

**Millikan Oil-Drop Experiment**  
**Physics 2150 Experiment 4**  
**University of Colorado<sup>1</sup>**

### **Introduction**

The fundamental unit of charge is the charge of an electron, which has the magnitude of  $e = 1.60219 \times 10^{-19}$  C. With the exception of quarks, all elementary particles observed in nature are found to have a charge equal to an integral multiple of the charge of an electron. (Quarks have charge  $e/3$ , but they are never observed as free particles). By convention, the symbol  $e$  represents a positive charge, so the charge  $q$  of an electron is  $q = -e$ .

The first person to measure accurately the electron's charge was the American physicist R. A. Millikan. Working at the University of Chicago in 1912, Millikan found that droplets of oil from a simple spray bottle usually carry a net charge of a few electrons. He built an apparatus in which tiny oil droplets fall between two horizontal capacitor plates while their motion is observed with a microscope. A voltage applied between the plates creates an electric field, which can exert an upward force on a charge droplet, counteracting the force of gravity. By studying the motion of a droplet in response to gravity, the electric field between the plates, and viscous drag due to the air, Millikan was able to calculate the charge on the droplet. He published data on 58 droplets and reported that the charge of an electron is  $e = (1.592 \pm 0.003) \times 10^{-19}$  C. Millikan's value was a bit low (three  $\sigma$  below the accepted value) because he used a slightly inaccurate expression for the drag force due to air. For this work and later research on the photoelectric effect, Millikan received the Nobel Prize in 1923.

In recent years, some historians have suggested that Millikan improperly threw out data which yielded charges of a fraction of an electron's charge; i.e. that he selected his data in order to get the answer he wanted. Fortunately for Millikan's reputation, he kept excellent lab notebooks and it has been possible to re-construct all his measurements and calculations. Millikan's notebooks do contain much data, which he never published, but there is no evidence that he fudged his results. All of his data, both published and unpublished, has been re-analyzed by CU physicist, Prof. Allan Franklin, who finds that all of Millikan's data, properly analyzed, yields a result which agrees perfectly with the modern value of  $e$ .

### **Theory**

If an oil drop with mass  $m$  and charge  $e$  enters the oil chamber, it falls down freely under the force of gravity when no voltage is applied between the parallel polar plates. When the force of gravity is balanced against air resistance (neglecting air buoyancy), the

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<sup>1</sup> Experimental apparatus and instructions come from *Lambda Scientific*:  
[www.lambdasys.com](http://www.lambdasys.com)

oil drop falls down at a uniform speed of  $V_g$ , as described by the following equation:

$$mg = f_a \quad (1)$$

Where  $f_a$  is the air resistance when the oil drop falls down at a uniform speed of  $V_g$ , and  $g$  is the acceleration due to gravity. According to Stokes' law, the air resistance exerted on the oil drop is:

$$f_a = 6\pi\eta a V_g \quad (2)$$

While the force of gravity exerted on the oil drop is:

$$G = mg = \frac{4}{3}\pi a^3 \rho g \quad (3)$$

Where  $\eta$  is the coefficient of viscosity for air;  $\rho$  is the density of oil drop; and  $a$  is the radius of the oil drop. By combining equations (1) to (3), the radius of the oil drop is:

$$a = 3 \sqrt{\frac{\eta V_g}{2g\rho}} \quad (4)$$

We will also make use of the more convenient expression for the radius:

$$a = \sqrt{\frac{9\eta l}{2\rho g t_g}} \quad (4b)$$

Where  $l$  is the fall distance of the droplet and  $t_g$  is the fall time of the droplet.

When an electric field is applied across the parallel polar plates, the oil drop is subject to the force of the electric field. If the strength and polarity of the electric field are controlled and selected properly, the oil drop can be elevated under the influence of the electric field. If the force of the electric field exerted on the oil drop is balanced against the sum of the force of gravity and the resistance of air exerted on the oil drop, the oil drop will rise at a uniform speed of  $V_e$ . Hence, we have:

$$mg + f'_a = qE \quad (5)$$

From equations (1) and (2), equation (5) can be rewritten as:

$$mg + f'_a = f_a + f'_a = 6\pi\eta a(V_g + V_e) = qE = q \frac{V}{d} \quad (6)$$

Thus,

$$q = \frac{6\pi\eta a d (V_g + V_e)}{V} \quad (7)$$

Where  $V$  and  $d$  are the voltage and separation between the parallel polar plates, respectively.

Equation (7) is derived assuming the oil drop is in a continuous medium. In this experiment, the radius of the oil drop is as small as  $10^{-6}$  m and therefore it is comparable to air molecules. Thus, air is no longer considered as a continuous medium, and viscosity coefficient  $\eta$  of the air needs to be corrected as:

$$\eta' = \frac{\eta}{1 + \frac{b}{pa}} \quad (8)$$

Where  $b$  is a correction constant;  $p$  is the atmospheric pressure;  $a$  is the radius of oil drop as given in equation (4). Assume the falling and rising distance of an oil drop with uniform speed are identical as  $l$  and the spent time are  $t_g$  and  $t_e$ , respectively, we have  $V_g = l/t_g$  and  $V_e = l/t_e$ . By substituting (8) and (4) into (7), we have:

$$q = \frac{18\pi d}{V\sqrt{2\rho g}} \left( \frac{\eta l}{1 + \frac{b}{pa}} \right)^{3/2} \left( \frac{1}{t_e} + \frac{1}{t_g} \right) \left( \frac{1}{t_g} \right)^{1/2} = \frac{K}{V} \left( \frac{1}{t_e} + \frac{1}{t_g} \right) \left( \frac{1}{t_g} \right)^{1/2} \quad (9)$$

Were we let  $K = \frac{18\pi d}{\sqrt{2\rho g}} \left( \frac{\eta l}{1 + \frac{b}{pa}} \right)^{3/2}$ . The above formula is used to calculate charge quantity in dynamic (unbalanced) measurement method. In the static (balanced) measurement method, which is the method we will use in the procedure below, we adjust the voltage between the parallel electrodes to hold the oil drop stationary ( $V_e = 0, t_e \rightarrow \infty$ ). Now equation (9) becomes:

$$q = \frac{K}{V} \left( \frac{1}{t_g} \right)^{3/2} \quad (10)$$

Important values for this experiment are summarized in the table below:

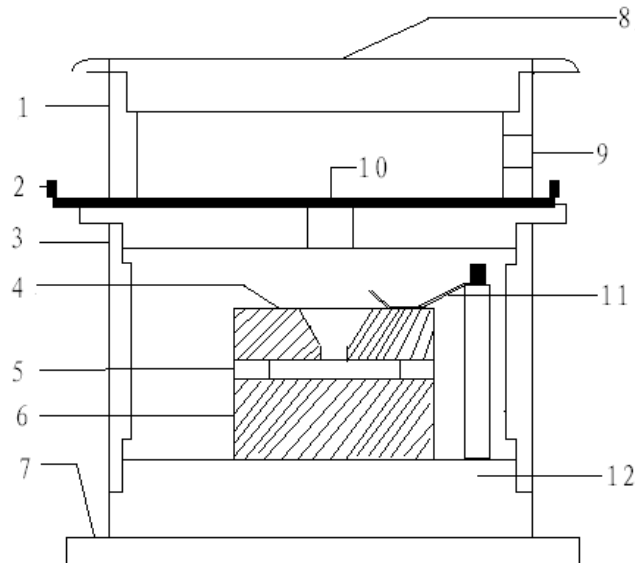
Density of Oil	$\rho = 981 \text{ kg}\cdot\text{m}^{-3}$
Gravitational Constant	$g = 9.8 \text{ m/s}^2$
Viscosity of Air	$\eta = 1.85 \cdot 10^{-5} \text{ kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$
Distance of Oil Drop Falling	$l = 1.5 \text{ mm}$
Atmospheric Pressure	Use the barometer
Distance of Parallel Electrodes	$d = 5 \text{ mm}$
Correction Constant	$b = 6.17 \cdot 10^{-6} \text{ m}\cdot\text{cmHg}$

### **Apparatus Structure**

This oil drop apparatus consists of oil drop chamber, CCD TV microscope, monitor screen, and electrical control unit. Among them, the oil drop chamber is a critical part with a schematic diagram shown in Figure 1.

As seen in Figure 1, the upper and lower electrodes are parallel plates with a

bakelite ring placed between them to ensure the parallelism between electrodes with high spacing accuracy (better than 0.01 mm). There is an oil mist hole (0.4 mm in diameter) in the center of the upper electrode. On the wall of the bakelite ring, there are three through holes, one for microscope objective, one for light passing and one as a spare hole.



*Figure 1 Schematic diagram of oil drop chamber*

- 1-oil mist cup    4-upper electrode    7-base stand    10-oil mist aperture  
 2-oil aperture switch    5-oil drop channel    8-cover    11-electrode clamp  
 3-windscreen    6-lower electrode    9-oil spray aperture    12-oil channel base

The oil drop cavity is covered by a wind proof screen on which a removable oil mist cup is placed. There is an oil drop aperture can be blocked with a stop. The upper electrode is secured with a clamp which can be toggled left and right for the removal of the upper electrode. Two illumination lamps are located on the side of the oil channel base with a light focusing mechanism. The intersection angle between the illumination light and the microscope objective is about  $150^{\circ}$ - $160^{\circ}$  to achieve high brightness, based on principles from light scattering theory. Unlike a conventional Tungsten filament bulb that can be blown out over time, focusing LEDs are used in this apparatus with extended lifetime and do not create convective currents due to heating. The CCD camera microscope is designed in an integral structure with stable and reliable operation. There are two sets of electronic scales, i.e. A and B. The former has 8 divisions in vertical direction and each division is 0.25 mm when the  $60\times$  objective is used; the latter is designed to observe Brownian movement of oil drops, with 15 small divisions in both X and Y directions. We will use scale A in our experiment.

## **Procedure**

### 1) Initial preparations:

- a. Make sure that the chamber is level by leveling the bubble level by adjusting the feet. Please do so carefully, if necessary.
- b. Get an oil filled atomizer from the lab coordinator. You must return the atomizer to the lab coordinator's office when you are finished. Please handle this item with care since it is quite delicate, and when it is not in use keep it upright in the cup.
- c. Turn on the apparatus first and then the monitor. You should see two red LED lights near the chamber light up and the screen should be grey and display a grid.
- d. Make sure the "Gang Switch" is depressed, that the "0 V" light is illuminated, and that the "Balance Voltage" knob is turned fully counterclockwise.

### 2) Measurement Rehearsal:

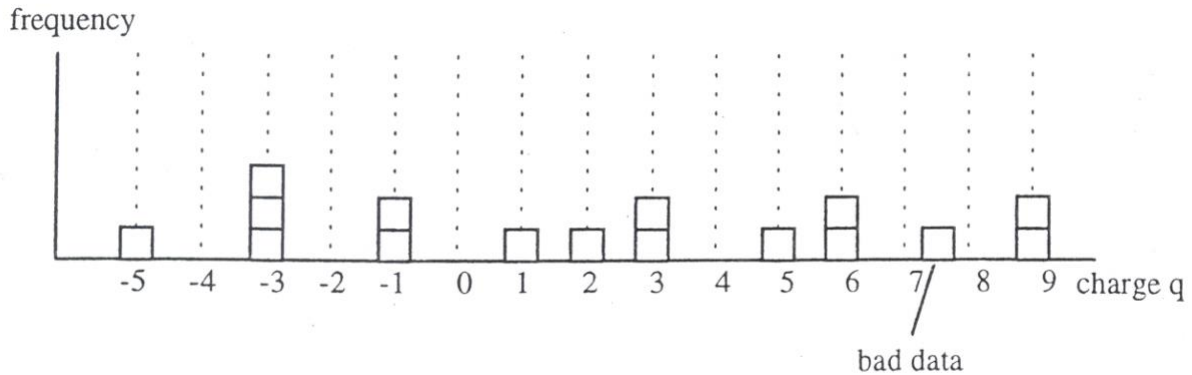
- a. Spray oil drops into the oil drop channel through the hole with the sprayer while observing the oil drops on the monitor. Do not spray too much oil into the chamber: one or two sprays should be sufficient. Close the oil mist aperture (sliding metal part) to avoid disturbance of air flow.
- b. Adjust the focus of the camera to observe the falling droplets. One of the key challenges in this experiment is to select proper oil drops. If the oil drops are too large, their uniform falling speed will be too fast, resulting in more charges carried by these large oil drops resulting in decreased measurement accuracy. (Why would the be the case?) On the other hand, if the oil drops are too small, they will be vulnerable to thermal motion and hence be difficult to control. In general, it is preferred to select oil drops in medium size and with slow rising or falling speed. Usually, if 200-300 V is applied to the electrodes (select "Balance" for and adjust "Balance Voltage" W to achieve this voltage), oil drops that move across 6 divisions (1.5 mm) in 8-20 s are the suitable oil drops for this experiment. Change "Balance" to "0 V", observe the falling speed of these oil drops, and select one proper oil drop as the measurement target. For reference, when viewing on a 10" display, the size of the oil drop image should be 0.5-1.0 mm. Get some practice selecting good drops before making a formal measurement.
- c. Time uniform motion of oil drops: select several oil drops at different speeds and use the criterion above to start/stop timing by looking at the scale on the monitor.
- d. Balance Oil Drops: adjust the balance voltage carefully to ensure that the oil drop truly stops moving and repeat the procedure several times.
- e. Once you feel comfortable using the apparatus begin taking data.

## 3) Formal Measurement:

- a. We will be using a static method to determine the charge on the oil drops.
- b. Begin by spraying oil in the chamber, close the mist aperture, and find a suitable drop.
- c. Press down the “Balance” button, and carefully balance an oil drop, so that it remains stationary on the screen. You may need to switch the polarity depending on the type of charge on the drop. You will also notice that the drop will jiggle slightly since the drop will undergo Brownian motion even under a force balance. Record the balancing voltage.
- d. Press down the “Elevation” button and bring the balanced oil drop to the second scale line from the top, then press “Balance” so it hovers on the second scale line.
- e. Ensure “Gang Switch” is depressed and that the timer is stopped. If the timer is timing press “Timing Start/Stop” button. Next, press the “0 V” button to remove the electric field balancing the drop; the timer will start incrementing. Notice that “0 V” and “Balance” buttons are synchronized with the timing since the “Gang Switch” is depressed. When the same oil drop reaches the eighth scale line (i.e. moves across 6 scale lines resulting in 1.5 mm of travel), press down the “Balance” button. Record the time.
- f. Repeat this process for 10-15 different oil drops only spraying oil in the chamber when necessary. Remember to open the oil mist aperture when you need to spray more oil.
- g. Once you’ve collected your data, record the air pressure of the room using the barometer in the G2B81 bay. If you are unsure how to read the barometer, ask your T.A. or appropriate staff person.
- h. Return your atomizer to the lab coordinator.

## 4) Data Analysis:

- a. **Use equation (10) to determine the charge** on your oil drops.
- b. We assume that charge can only come in a quantized number i.e. integer values of charge. Invent a scheme to match the measured charge from step 4a to the integer value of charge on the drop. (Hint: your lowest value of charge should correspond to what integer value?)
- c. **Make a histogram of the charges you measured, as in the figure below.** If the measured charges appear to be multiples of a minimum charge ( $e$ ) then you can safely identify the charge number  $n$ . Use your histogram to assess if the statement “charge can only come in a quantized amount” is true or not. Your histogram might look something like this:



- d. Pair the values of  $n$  and  $q$  and make a **plot of charge  $q$  vs integer charge  $n$  and perform a least square fit on your data.** *Linfit.nb* might be helpful for you in this process. What should the slope of this line be?
- e. **What is your measured value for the charge of the electron?** What is the random error on your measurement. Next we'll estimate the systematic errors. The uncertainty of the slope of ( $q$  vs.  $n$ ) gives the uncertainty in the charge  $e$  due to random measurement errors only. This random error must be combined with an estimate of the systematic error to get the total error. In this lab, we will **determine the uncertainty due to systematic errors numerically, rather than analytically.** Ordinarily, if one has some computed quantity  $f$  which is a function of several experimentally determined quantities  $x, y, \dots$  each with a systematic uncertainty  $\delta x, \delta y, \dots$ , then the systematic uncertainty in  $f$  is computed from

$$\delta f_{sys} = \sqrt{\left(\frac{\partial f}{\partial x} \cdot \delta x\right)^2 + \left(\frac{\partial f}{\partial y} \cdot \delta y\right)^2 + \dots}$$

In this experiment, the computed charge of  $q$  (from which we get  $e = q/n$ ) is a function of several variables ( $\rho, d, \dots$ ) with systematic uncertainties. However, a few moments contemplation of eqns. (3), (6), and (8) quickly leads to the conclusion that computing the necessary derivatives  $\frac{\partial q}{\partial \rho}, \frac{\partial q}{\partial d}, \dots$  is an extremely messy task. Fortunately, we can use Mathematica to compute numerically the systematic error in  $q$  due to each of the variables. For instance, consider the error in  $q$  due to the error in  $\rho$ ,  $\delta q_p = \frac{\partial q}{\partial \rho} \cdot \delta \rho$ . To compute this numerically, simply change the value of  $\rho$  in your Mathematica document to its extreme (maximum or minimum) value, update the document, and observe how much the compute  $q$  changes. The change is  $\delta q_p$ . Repeat this procedure for each variable and add the errors in quadrature to get the total systematic error,

$$\delta q_{sys} = \sqrt{(\delta q_p)^2 + (\delta q_d)^2 + \dots} = n \cdot \delta e_{sys}.$$

Do your best in estimating the systematic error. **Report your value of the electron charge with the random and systematic errors in standard format.** How does your value compare with the accepted value? Did you do better than Millikan?