Wave nature of matter

Announcements:

• lecture 10 is posted

• homework 6 (*due Feb 25, in class*) solutions are posted on CULearn

• homework 7 (*due March 4, in class*) is posted on CULearn

• reading for this week is:
  o Ch 6 in TZD
Last Time

recall lecture 10:

Problems with classical physics: atomic spectra

• Atomic spectra:

Balmer series of Hydrogen (n→2 transitions):

\[ E_n = -\frac{Z^2e^4m}{2\hbar^2n^2} = -\frac{13.6eV}{n^2}Z^2 \]

• Atomic instability in classical theory

• Bohr’s theory of atomic spectra:
Wave nature of matter

- waves primer
- Young’s double-slit experiment
- electron diffraction Davisson–Germer experiment
- deBroglie matter waves
- wavefunction and its interpretation
- Heisenberg uncertainty principle
Waves primer: basics

- periodic (spatially-temporally extended) disturbance
  
e.g., sound, water, EM waves (in gas, liquid, solid, vacuum)

\[
\mathcal{E} = \mathcal{E}_0 \cos(kx - \omega t) = \mathcal{E}_0 \cos[k(x - vt)]
\]

- frequency: \( \omega = 2\pi v \)
- wavevector: \( k = 2\pi/\lambda \)
- phase velocity: \( \omega = v_p k \)
Interference

- key wave property: *interference*

- constructive
destructive
Mathematics of interference (I)

- wave interference: \( I_{12} = \mathcal{E}_{12}^2 = (\mathcal{E}_1 + \mathcal{E}_2)^2 \)
  
  \[ = \mathcal{E}_1^2 + \mathcal{E}_2^2 + 2\mathcal{E}_1\mathcal{E}_2 \]
  
  \[ = I_1 + I_2 + 2\mathcal{E}_1\mathcal{E}_2 \neq I_1 + I_2 \]

  - adding two phase-shifted waves:

    - **in-phase**
      \[ \mathcal{E}_{12} = 2 \cos(kx) \]
      \[ \mathcal{E}_1 = \cos(kx) \]
      \[ \mathcal{E}_2 = \cos(kx) \]
      \[ I_{12} = 4 \cos^2 kx \]
      \[ = \cos^2 kx + \cos^2 kx + 2 \cos^2 kx \]
      \[ \text{constructive interference} \]

    - **out-of-phase**
      \[ \mathcal{E}_{12} = 0 \]
      \[ \mathcal{E}_1 = \cos(kx + \pi) \]
      \[ \mathcal{E}_2 = \cos(kx) \]
      \[ I_{12} = 0 \]
      \[ = \cos^2 kx + \cos^2 kx - 2 \cos^2 kx \]
      \[ \text{destructive interference} \]
Mathematics of interference (II)

• wave interference: \[ I_{12} = \mathcal{E}_{12}^2 = (\mathcal{E}_1 + \mathcal{E}_2)^2 \]
  \[ = \mathcal{E}_1^2 + \mathcal{E}_2^2 + 2\mathcal{E}_1\mathcal{E}_2 \]
  \[ = I_1 + I_2 + 2\mathcal{E}_1\mathcal{E}_2 \neq I_1 + I_2 \]

  ○ adding two different wavelengths, \( k_1, k_2 \) waves:

\[
\mathcal{E}_{12} = \cos(k_1 x) + \cos(k_2 x) = 2 \cos \left[ \frac{1}{2} (k_1 + k_2) x \right] \cos \left[ \frac{1}{2} (k_1 - k_2) x \right]
\]

**beating phenomena** (tuning piano, FM modulation,...)
Diffraction and refraction

- Bragg diffraction:
  - spectroscopy
  - crystallography:
    - crystals
    - DNA
    - Proteins

Bragg condition: \(2dsin\theta = n\lambda\)

- refraction:
  - rainbow
  - prism
  - lens
Young’s double-slit experiment

wave character of light:

1. A plane wave is incident on the double slit.
2. Waves spread out behind each slit.
3. The waves interfere in the region where they overlap.
4. Bright fringes occur where the antinodal lines intersect the viewing screen.

Pattern produced from a single slit.
Pattern produced from a double slit.

T. Young
1773–1829
Interference applets

http://phet.colorado.edu/new/index.php
deBroglie waves

• particle-wave duality:

  • photon: \[ E = pc = h\nu = \frac{hc}{\lambda} \Rightarrow p = \frac{h}{\lambda} \]

  relativity Planck \( \nu = c/\lambda \)

• particle: true as well (deBroglie, 1924)

Does this relationship apply to all particles? Consider a pitched baseball:

\[ \lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(0.15 \text{ kg})(40 \text{ m/s})} = 1.1 \times 10^{-34} \text{ m} \]

For an electron accelerated through 100 volts: \( v = 5.9 \times 10^6 \text{ m/s} \)

\[ \lambda = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(5.9 \times 10^6 \text{ m/s})} = 1.2 \times 10^{-10} \text{ m} = 0.12 \text{ nm} \]

This is on the order of atomic dimensions and is much shorter than the shortest visible light wavelength of about 390 nm.
Electron diffraction: Davisson-Germer experiment

- wave character of electrons (and all matter) (1927)
  - diffraction of e’s off nickel crystal:

\[
\frac{1}{\lambda} = \frac{n}{2d \sin \theta} = \frac{p}{h} = \frac{\sqrt{2mE}}{h} = \frac{\sqrt{2meV}}{h}
\]

- estimate of e wavelength: 1 Angstrom for 100 eV
- if detect which slit, diffraction pattern disappears
- double-slit experiment with: e, p, n, molecules,…

- electron microscope:
Q: To further prove the de Broglie wave hypothesis, D-G increased the Electron energy. If de Broglie's theory is correct, what will happen?

Bragg condition:

\[ 2d \sin \theta = n \lambda \]

A: (c) Increasing energy increases momentum, which decreases the angle: \( \lambda = h/p \) and \( \theta \approx \lambda/d = h/(pd) \)
“What do you care what other people think?”
Complex numbers

- complex number: $z = x + iy$
  
  - $i = \sqrt{-1} \iff i^2 = -1$
  
  - complex conjugate of $z$: $z^* = x - iy$ (change $i \rightarrow -i$; $i^* = -i$)
  
  - magnitude of $z$: $|z|^2 = z^*z = (x + iy)(x - iy) = x^2 + y^2$ (no cross term)
  
  - $z = x + iy = |z|(\cos \theta + i \sin \theta) = |z|e^{i \theta}$

- analogous to a 2D vector: $z = (x, y)$
deBroglie matter waves and wavefunction interpretation

- familiar waves:
  - sound (pressure wave in a gas): $P(r,t)$ - pressure/molecular displacement
  - water (transverse wave in liquid): $h(r,t)$ - up/down displacement
  - string (transverse wave in a string): $y(s,t)$ - up/down displacement
  - EM wave (E, B oscillating in vacuum): $E(r,t)$, $B(r,t)$ - fields
  
  \[ I = |E|^2 \] - intensity = number of photons per second landing on a unit area probability of photon arrival

- deBroglie matter waves:
  - $\psi(r,t)$ - not physical, complex wavefunction ($\psi$)

  \[ P(r,t) = |\psi(r,t)|^2 \] - probability density of finding a particle at $r$, at time $t$

  Copenhagen interpretation, Max Born (1926)

  \[ \int |\psi(r, t)|^2 d^3r = 1 \] - probability normalization = particle is somewhere

- classical trajectory:

  quantum wavepacket: $\psi(r,t)$
Localization of waves in space

small $\Delta p$ – only one wavelength

medium $\Delta p$ – wave packet made of several waves

large $\Delta p$ – wave packet made of lots of waves
Localization of waves in time

\[ \Delta t \]

small $\Delta E$ – plane-wave in time, only one frequency (energy, E)

\[ \Delta t \]

medium $\Delta E$ – wave packet in time made of several frequency (E) waves

\[ \Delta t \]

large $\Delta E$ – wave packet in time made of lots of frequency (E) waves
**Plane-waves vs wave packets**

- plane wave: \( \Psi(x, t) = A e^{i(kx - \omega t)} \)

- wave packet: \( \Psi(x, t) = \sum_{n=0}^{\infty} A_n e^{i(k_n x - \omega_n t)} \)

Adding several waves of different wavelength together will produce an interference pattern which begins to localize the wave. But that process spreads the momentum values and makes it more uncertain. This is an inherent and inescapable increase.
You are cordially invited to attend an Informational Session to learn more about becoming a Learning Assistant.

**When:** Wednesday, March 9, 2011, at 6 p.m.

**Where:** UMC 235 (hall right of Reception Desk)

**RSVP:** By March 4 to olivia.holzman@colorado.edu

Refreshments will be served, while they last.

Applications for Fall 2011 available March 9 - 23

Goto: http://laprogram.colorado.edu/applications

Get more information from faculty and LAs in these departments:

- Applied Math
- Math
- Mechanical Engineering
- MCDBiology
- Chemistry
- Geological Sciences
- Physics
- Astronomy
- And MORE!!
Heisenberg uncertainty principle

- position-momentum and time-energy:

\[ \Delta x \Delta p > \frac{\hbar}{2} \]
\[ \Delta E \Delta t > \frac{\hbar}{2} \]

The position and momentum cannot both be determined precisely. The more precisely one is determined, the less precisely the other is determined.

- true for all waves, consequence of Fourier transformation
Heisenberg microscope

- **thought-experiment:** *observation of electron's position by a photon gives it a kick and thereby perturbs its momentum*
  - optical resolution: $\Delta x = \lambda \frac{f}{D} = \frac{\lambda}{\text{N.A.}} \approx \frac{\lambda}{\alpha}$
  - momentum kick from photon: $\Delta p \approx \frac{\hbar k}{\alpha} = \frac{\hbar \alpha}{\lambda}$
  - $\Delta x \Delta p > \frac{\hbar}{2}$

Smaller wavelengths allow a better measurement of $x$ but the photons have larger momentum giving larger kicks to the particle, making the momentum more uncertain.
Q: For which type of a wave is the momentum and position most well defined?

**Plane wave:**\[\Psi(x,t) = Ae^{i(kx - \omega t)}\]

**Wave packet:**\[\Psi(x,t) = \sum_{n=0}^{\infty} A_n e^{i(k_n x - \omega_n t)}\]

**Answer:**

a) p is well defined for plane wave, x is well defined for wave packet

b) x is well defined for plane wave, p is well defined for wave packet

c) p is well defined for one, but x is well defined for both

d) p is well defined for both, but x is well defined for one

e) both p and x are well defined for both

A: (a) plane wave has a well-defined wavelength and therefore describes particle with well-defined momentum. However, its position is not well defined as the particle is infinitely delocalized.
Zero-point energy

- $E = p^2/2m + V(x)$

**uncertainty:**  $\Delta x \Delta p > \hbar \quad \iff \quad \Delta p \approx n\hbar / \Delta x$

**cannot settle down to classical energy minimum:**

**zero-point energy:**

$$E \approx \frac{\hbar^2}{2mx^2} + V(x)$$