Introduction

In the experiment, two different types of measurements will be made with a Michelson interferometer. The first will be a direct measurement of the wavelength of the 6328-Angstrom radiation from a helium-neon laser. The second will be a measurement of the difference in wavelength between the two components of the well-known intense yellow sodium doublet at 5889.9 and 5895.9 Å. Part three is optional, however makes this experiment a "starred" experiment. In this part you will use an interferometer and vacuum cell to determine the index of refraction of air.

The Michelson interferometer is one example of an optical interferometer, including Fabry-Pérot, Mach-Zender, etc... The operation of an interferometer of this type is based upon the division of a light wave into two beams. The Michelson interferometer has been used to compare accurately the wavelength of light with the standard meter. It was also used in the famous Michelson-Morley experiment on the ether drift and in a number of other important experiments. Because of its use in the Michelson-Morley experiment in 1887, many physicists feel that the Michelson interferometer has influenced the course of 20th century physics more than any other instrument.

Before discussing the explicit measurements, the principles and operation of a Michelson interferometer and function of a gas laser will be explained.

Theory and Operation of a Michelson Interferometer

Fig. 1 shows a diagram of a Michelson interferometer. The objects marked M₁ and M₂ are glass mirrors and the objects marked G₁ and G₂ are glass plates of the same thickness which are tilted at the same angle. The back side of plate G₁ is partially silvered so that light coming from the source S is divided into a reflected beam shown as (1) and a transmitted beam shown as (2) of equal intensity. The object L (either a lens or a screen) produces an extended source for certain applications. The reflected beam (1) will be reflected at M₁ and then pass through G₁ a third time before reaching the observer. The
transmitted beam (2) will pass through \( G_2 \), be reflected by \( M_2 \), pass through \( G_2 \) again and reach the observer upon reflection from the back surface of \( G_1 \). The purpose of the compensating plate, \( G_2 \), is to provide for equal path lengths in glass for the two beams. This equality is not needed for observing fringes produced by monochromatic light but is needed if white light fringes are to be observed.

Since the wave fronts/rays 1 and 2 originate from the same source (i.e. they start in phase with each other) and travel over different paths before being recombined, they may interfere constructively or destructively as they form the image seen by the observer.

In order to quantitatively study the phase interference of the two wave fronts/rays consider the setup in figure (2). This setup is somewhat simplified from the Michelson interferometer you will encounter in the lab, however the analysis is the same. The applications in this laboratory involve the observation of circular fringes produced either by spherical wave fronts from a diverging point source or an extended source (sodium light).

This pattern of alternating constructive and destructive interference produced by moving mirror \( M_1 \) is exactly what is seen for the central optical ray passing through the instrument. These circular fringes can be explained for the extended source by reference to Fig. 2, which shows \( M_1 \) and \( M_2 \) as seen at \( E \). \( M_2 \) is the reflected image of \( M_2 \) in the half slivered mirror. If \( t \) is the separation
between \( M_1 \) and \( M_2 \), and \( \theta \) is the angle of observation relative to the central ray, then simple trigonometry shows that the optical path length difference between incident rays 1 and 2 before they reach the observer's eye is:

\[
\text{Path length difference} = \frac{t}{\cos \theta} + \frac{t \cos 2\theta}{\cos \theta} = \frac{t}{\cos \theta} (1 + \cos 2\theta) = 2t \cos \theta. \tag{1}
\]

Since the ray 1 is reflected once from a mirror in air and once from a mirror in glass, and ray 2 is reflected twice from a mirror in air, there will be some additional phase difference in the two wave trains as they enter the eye. Thus the effective optical path length difference is

\[
2t \cos \theta + \epsilon \tag{2}
\]

where \( \epsilon \) is the equivalent path difference due to this extra phase difference. If

\[
2t \cos \theta + \epsilon = n\lambda \tag{3}
\]

where \( \lambda \) is the wavelength and \( n \) is an integer, the waves will interfere constructively and the eye at \( E \) will see a bright spot. Thus, the interference fringes will be circular, for if \( E \) is on the axis of a cone of some-vertical angle \( \theta \), the above equation will hold and a bright ring will be seen. If we should vary the angle \( \theta \) so that
then the wave trains will interfere destructively and a dark ring will be seen. The interference pattern will thus consist of alternatively bright and dark concentric rings.

Let us suppose that we are observing the center of the interference pattern, so the \( \cos i = 1 \). If the mirror \( M_1 \) is moved a small distance \( d \), the optical path of ray 1 is changed by \( 2d \), and there will be a shift in the pattern of interference fringes – that is, the rings will seem to grow out of or shrink into the center. If \( n \) fringes undergo such a shift while \( M_1 \) is moved a distance \( d \), then

\[
n\lambda = 2d
\]

or

\[
\lambda = 2d/n
\]

since we may easily count the number of fringes which pass while the mirror is being moved, and since \( d \) may be measured, Eq. (5) may be used to determine the wavelength of light.

A diagram similar to Fig. 2 may be drawn to show how a projected pattern of interference rings results when a point source of light rather than an extended source is used. In this case, the rings are observed at off-axis locations.

**Determination of the Difference Between Two Wavelengths That Are Nearly Equal with a Michelson Interferometer**

If the two wavelengths of light are present, such as in a sodium light source which has two yellow lines at approximately 5889.9 Å and 5895.9 Å, then the fringe pattern observed in the interferometer consists of the sum of intensities which would be seen using either line alone.

Suppose that the moveable mirror of the interferometer is set so that the two fringe patterns are at the same positions, in which case the two component patterns will superimpose to resemble the pattern of a single wavelength. If the mirror is now moved from this position, fringes will pass by. However, the fringes due to each line move at slightly different rates, so that the agreement between the component patterns will get progressively worse. When they disagree by just \( \frac{1}{2} \) fringe, the two patterns will just cancel each other, and the total pattern will become very fuzzy (depending on the extent to which the two lines are of equal intensity). If the mirror is moved the same distance again, the component patterns will differ by a full fringe, which is indistinguishable from complete agreement, so that the pattern will be again sharp.
If the mirror must be moved a distance $d$ to go from the position where the pattern is sharp to the position where the pattern is again sharp, the number of half-wavelengths traversed differs by just one fringe between the two wavelengths. If the two wavelengths are $\lambda_1$ and $\lambda_2$,

$$2d = n\lambda_1 \text{ and } 2d = (n + 1)\lambda_2$$

where $n$ is an integer. Elimination of $n$ between these two equations yields $2d2d \Delta \lambda = \lambda_1\lambda_2$ where $\Delta \lambda = \lambda_1 - \lambda_2$. With the assumption that the two wavelengths do not differ greatly, so that we can speak of an average wavelength $\bar{\lambda}$ the above equation becomes

$$2d\Delta \lambda = \lambda_1\lambda_2 = \bar{\lambda} - \frac{\Delta \lambda}{2} \bar{\lambda} + \frac{\Delta \lambda}{2} \bar{\lambda} = \bar{\lambda}^2 - \frac{\Delta \lambda}{2} \bar{\lambda} + \frac{\Delta \lambda}{2} \bar{\lambda} - \frac{\Delta \lambda^2}{2} \approx \bar{\lambda}^2$$

or

$$\Delta \lambda \approx \bar{\lambda}^2 / 2d.$$  \hspace{1cm} (6)

**Measuring the Index of Refraction of Air**

In order to measure the index of refraction of air, the pressure in a vacuum cell located in one of the arms of the interferometer will be reduced and the number of fringes will be counted, however in this case the moving mirror will remain fixed as the pressure is varied. Keep in mind that the index in vacuum is exactly 1 and the index of air is typically 1.00026-1.00029, so you will need to be very careful to minimize any additional vibrations and movements that may throw off the counting of the fringes.

Let's assume that the index of air as a function of pressure can be written as,

$$n(p) = 1 + kp$$  \hspace{1cm} (7)

Where $k$ is a constant, to be determined, and $p$ is pressure. Suppose the refractive index in the cell changes by $\Delta n$, which in turn causes the optical path length to change by $2\Delta nL$. $L$ is the length of the vacuum cell which the manufacturer gives as 3.0 cm. (Think: where does the factor of 2 come from?) We can relate the number of $m$ fringe shifts to the change in index with the following,

$$\Delta n = \frac{m\lambda}{2L}$$  \hspace{1cm} (8)

A change in pressure $\Delta p$ causes a change in the index, namely $\Delta n = k\Delta p$, therefore we can write,

$$\Delta p = \frac{\Delta n}{k} = \frac{m\lambda}{2L\Delta p}$$  \hspace{1cm} (9)
Hence, the constant $k$ is,

$$k = \frac{m\lambda}{2L\Delta p}$$  \hfill (10)

Lastly, the index of air can be computed with the following formula,

$$n_{air} = 1 + \frac{m\lambda}{2L\Delta p} p_{room}$$  \hfill (11)

**Theory and Operation of a Helium-Neon (He-Ne) Laser**

A laser is a very special source of monochromatic radiation and the He-Ne gas laser has found a number of important applications in research laboratories and elsewhere. Before becoming directly involved in measuring the wavelength of the orange-red light from the laser, it is instructive to find out how a laser works and what makes its light so special.

Figure 3 models the energy levels of an atom. In the normal state, almost all of the atoms would be in their ground state, $E_1$. If, on the other hand, an intense source of radiation of frequency $f = (E_3 - E_1) / h$ is incident on the atoms, a certain number will absorb this radiation and be excited to level 3. This process called optical pumping can leave more atoms in state 3 than are in the ground state. From level 3 two general types of processes can occur: either they can by spontaneous emission return to the ground state or they can decay (also by spontaneous emission) to a third level (state 2 in Fig. 3). If state 2 is metastable, meaning that the rate of spontaneous emission from state 2 back to that ground state is very small, then the possibility of lasing is present through optical pumping and the subsequent decay. The population of state 2 can build up and remain in that condition for some time. With sufficient pumping, there can be population inversion. That is, there are more atoms in state 2 than there are in the ground state (state 1).
With a population inversion, incident photons of energy $E_2 - E_1$ from some other atom can cause stimulated emission from state 2 to state 1. With stimulated emission, the incident photon will trigger the emission of a second photon and the atom will go from state 2 to state 1. Note that the phase on the emitted photon will be just the same phase as the incident photon, moreover it will also have the same direction, polarization, and energy. Under these conditions the photons are said to be coherent. As stimulated emission takes place from many atoms there will be a build-up of many photons. A laser is usually in a long cylindrical tube with highly reflecting ends so that the photons can surge back and forth in the tube and build up intensity. A small hole in the mirror at one end or a partially reflecting mirror provides a place where the intense, highly collimated, and coherent laser beam can be brought outside the tube and used.

In the particular laser used in this experiment, helium gas at a pressure of about 1 torr is present with neon gas at about 0.1 torr. A gaseous discharge takes place in the gas mixture when sufficiently high voltage is present on the two terminals shown in Fig. 4. In the discharge, some of the neutral helium atoms are excited from the $^1S_0$ ground state to the $^3S_1$ and $^1S_0$ states formed when one of the helium electrons is taken to the next higher orbital (from $1s^2$ to $1s2s$). Both of these excited states of helium are metastable since the downward transitions are forbidden by “radiation selection rules” (see a physics text or perform a Google search for these rules). These states, along with the pertinent states of neon, which has 10 electrons, are shown in the level diagram of Fig. 5. When one of the metastable helium atoms collides with a neon atom in its ground state, there is a high probability that the excitation energy will be transferred to the neon atom leaving it in one of the $^1P$ or $^3P$ states shown. The helium atom in the process returns to its ground state.
Lasing requires that the $2p^54s$ and $2p^55s$ Levels of neon be populated more than the $2p^53p$ levels. Since there is no likely mechanism for population of the 3p states of neon, the excitation in the helium-neon discharge leaves a population inversion between the 5s, 4s, and 3p states. The three transitions shown (6328, 11523, and 11177 Å) are the ones permitted by the selection rules and lasing can take place in any one of them. The particular laser used in this experiment has its cavity arranged to produce primarily the 6328 Å radiation.

Figure 5: Energy Level Diagram for Helium and Neon
**Procedure for Part I: Measuring the Wavelength of Light from a He-Ne Laser**

The 0.5 mW He-Ne gas laser used in this experiment puts out an intense beam of radiation over an area of 2 mm$^2$. While the beam will not harm your skin or clothes you should never look into the beam as eye damage could result. The apparatus is arranged so that the fringes from Michelson interferometer can be seen as they are projected onto a small screen that is shielded from room light, so there is no occasion when you need to look directly into the laser beam.

Before entering the interferometer, the laser beam travels through a device called a spatial filter. This device is basically a focusing lens followed by a pinhole in a metal wafer to remove spurious frequencies from the focused laser beam. It also provides a diverging point source for the interferometer so that a projected circular fringe pattern can be seen.

The laser can be switched on and one can check to see if a useable fringe pattern is present on the observing screen. If no pattern is present, it may mean that either the spatial filter is badly positioned or one of the mirrors is out of alignment. If after some small adjustments you do not see the fringe pattern, it is best to contact your instructor/lab coordinator for help.

There is a one to one scaling of mirror movement to the measurement on the micrometer e.g. measuring 25 marks/divisions off the dial corresponds to 25 micros of mirror travel. The manufacturer states: "...turning the dial clockwise moves the mirror toward the right (looking from the micrometer side); 25 microns per micrometer dial revolution (±1% near center of movement); movement through full distance of travel is linear to within 1.5%." Take note that the same model of interferometer is used for all three parts of the experiment, thus the errors stated here can be used for parts two and three.

After obtaining clear fringes from the laser light, determine the value of $\lambda$ for the He-Ne laser should be made using Eq. (5). It is good practice to measure the distance $d$ over which 100 fringes pass by. The mechanical motion should always be advanced in the same direction in order to avoid effects of backlash in the gears. Ten determinations of 100 fringes each should provide a reasonable measurement of the wavelength of the radiation from the laser. Compare your average value for the 10 determinations and its associated uncertainty with the known value of 6328 Å.

**Procedure for Part II: Measuring the Difference in Wavelength of the Sodium Doublet**

This part of the experiment is done with a different interferometer and with the use of a sodium lamp. The fringe pattern that is observed, when looking directly into the interferometer, should be similar to that seen with the He-Ne laser. As the distance $d$ is changed, the fringes should go through a region of maximum sharpness (high contrast). As you continue changing $d$, the fringes should then become quite fuzzy (low contrast) and then sharp again. Turn on the sodium lamp and allow it to warm up for 10-20 minutes. Also
be sure to turn on the mini-fan and circulate air around the power supply/ballast. If it overheats, it will automatically turn off. **Determine the distance, \(d\), to go from fuzzy fringes to fuzzy fringes.** Eq. (6) can then be used to calculate the wavelength difference \(\Delta \lambda\). Assume a value of \(\lambda\) taken as the average of the two values of \(\lambda\) for the sodium doublet. About six independent determinations of \(\Delta \lambda\) should be made and the average value with its uncertainty compared to the difference between 5889.9 and 5895.9 Å.

**Procedure for Part III: Measuring the Index of Refraction of Air**

*Remember...this part is optional*, but by doing this part you will have completed one of the two required “starred” labs.

Begin by moving the “moving mirror” with the knob to ensure that the interferometer is aligned and you are able to count fringes. If you have trouble aligning the mirrors, contact a lab coordinator or your instructor. The endplates of the vacuum cell need to be perpendicular to the laser beam, so rotate the cell and observe the fringes. Think: how can you be sure the vacuum cell is properly aligned? Be sure that the vacuum cell is at atmospheric pressure by flipping the vacuum release toggle switch. **Record the initial pressure reading on the vacuum pump gauge.** Slowly pump down the air in the vacuum cell and **count the number of fringes and record the final pressure reading on the gauge.** You will need to **record the atmospheric/room pressure**, which can be done on the barometer in the G2B81 bay. This will allow you to compute \(n_{\text{air}}\). **Make 5-10 determinations of \(n_{\text{air}}\) and report your result as \((n_{\text{air}}-1) +/-\) errors.** Essentially, how much does air increase the index of refraction compared to vacuum?

Answer the additional questions for part III:
1) What factors might contribute to changing the index of air, or any gas, other than pressure?
2) In the formula for \(n_{\text{air}}\) we assume the index varies linearly with pressure. How could you test to determine if this indeed the case?
3) Suppose you are asked to measure the index of refraction of a slab of mystery glass. How would you use a Michelson interferometer to do this? (Hint: think about how you can increase or decrease the path length through the slab.)