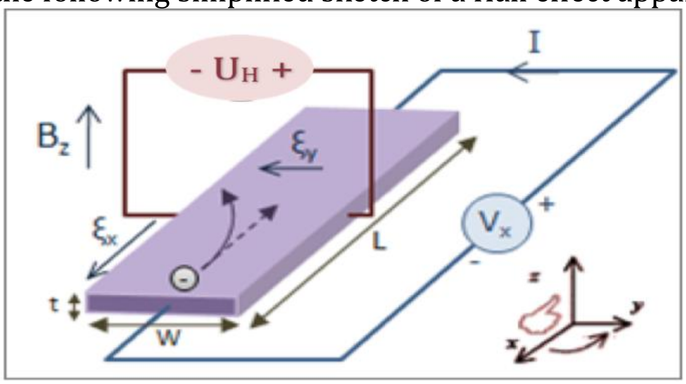
The Hall Effect
Physics 2150 Experiment 3
University of Colorado[1]

Introduction

The Hall Effect can be used to illustrate the effect of a magnetic field on a moving charge to investigate various phenomena of electric currents in conductors and especially semi-conductors. When a current-carrying conductor is placed in a magnetic field such that the field and current directions are perpendicular to each other, a voltage difference will appear as a result of the magnetic field. This voltage is called the Hall voltage and is the discovery of E.H. Hall in 1879. This Hall voltage is proportional to the product of the current and component of the magnetic field perpendicular to the current. More recently, the Hall Effect is widely employed throughout industry in modern Hall Effect gauss-meters, automotive speedometers, fluid flow sensors, and pressure sensors to name a few.

Hall Coefficient/Sensitivity for Negative Charge Carriers

Consider the following simplified sketch of a Hall effect apparatus:



(Figure 1: Hall Effect Diagram)

If the carriers are negatively charged, they are then moving in the negative x-direction in Fig. 1. The magnetic field exerts a force on them that will be in the positive z-direction with magnitude given by

$$F_B = F_y = -q\ \xi_x\ B \tag{1}$$

where ξ_x is the average velocity of the carriers in the x-direction. These carriers will thus be forced toward the left edge of the slab, which will then develop a lower potential than the right edge. An electric field E_H will grow until the force on a

[1] Experimental apparatus and instructions come from ***Lambda Scientific***: www.lambdasys.com

charge carrier due to the magnetic field is just canceled out, preventing further buildup of charge.

This electric field will be given by

$$E_H = E_y = -|q\ \xi_x|\frac{B}{|q|} = -|\xi_x|\ B\ . \qquad (2)$$

The current density in the x-direction is given by

$$j_x = n\ |q\ \xi_x| \qquad (3)$$

where n is the average density of the carriers.

Thus, the Hall field is given by

$$E_H = -j_x\ \frac{B}{n|q|}\ . \qquad (4)$$

This is customarily written

$$E_H = R_H\ j_x\ B \qquad (5)$$

and the “Hall Coefficient” R_H is thus given by the positive number

$$R_H = -\frac{1}{n|q|}. \qquad (6)$$

Typical units for R_H are cubic meters per Coulomb. The “sensitivity” of the Hall element (with units of mV mA^{-1}T^{-1}) is given by:

$$K_H = \frac{R_H}{d} = \frac{-1}{n|q|d} \qquad (7)$$

where d is the is the thickness of the Hall material. When determining the sensitivity experimentally, we will make use of the following formula:

$$V_H = I_H K_H B \qquad (8)$$

where V_H is the measured Hall voltage and I_H is the measured Hall current. (Note that eq. 8 resembles eq. 5. We can derive eq. 8 if we consider that the speed of the charge carriers is: $v = I_H/nqwd$ where v is the velocity and w is the width of the sample. Try it!)

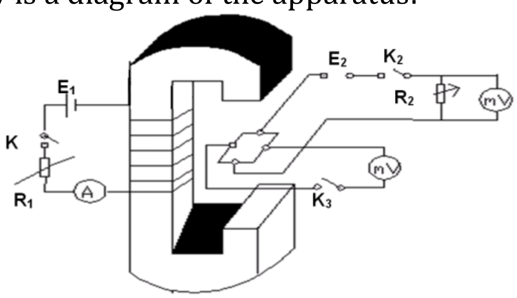
If positive charge carriers are considered, then one will find that the Hall coefficient and sensitive are positive values with the same magnitudes found in the analysis for negative charger carriers.

An experimental measurement of the Hall coefficient thus allows one to determine both the sign and the density of the charge carriers in a material. Historically, it was this effect that first conclusively established that current is carried in metallic conductors by negatively charged particles. A material, in particular semiconductor materials, that consists of negative charge carriers are referred to as “n-type.” Materials that predominantly have positive charge carriers are referred to as “p-type”. The positive charge carriers are referred to as electron “holes” since they aren’t drifting particles, but regions with a lack of an electron(s) that drift through the material.

In metals, which have a large density of conduction electrons, R_H is small and negative. In a semiconducting material with a small excess of positive charge carriers over negative charge carriers, the Hall coefficient can be large and positive. For example, copper typically has a value of -6 x 10^{-11} m^3/C, whereas the semiconductor bismuth, -4 x 10^{-7} m^3/C, nearly 10,000 times larger, both of which are elements.

<u>Experimental Apparatus</u>

The figure below is a diagram of the apparatus:



(Figure 2: Cartoon of the Hall Apparatus)

The two switches, K and K_2, allow us to change the direction of current flowing through the electromagnet and the direction of current flowing through the sample. The switch K_3 toggles between measuring the Hall voltage and strip current. We do not measure the strip current directly, but measure the voltage across a 300 Ω resistor and compute the current. In the I_H setting we are measuring the strip current and in the U_H setting we are measuring the Hall voltage across the strip.

We need to be able to toggle K and K_2 to eliminate unwanted magnetic and thermal side effects. These include: Ettingshausen effect for the creation of a thermoelectric potential due to the existence of a temperature difference at the two sides of a Hall element. It is related to the directions of Hall current and magnetic field; (b) Nernst effect for the creation of a potential between the two sides of a Hall element when heat flows through the Hall element. It is only related to magnetic field B and heat flux. Righi–Leduc effect for the creation of a thermoelectric potential U_R due to a temperature difference at the two ends of a Hall element created by heat flowing through the Hall element. It is related to magnetic field B and the heat field. These effects are often small, may become apparent (epically those that involve heat), when the apparatus has been on for a long time. To reduce error, in particular a systematic bias, we toggle K and K_2 in four different combinations and compute an average for each measurement.

Procedure

1) **Measure the relationship between the Hall current and Hall voltage. In this part we will measure the relationship to ensure a linear relationship between these two quantities**. Read the set-up steps a-e before taking data.
    a. Place the Hall element at the center of the electromagnet by turning the adjustment knob so the pointer is at 0mm on the ruler scale.
    b. Before you begin to take data prepare the following data table in your lab notebook (with space for additional rows):

I_H (mA)	Nom. V_x (mV)	Meas. V_x (mV)	U_{H1} (mV)	U_{H2} (mV)	U_{H3} (mV)	U_{H4} (mV)	U_H (mV)
0.5	150	?	?	?	?	?	?
…	…	…	…	…	…	…	…

    c. Choose 7-10 values of I_H to measure and write these down in the first column. Notice that we cannot measure the current directly using this apparatus; rather we set a voltage. The manufacturer has placed a 300-Ohm resistor (+/- 5%) in series with the Hall element so that current can be set based on a voltage measurement. Compute the nominal voltage (by Ohms Law) that you will set the " Hall Current Adj" knob to at each measurement. Put the nominal values in the "Nom. V_x" column.
    d. To eliminate side effects we need to measure the Hall voltage U_H in four different configurations using switches: I_M and I_H (these change the direction of current flowing through the electromagnet and Hall element respectively). The switches will be oriented via the following scheme:

	I_M	I_H
U_{H1}	+	+
U_{H2}	-	+
U_{H3}	+	-
U_{H4}	-	-

    e. The value of U_H is the average of the absolute values of the four different hall voltages: $U_H = -\frac{1}{4}\sum_{i=1}^{4} |U_{Hi}|$ (Note: we know that U_H should be negative for an n-type semiconductor).

f. Set the magnetizing current of the electromagnet to 400 mA using the "Current Adj" and record the B-field strength with the Tesla meter.
g. The process for acquiring data goes a follows:
    i. Switch the "I_H/U_H" to the I_H position and match this voltage reading to your nominal V_x and record your reading in the third column.
    ii. Switch the "I_H/U_H" to the U_H and record the voltages for the four different switch configurations listed in step (d).
    iii. Compute the average U_H using the formula from step (e).
    iv. Return to step (i) and repeat for a different I_H/V_H value.
h. **Verify the linear relationship between Hall current and Hall voltage. Do so qualitatively and quantitatively** (i.e. plot and correlation coefficient). **Why might it be important to ensure a linear relationship between these two quantities?**

2) **Measure the sensitivity of the GaAs Hall element**.
    a. Set the electromagnet DC Current to zero using the "Current Adj" knob, and zero the teslameter to zero using the "Teslameter Zero" knob.
    b. Set the Hall current I_H to 1mA or 300 mV.
    c. Set up a data table similar to the one in step (1b), however instead of varying I_H you will vary the magnetizing current I_M with the "Current Adj." knob. You also will need an additional column to record the magnetic field strength with the teslameter. You will need two extra columns for the B-field. One for a positive magnetizing current and the other for a negative magnetizing current. You may or may not see a change in current or B-field magnitude when you toggle the direction of I_H.
    d. Fill in your data table for magnetizing currents from 50 mA to 500 mA in 50 mA increments. Be sure to toggle the switches according to the scheme listed in step (1d).
    e. Recall that the sensitivity of the Hall element can be calculated with $K_H = \frac{U_H}{I_H B}$. **Compute the "best" value of K_H (i.e. mean) and the statistical uncertainty (i.e. standard deviation of the mean).**
    f. Justify/verify the sign of the Hall coefficient/sensitivity.
    g. **Use your measured value of sensitivity to calculate the intrinsic charge carrier density n** using eq. 7. The manufacturer gives $d = 0.2$ mm. Compare your value to the calculated value at 300 K (about 80 Fahrenheit –a hot room!) of $2.03*10^6$ cm^{-3}.[2]
3) **Measure the magnetic field distribution of the electromagnet** along the horizontal direction.

[2] https://ecee.colorado.edu/~bart/book/book/chapter2/ch2_6.htm see section 2.6.3

Experiment 3

a. Set the magnetizing current to 400 mA using the “Current Adj” knob and the Hall current to 1 mA or 300 mV, using the “Hall Current Adj”.
b. Use the knob on the probe mount to move the probe to the -12 mm position and take measurements through the +10 mm position. Do this in increments of 1mm. Make sure the gray wire connecting the Hall element to the apparatus does not get caught while moving, the wire can be gently pulled out for more slack. Notice that there is some gear backlash in the probe mount carriage.
c. **Measure the Hall voltage and calculate the magnetic field strength B at each location using your best value for the sensitivity of the Hall probe**: i.e. $B = \frac{U_H}{I_H K_H}$. **Also record the value of B that the teslameter displays** at each position. In this case we do not need to do the toggling procedure discussed in step (1d). You may want to probe around the electromagnet before taking data –in particular we want to characterize the fringe field outside of the magnet.
d. **Plot B vs. position**. (Combining ListPlot[] and ListLinePlot[] might be a nice way to visualize your data.) Do you notice a discrepancy between your B-values measured with the Hall voltage and the direct measurement of B with the teslameter.