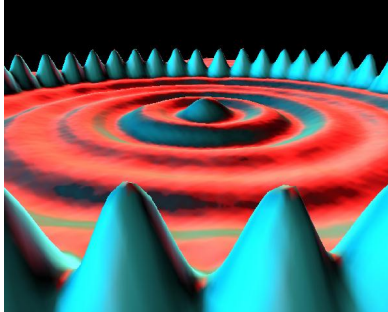


Why is this picture so cool and what does it have to do with John Travolta?



Here they have positioned 48 iron atoms into a circular ring in order to "corral" some surface state electrons and force them into "quantum" states of the circular structure. The ripples in the ring of atoms are the density distribution of a particular set of quantum states of the corral - IBM Research

Day 32:

Questions?

STM


Reminders:

HW

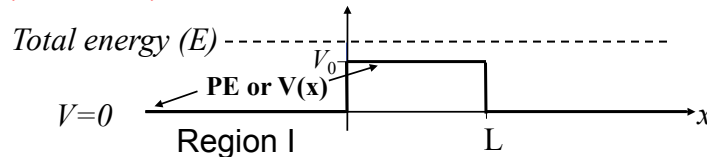
Next up:

Alpha Decay and Nooks

2. Find functional form... guess and make sure it works.

(On tutorial 

Easy way (for math):



The general solution for total energy (E) $>$ potential energy (V):

$$\Psi(x, t) = (Ae^{ikx} + Be^{-ikx})e^{-iEt/\hbar}$$

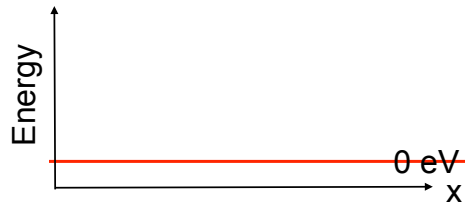
Schrodinger Eqn:
$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + V(x, t)\Psi(x, t) = i\hbar \frac{\partial \Psi(x, t)}{\partial t}$$

Plugging into the Schrodinger Equation gives:

$$-\frac{\hbar^2}{2m} (-k^2)\Psi(x, t) + 0 = i\hbar \left(\frac{-iE}{\hbar}\right)\Psi(x, t)$$

This simplifies to: $\hbar^2 k^2 / 2m = E$ OK

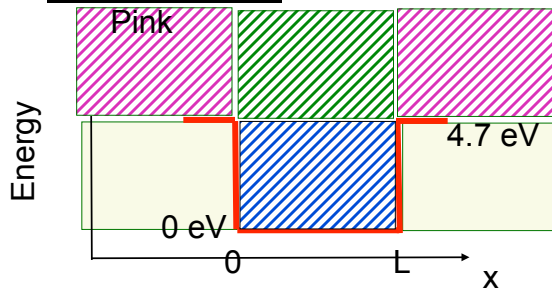
In free space:



For the case of an electron in free space ($E > V=0$), what energies are allowed?

- A. Any energy is allowed
- B. Only certain specific energies are allowed.

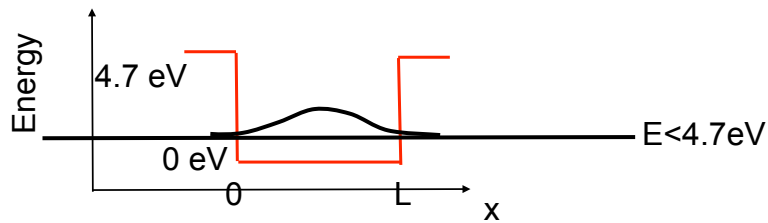
In wire length, L :



In which regimes is an electron allowed to have only specific fixed energies (quantized energy levels)? (In the other regimes... any energy is allowed)

- A. Green only
- B. Blue only
- C. Pink only
- D. Green and Blue
- B. Green, Blue, and Pink

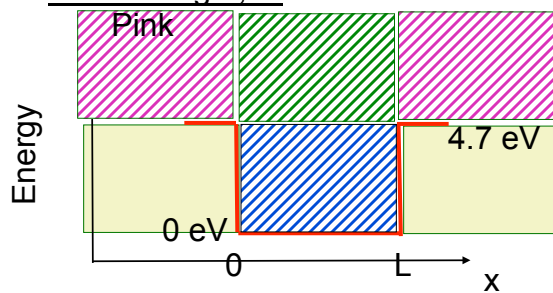
In wire length, L:



If only specific energy levels are allowed:

- A. This outcome is only true in QM
- B. This is true in QM and Classical instances (e.g. violin)
- C. I have no idea

In wire length, L:



Which areas not possible classically?

- A. Green
- B. Blue
- C. Pink
- D. Yellow

Today:

other applications of tunneling in real world

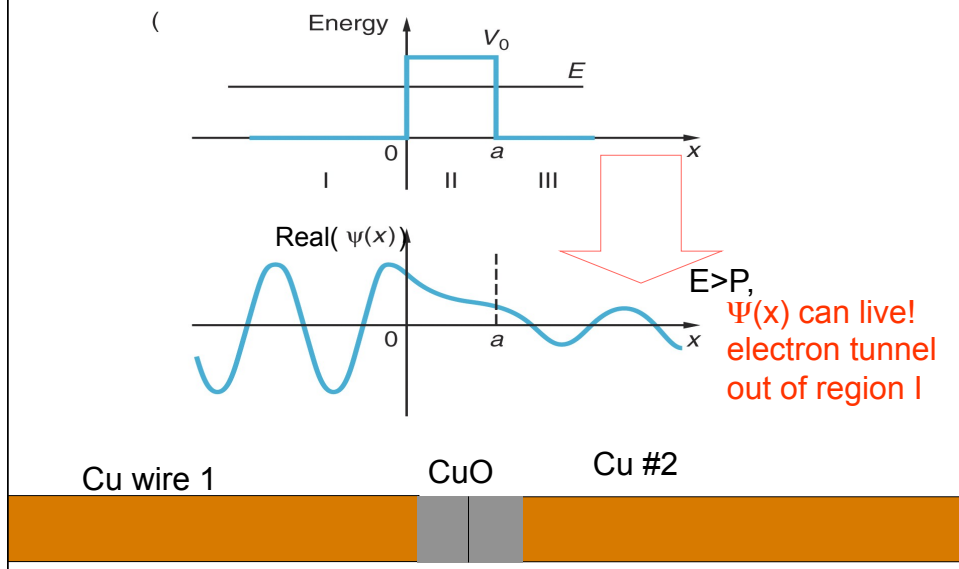
Scanning tunneling microscope (STM):

how QM tunneling lets us map individual atoms on surface

Interesting example may not have time to cover but may be in notes:

- Sparks and corona discharge (also known as field emission) electrons popping out of materials when voltage applied.
- Many places including plasma displays.

Can have transmission only if third region where solution is not real exponential! (electron tunneling through oxide layer between wires)



Tunneling Probability as in HW

$$\text{Prob.} \sim \Psi^2 \approx e^{-2\alpha D} \quad \alpha = \text{decay constant} = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

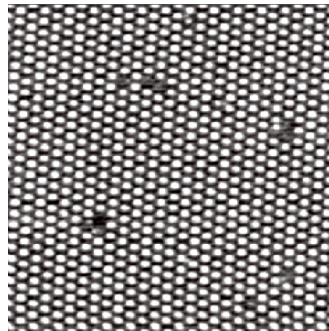
D = width of barrier

From homework, what is distance wave function of an electron of 1eV penetrates from copper into an air / vacuum?

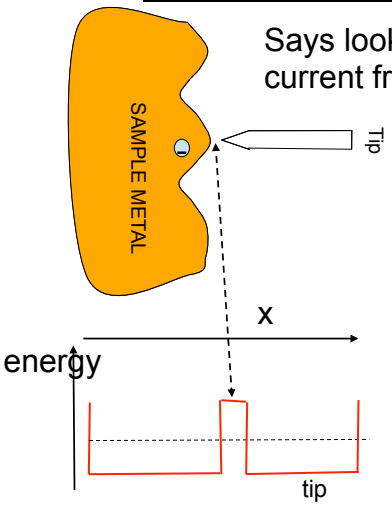
$$\begin{aligned} \text{penetration depth} &= \hbar/[2m(V-E)]^{1/2} \\ &= 10^{-34} \text{ Js}/[2 \times 9 \times 10^{-31} \text{ kg} \times (4.7-1) \text{ eV} \times 1.6 \times 10^{-19} \text{ J/eV}]^{1/2} \\ &= 1.01 \times 10^{-10} \text{ m} = 2 \text{ Bohr radii} \sim 1 \text{ atom diameter} \end{aligned}$$

implies that if have barrier of few eV, and change distance by one atom diameter will change tunneling current by large factor ($1/e = 1/2.7$)

Use tunneling to measure small changes in distance.
Nobel prize winning idea- invention of “scanning tunneling microscope (STM)”. Measure atoms on surfaces.



Book description of STM wrong in earlier versions.



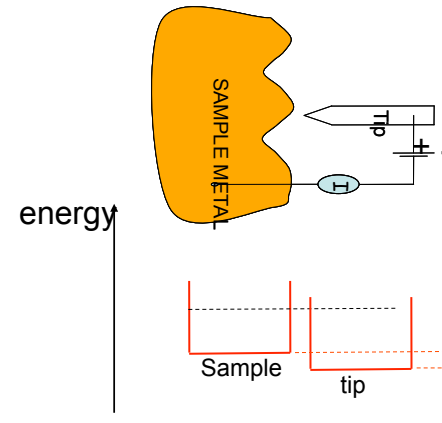
Says looks like this, and one looks at tunneling current from sample to tip to measure gap.

What is wrong with this?

Electron tunnels from sample to tip.
 What would $V(x)$ look like then?

- same as before.
- V in tip higher, V sample lower.
- V in tip lower, V sample higher.
- V same on each side as before but barrier higher.

Correct picture of STM-- voltage applied between tip and sample. Holds potential difference constant, electron current.
Figure out what potential energy looks like in different regions so can calculate current, determine sensitivity to gap distance.



What does V tip look like?

- higher than V sample
- same as V sample
- lower than V sample
- tilts downward from left to right
- tilts upward from left to right

cq. if tip is moved closer to sample which picture is correct?

a. b. c. d.

tunneling current will go: (a) up, (b) stay same, (c) go down

STM (picture with reversed voltage, works exactly the same)
end of tip always atomically sharp

Mini-tip

e^-

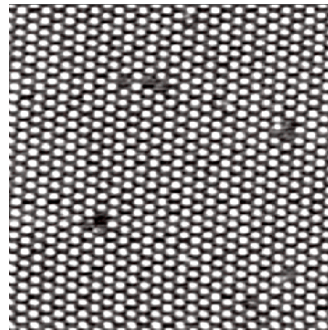
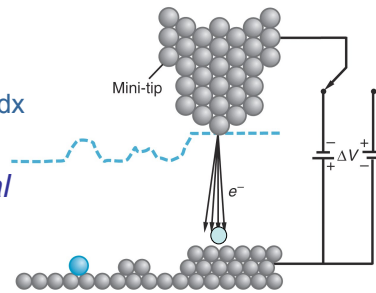
ΔV

How sensitive to distance
Need to look at numbers.

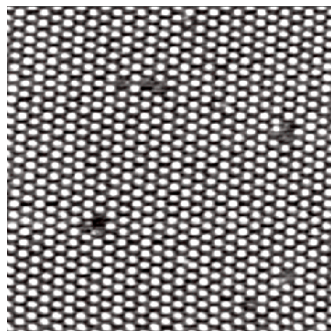
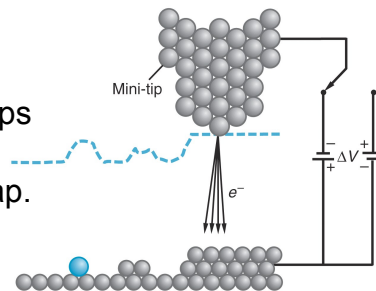
$T \sim e^{-2\alpha D}$ from $P(x,t) = |\Psi(x,t)|^2 dx$
how big is α ?

(= 1/(how long is exponential
tail of wave function =
how far can tunnel).

$$\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$



actual STM moves tip across
surface, adjusts distance
to keep distance constant, keeps
track of how much has
to move in and out to make map.



a more common manifestation of QM tunneling

1. understanding discharges- electrons popping out of surface when voltage applied.

What electric field needed to rip electron out of solid if no tunneling?

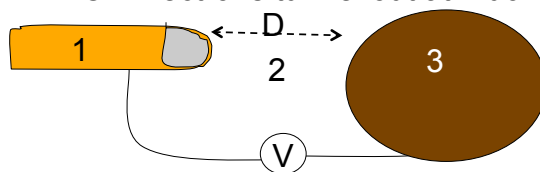
Applied E must exceed E of nucleus.
Using H to estimate E_{nuc} .
 $E \sim kq/r^2$
 $\sim 13 \text{ V} / 0.053 \text{ nm}$
 $\sim 3 \times 10^{11} \text{ V/m}$
 $\sim 3 \times 10^9 \text{ V/cm}$

solid

so would need around $5 \times 10^9 \text{ V/cm}$ J. Travoltage sim

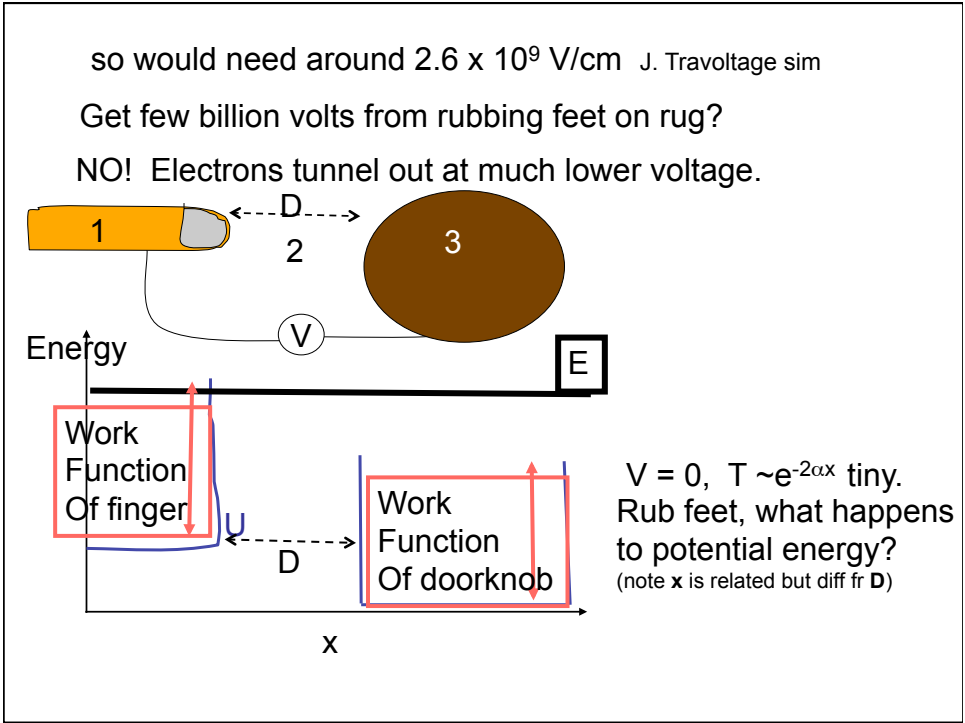
Get few billion volts from rubbing feet on rug?

NO! Electrons tunnel out at much lower voltage.



What is the minimum we need to know to figure out tunneling probability?

- Only D
- only V
- V and D
- V, D, and work functions of finger and doorknob
- none of the above, or need additional information



Travolta knows this!



<http://phet.colorado.edu/en/simulation/travoltage>