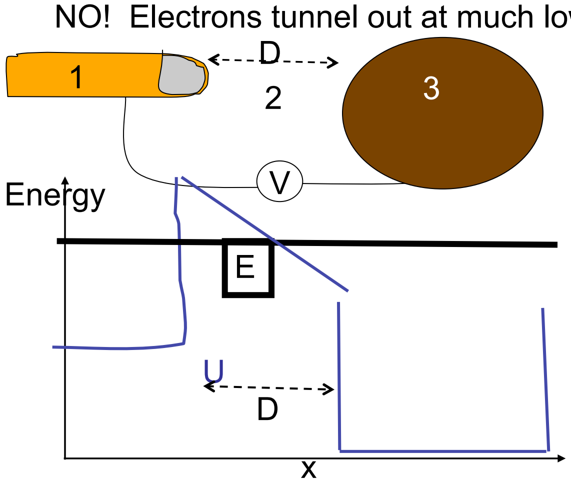
V = 0, T ~$e^{-2\alpha x}$ tiny.
Rub feet, what happens to potential energy?
(note **x** is related but diff fr **D**)

BEFORE RUBBING

so would need around 5 x 10^9 V/cm J. Travoltage sim

Get few billion volts from rubbing feet on rug?

NO! Electrons tunnel out at much lower voltage.



V = 0, T ~$e^{-2\alpha x}$ tiny.
Rub feet, what happens to potential energy?
(note **x** is related but diff fr **D**)

AFTER: Distance to tunnel much smaller. Big V ⇒ a small, so $e^{-2\alpha x}$ big enough, e's tunnel out!

Travolta knows this!

http://phet.colorado.edu/en/simulation/travoltage