physicscourses.colorado.edu/phys2150

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Monday, 2-4 pm
Also by appointment
Usually ok without an appointment

Is this your clicker?

- 4A A8 80 62
- 4A B2 AB 53

Propagation of uncertainty - 1

- $y = a_1x_1 \pm a_2x_2 \pm \ldots \pm a_nx_n$

$$\delta y = \sqrt{(a_1\delta x_1)^2 + (a_2\delta x_2)^2 + \cdots}$$

$y = ax^n$ $$\frac{dy}{y} = n\frac{dx}{x} \qquad dy = anx^{n-1}dx$$

$$y = \frac{pqr}{xwz}$$

$$\frac{\delta y}{y} = \sqrt{\left(\frac{\delta p}{p}\right)^2 + \left(\frac{\delta q}{q}\right)^2 + \left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta w}{w}\right)^2 \ldots}$$

General Formula

$$q = q(x, y, z, \ldots)$$

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + \left(\frac{\partial q}{\partial y}\delta y\right)^2 + \cdots}$$

Assumes δx, δy, … are independent

General result: independent errors add in quadrature

Example (text, page 62)

- $x = 200 \pm 2$, $y = 50 \pm 2$, $z = 20 \pm 1$
- $q = x+y-z = 200+50-20 = 230$
- $\delta q = \sqrt{(4+4+1)} = 3$
- $q = 5x+10y-20z = 1000+500-400 = 1100$
- $\delta q = \sqrt{(25\times4+100\times4+400\times1)} = 30$

Example (text, page 62)

- $x = 200 \pm 2$, $y = 50 \pm 2$, $z = 20 \pm 1$
- $q = xy/z = 200 \times 50/20 = 500$
- $\delta x/x = 2/200 = 0.01$
- $\delta y/y = 2/50 = 0.04$
- $\delta z/z = 1/20 = 0.05$
- $\delta q/q = \sqrt{(10^{-4} + 16 \times 10^{-4} + 25 \times 10^{-4})}$
- $\delta q/q = \sqrt{(42 \times 10^{-4})} = 0.065$
- $\delta q = 0.065 \times 500 = 32.4$

Clicker Question

- $y = a_1x_1 \pm a_2x_2 \pm \ldots \pm a_nx_n$

$$\delta y = \sqrt{(a_1\delta x_1)^2 + (a_2\delta x_2)^2 + \cdots}$$

x=100±2, y=250 ± 5, z= 50 ±1
q= 2x + 3y - 4z=200+750-200
δq=
A: 16
B: 5.5
C: 650
D: 25.5

Answer is A

√(16+225+16)=16

A little more complicated

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + \left(\frac{\partial q}{\partial y}\delta y\right)^2 + \cdots}$$

x=12±2 y=8±1

$q = 2x^2 + 3y^3$ =1824

x contribution= 288, uncertainty= 96, 33%
y contribution= 1536, uncertainty= 576, 38%
δq= 584, total uncertainty about 32%

Details on web in class notes

Systematic Errors

- Contributions to the uncertainty that are not improved by averaging
- Often called “Type B” errors in the literature
- Example: calibration of instruments
  - Typically estimated by secondary measurements
- Systematic errors often vary slowly

Calibration Errors

- Compare measuring tool with a standard
- What are the uncertainties in this comparison measurement?
- How do I know that the standard is correct?
  - This is a recursive problem
    - traceability
  - What does it mean for the ultimate standard to be wrong?

Reporting systematic errors

- Report separately:
  - $\delta q_{total} = \delta q_{statistics} \pm \delta q_{systematic}$
- Combine in quadrature:
  - $\delta q_{total} = \sqrt{(\delta q_{stat}^2 + \delta q_{sys}^2)}$
- Table of systematic errors
  - Uncertainties of calibrations

Definition of the kilogram - 1

- Louis XVI in 1791, 1795, 1799
  - Mass of 1 L of water at 4° C
    - Large systematic uncertainties
  - "Kilogram of the Archives" (KA)
    - Platinum "sponge"
- Measurement uncertainty about 2 mg
  - $2\times10^{-3}/10^{3} = 2\times10^{-6}$
- KA is effective legal definition at that time

Definition of the kilogram - 2

- Treaty of the meter – 1885
  - International Prototype Kilogram (IPK)
    - 90% platinum, 10% iridium
    - Mass set equal to KA in 1880
      - Uncertainty 0.015 mg
    - Legal definition in 1889
    - 6 official copies made at the same time
- Kept at the International Bureau of Weights and Measures near Paris
    - www.bipm.org

The troubles - 1

- In 1939 m(KA)= m(IPK) – 0.43 mg
  - What changed?
- In 1946 comparison of IPK with its official copies
  - M(IPK) loses mass with respect to the 6 official copies

The troubles - 2

- Masses of official copies slowly increase because of a film of oil and dirt?
  - Can be minimized but not eliminated
  - Official washing procedure
- After washing, copies of the kilogram *still* disagree
  - 50 μg in about a century
  - $\delta m/m = 50\times10^{-6}/10^{3} = 5\times10^{-8}$
  - Cause is not known

Change in definition

- Definition in terms of fundamental constants: e, m, N_a
- Realization:
  - Compare mg against magnetic force
  - Measure sphere of known density and size
- Realization vs. practical metrology
- Prototype kilograms are not going to disappear

The Bottom Line

- Estimating and removing systematic errors is hard
  - Much harder than random stuff
  - Some sources are never known
- Estimation process has its own uncertainties
- Laboratory notebooks are very important
- It's an imperfect world