One introduces now (to simplify)

wave number $k = 2\pi/\lambda$

angular frequency $\omega = 2\pi/T$

$$\Rightarrow \quad E(x,t) = E_0 \sin(kx - \omega t)$$

Now we will convince ourselves that such electromagnetic waves exist. Any (real) electric field has to fulfill the Maxwell equation or here the wave equation (which can be derived from the Maxwell equations)

$$\frac{\partial^2 E(x,t)}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E(x,t)}{\partial t^2}$$

We will not work through the full proof that

$$E(x,t) = E_0 \sin(kx - \omega t)$$

indeed fulfills this equation, but we will motivate that it is the case. To this end, we will understand the left hand side of the equation. Let's start with

$$\frac{d}{dx} \sin(kx) = \cos(kx) \frac{d}{dx}(kx) = k \cos(x)$$

Next we consider the sin-function depending on x and t

$$\frac{\partial}{\partial x} \sin(kx - \omega t) = \cos(kx - \omega t) \frac{\partial}{\partial x}(kx - \omega t)$$

$$= k \cos(kx - \omega t)$$

partial derivative since function depends on x and t

And then we take the second derivative

$$\frac{\partial^2}{\partial x^2} \sin(kx - \omega t) = \frac{\partial}{\partial x}\Big(\underbrace{\frac{\partial}{\partial x} \sin(kx - \omega t)}_{= k \cos(kx - \omega t)}\Big)$$

$$= \frac{\partial}{\partial x}\big(k \cos(kx - \omega t)\big)$$

$$= -k^2 \sin(kx - \omega t)$$

Do you "see" that we get a similar expression on the RHS of the wave equation for the 2nd derivative in time.

-> Looks promising! Seems that such a sin-function fulfills the wave equation -> Work it out!

Analogous you can show that $E(x,t) = E_0 \cos(kx - \omega t)$ is also a solution of the wave equation.

Let's summarize the properties of electromagnetic waves

- EM waves are coupled oscillating electric and magnetic waves
- They can be represented by function of position and time: $\vec{E} = \vec{E}_0 \sin(kx \pm \omega t)$ or $\vec{E} = \vec{E}_0 \cos(kx \pm \omega t)$

with $k = \frac{2\pi}{\lambda}$ and $\omega = \frac{2\pi}{T}$

λ: wavelength and T: period

Furthermore: $f = \frac{1}{T}$ is frequency of wave

Other properties are:

- The speed of (EM) waves depends only on the properties of the medium and is independent of the wavelength or frequency of the wave

$$v = \frac{\lambda}{T} = \lambda f$$

In case of EM waves the speed is

$$v = c = 3 \times 10^8 \, m/s \quad \text{in vacuum}$$

[in fact, the speed of EM waves is slower in materials]

- The spectrum of EM waves ranges in wavelength from radio waves (10^3 m) over visible light (500×10^{-9} m) to gamma rays (10^{-12} m)

- In an EM wave the electric and magnetic fields are perpendicular to each other and to the direction of propagation. Such waves are also called transverse waves, since the fields are in the direction transverse to the propagation direction

- Electric and magnetic field oscillate "in phase", that means the $\vec{E}$-field reaches the maximum at the same time and place as $\vec{B}$ reaches its maximum ...

- The magnitude of the electric field is much larger than the magnitude of the magnetic field, namely

$$B_0 = \frac{E_0}{c}$$

- Every wave, i.e. also EM waves transfer energy. The energy stored in an EM wave is proportional to the square of the amplitudes of the fields

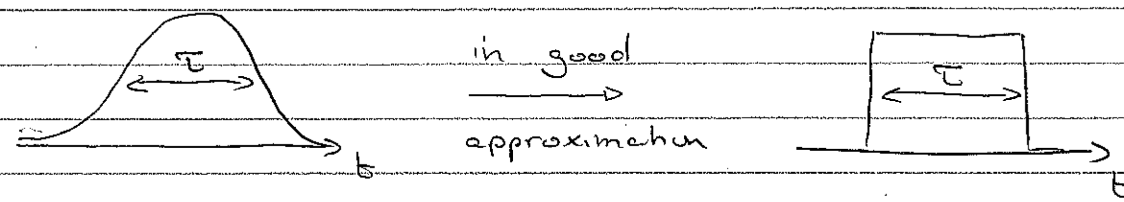
$$W = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

$$= \varepsilon_0 E^2 \qquad \left(\text{use } B = \frac{E}{c} \text{ and } c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}\right.$$

To characterize EM waves one also uses

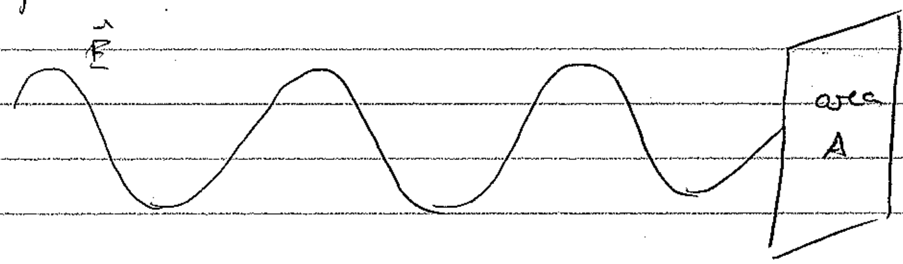
Power P = Rate at which wave transfers energy

For example : In modern technology with laser pulses



If we know the energy in such a light pulse and its pulse duration, we can calculate the laser power.

One further uses the intensity of a wave impinging on a surface



as $$I = \frac{\text{Power}}{\text{Area}} \sim \text{Energy in wave} \sim E^2$$