

**Emission Spectra and the Balmer Series of Hydrogen**  
**Physics 2150 Experiment 7**  
**University of Colorado**

**Introduction**

The hydrogen atom consists of a nucleus, a proton (having a positive charge), and an electron (having an equal but opposite charge of magnitude  $e$ ). According to Bohr's original theory, this atom may exist in any one of a number of stationary states. These states are characterized by the requirement that the orbital angular momentum  $L$  of the atom must be

$$L = n \hbar \quad (1)$$

where  $n$  is a positive integer and  $\hbar = h/2\pi$ ;  $h$  is Planck's constant. It follows from this condition (see any text on modern physics) that the energies of the stationary states are quantized and have the values:

$$E_n = -\frac{1}{2} \left( \frac{e^2}{4\pi\epsilon_0} \right)^2 \frac{m}{\hbar^2 n^2} \quad (2)$$

where  $m$  is the mass of an electron and  $\epsilon_0$  is the permittivity of free space. Here the proton is assumed to be infinitely massive, as compared to an electron.

When a hydrogen atom in a state of energy  $E_1$  makes a transition to a state of lower energy  $E_2$ , a photon is emitted which by conservation of energy has a frequency  $\nu$  given by:

$$h\nu = E_1 - E_2 = \left( \frac{e^2}{4\pi\epsilon_0} \right)^2 \frac{m}{2\hbar^2} \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \quad (3)$$

and the wavelength of the photon will be:

$$\frac{1}{\lambda} = \frac{\nu}{c} = \left( \frac{e^2}{4\pi\epsilon_0} \right)^2 \frac{2\pi^2 m}{ch^3} \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right). \quad (4)$$

Atoms of hydrogen are ordinarily found in the lowest energy state  $E_{n=1}$ , but if hydrogen gas is placed in an electric discharge, many of the atoms will be excited to higher energy states by collisions. They may then radiate photons of the wavelengths given by Eq. (4). Most of these wavelengths are not in the visible portion of the electromagnetic spectrum. However, it happens that the particular wavelengths characterized by

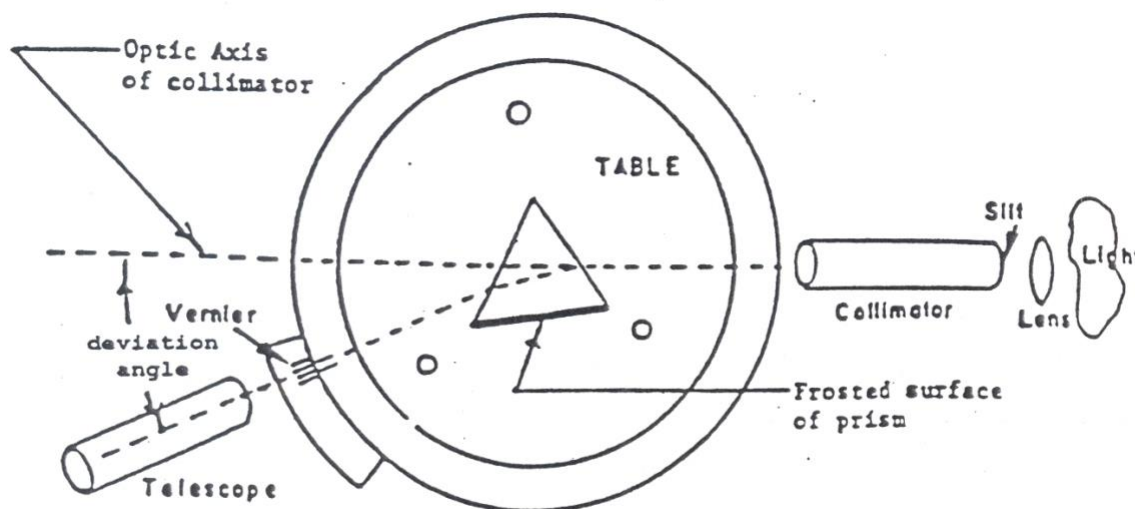
$$n_2 = 2; n_1 = 3, 4, 5, 6$$

are visible. These are the first four members of a spectral series called the Balmer Series. The lines are commonly labeled  $H_\alpha, H_\beta, H_\gamma, H_\delta, \dots$  in order of decreasing wavelength. In this experiment, the wavelengths of the visible lines in the Balmer series will be measured by

comparing the spectrum of hydrogen with those of mercury and helium. The wavelengths of the visible lines in the Hg and He spectra will be assumed to be known.

### **Procedure**

The measurements will be taken with a prism spectrometer, which is diagrammed in Fig. 1. Light from an intense source falls on an adjustable vertical slit which is at one end of a collimator tube. Light rays passing through the tube thus emerge from the lens in a



*Figure 1: Schematic Diagram of a Prism Spectrometer*

parallel beam. These rays pass through a glass prism, which is placed in the center of the apparatus on an adjustable table. Since the index of refraction of glass is a function of the wavelength of the incident light each wavelength will be refracted by the prism at a slightly different angle. The higher frequencies (shorter wavelengths) are refracted through greater angles than the lower frequencies. These refracted rays pass into a telescope focused on infinity, and the eye upon looking into the telescope sees a series of images of the slit, one for each wavelength of light emitted by the source. The angles through which the various rays are bent may be accurately measured by means of a fixed scale and vernier, which are attached to the instrument base and telescope, respectively. If you do not know how to read a vernier scale ask your instructor or lab coordinator.

Arrange the Hg source in front of the slit and rotate the telescope until the spectrum comes into view. The eyepiece on the telescope can be adjusted to find a good image of its cross-hair. The telescope tube can then be adjusted to provide a sharp image of the Hg spectrum (colored lines) and the slit on the collimator can be adjusted to provide a narrow but bright image of the slit. If the spectrometer is properly adjusted, the yellow doublet of Hg will be resolved into two distinct yellow lines.

Before beginning to make measurements, rotate the prism until the total angle of deviation is a minimum. Under this condition the rays will go symmetrically through the

prism as shown in Fig. 2; that is, the angle  $\Phi_1$  will equal the angle of  $\Phi_2$ . The total angle of deviation  $\delta$  is equal to  $\Phi_1 + \Phi_2 - \alpha$ . Prism spectrometers are normally used in the minimum deviation configuration because otherwise a divergence or convergence of the incident light causes astigmatism in the final image.

In order to set the prism at the angle of minimum deviation, locate the yellow doublet in the mercury spectrum by rotating the telescope. The prism should be in approximately the position shown in Fig. 1.

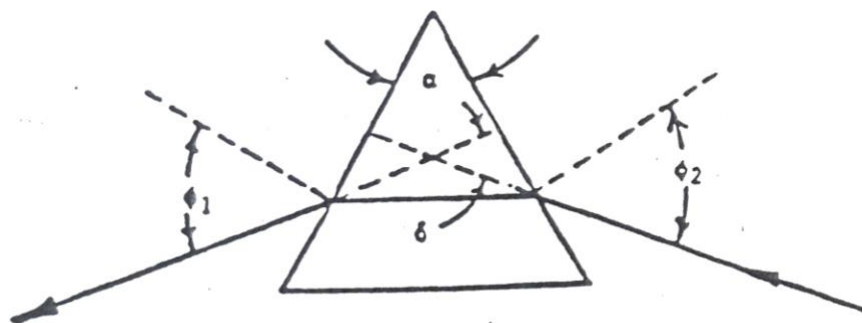


Figure 2: Prism Spectrometer Operated under Condition of Minimum Deviation

With the telescope in focus and the cross-hair set to one of the yellow lines in the Hg spectrum, loosen the prism table lock and rotate the prism (and the telescope, if necessary) until the minimum deviation angle is achieved. That is the smallest angle between the yellow line (telescope) and the optic axis of the collimator tube (again, see Fig. 1). Note that the yellow line can move in two directions in response to the prism rotation in just one direction. You want the yellow line as close to the optic axis as possible to achieve the minimum deviation condition. After you finish this adjustment, lock the prism table and leave it locked throughout the rest of the experiment.

Using the Hg source, carefully read each of the angular positions of the telescope (using the vernier scale) when the cross-hairs in the telescope are positioned on each of the various lines of mercury. You may not be able to see the red line at  $6907\text{\AA}$ . A black cloth over the spectrometer helps in observing the weak lines. **Record the angular positions of all of the mercury lines that you see and make a diagram similar to the one shown in Fig. 3 to help identify the lines with the wavelengths shown in Table 1. The number in parentheses below each spectral line is its relative intensity. These, along with actual color, can be used to identify the lines.**

**Now replace the Hg source with He, and repeat the entire calibrating process (except rotating the prism!!) for He, reading and recording the angles corresponding to the He spectrum given in Table 2. The angular measurements for Hg and He should be converted from degrees and minutes to decimal degrees.**

**Now, take similar angle readings should for the unknown wavelengths in the hydrogen spectrum; record each angle and its corresponding color. Three of its**

spectral lines should be bright and quite apparent; the fourth, the far violet line is typically faint and therefore is somewhat harder to see. It is absolutely necessary to measure the angular position of the lines for the two calibration sources (Hg and He) and hydrogen on the same day. Thus, calibration of the prism and angle readings for the unknown wavelengths of H are taken with no risk that the prism on its table will have been moved, which would necessitate starting over.

**Use the Balmer least-squares analysis Mathematica program to determine a calibration fit using both the Hg and He data sets located in the Scratch folder on the lab computers.** This program will construct a calibration curve based on your data and draw a “best fit” curve through it. It will also list a table of the original data and the deviation in wavelength for each ordered pair, along with the RMS deviation for the entire data set that is to be used in determining the systematic error in the experiment. When you examine the quality of the data, this curve used in conjunction with the table will enable you to correct or eliminate extraneous data, if necessary.

6907	(20)	Red
5791	(100)	yellow
5770	(20)	yellow
5461	(200)	green
4916	(5)	blue-green
4358	(50)	blue
4078	(15)	violet
4047	(30)	violet

*Table 1: Prominent Lines in the Spectrum of Mercury*  
Wavelengths are in Angstroms; numbers in parentheses are relative intensities

7065	(80)	red
6678	(100)	red
5876	(1000)	yellow
5048	(15)	green
5016	(100)	green
4922	(50)	blue-green
4713	(40)	blue
4471	(100)	blue
4388	(30)	blue
4121	(25)	violet
4026	(70)	violet
3965	(50)	violet

*Table 2: Prominent Lines in the Spectrum of Helium*  
Wavelengths are in Angstroms; numbers in parentheses are relative intensities.

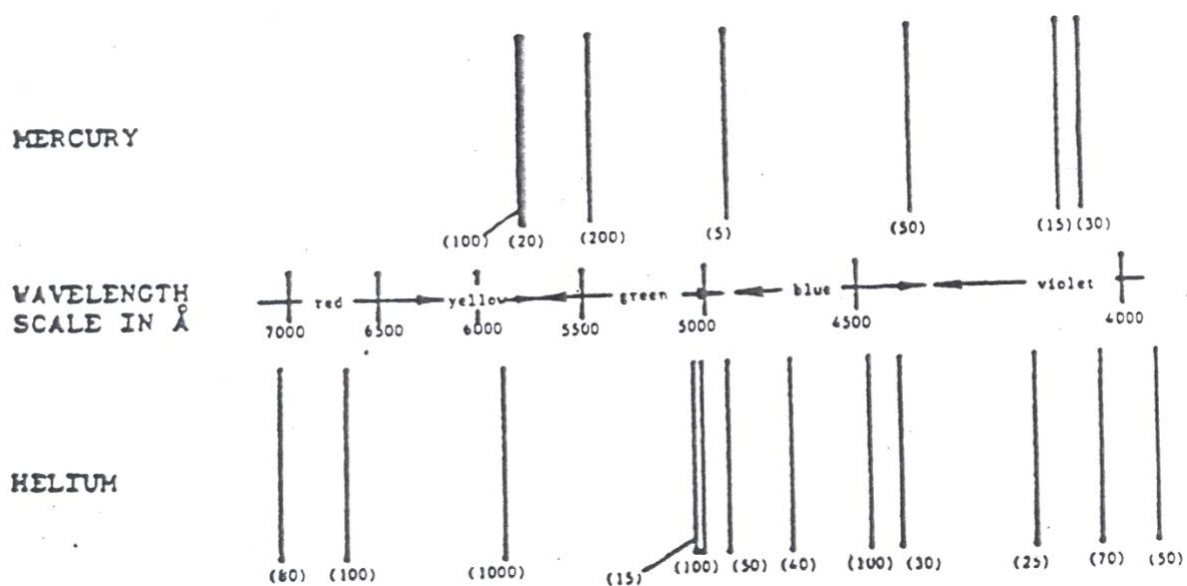


Figure 3: Appearance of Helium and Mercury Spectra in a Prism Spectrometer

Use Eq. (4) to calculate Planck's constant from each of the four H lines of the Balmer Series. Be sure that all the constants used in this equation have at least four significant figures. The average of your four values of Planck's constant should be compared with the standard value.

The random uncertainty in your value of Planck's constant can be approximated from the standard deviation of the mean for your four values. The systematic errors can be estimated from the computer output. Imagine that you obtain from the computer a calibration curve, which, if actually graphed, would look like the graph in Fig. 4.

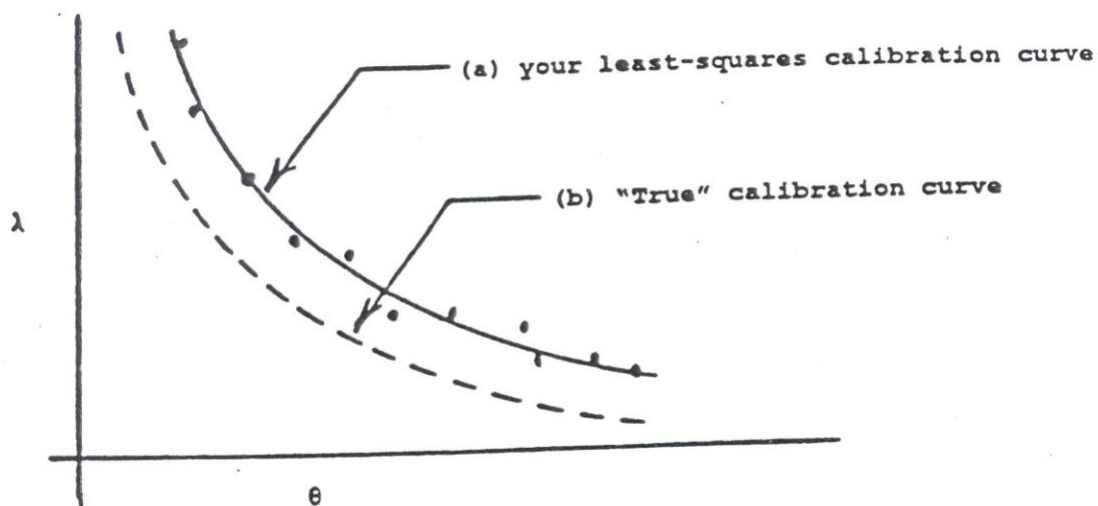


Figure 4: Calibration of the prism spectrometer by a computer least-squares fit program.

It is statistically possible that the curve (a) is not the same as curve (b) because only a limited data set has been fit. Curve (b) is what would be obtained in the limit of fitting a very large number of highly accurate data points. Once the calibration curve is obtained, it can be regarded as an instrument that is used for obtaining  $\lambda$ 's from measured  $\theta$ 's. Thus, if the curve is systematically high or low, its use will produce systematic errors in the wavelengths determined from it. The amount by which the curve may be systematically high or low can be estimated to be about equal to  $\left(\frac{\text{R.M.S. deviation}}{\sqrt{N-5}}\right)$  where the R.M.S. deviation is shown on the computer printout and where  $N$  is the number of calibration points (Hg + He) given to the computer to generate the calibration curve. Note: we use the  $\sqrt{N-5}$  term in the denominator due to the fact that we fit our data to a fourth-order polynomial with five unknown parameters. This restricts the number of "degrees of freedom" we have. If we have four or fewer points to fit, the error should be undefined since we don't have enough information to perform the fitting. The result of this is a slight increase in the estimated systematic error.

**Once the systematic error in the computer-generated wavelengths is thus estimated, calculate the systematic error in your average value of Planck's constant by noting that  $\frac{\Delta h}{h} = \frac{1}{3} \cdot \frac{\Delta \lambda}{\lambda}$ . Derive this last statement and include the derivation in your report.**