3220 Exam 1 Cribsheet

Spin ½ systems: $|\pm\rangle_x = \frac{1}{\sqrt{2}}(|+\rangle \pm |-\rangle) \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm 1\end{pmatrix}$

$$|\pm\rangle_y = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle) \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm i\end{pmatrix}$$

$S_x \doteq \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, S_y \doteq \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, S_z \doteq \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, S_n \doteq \frac{\hbar}{2}\begin{pmatrix}\cos\theta & \sin\theta e^{-i\phi}\\ \sin\theta\, e^{i\phi} & -\cos\theta\end{pmatrix}$,

$S^2 \doteq \frac{3\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$. Eigenvectors of S_n are $|+\rangle_n \doteq \begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}e^{i\phi}\end{pmatrix}$, $|-\rangle_n \doteq \begin{pmatrix}\sin\frac{\theta}{2}\\ -\cos\frac{\theta}{2}e^{i\phi}\end{pmatrix}$.

Postulate #4: If you make a quantum measurement on state $|\psi\rangle$ of an operator A the probability of getting one of its eigenvalues a_n as your outcome is $|\langle a_n|\psi\rangle|^2$

Postulate #5: If you make a quantum measurement on state $|\psi\rangle$ of an operator A and get one of its eigenvalues a_n as your outcome, then you "collapse" into state:

$|\psi_{new}\rangle = \frac{P_{a_n}|\psi\rangle}{\sqrt{\langle\psi|P_{a_n}|\psi\rangle}}$, where P_{a_n} (the "projection operator into state a_n") is defined as:

$P_{a_n} = |a_n\rangle\langle a_n|$

Ex: The projection operator onto a state of S_z "spin up" is $P_{+z} = |+\rangle\langle +| \doteq \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$,

Completeness (or "closure") says: $\sum_n P_{a_n} = \mathbf{1}$.

Quantum expectation value $\langle A\rangle = \langle\psi|A|\psi\rangle = \sum_n P_n \,\mathrm{Prob}(a_n)$

Quantum uncertainty: $\Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2}$

QM uncertainty principle: $\Delta A\, \Delta B \geq \frac{1}{2}|\langle[A,B]\rangle|$

Spin 1 systems:

$|1\rangle_x = \frac{1}{2}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle$, $\quad |1\rangle_y = \frac{1}{2}|1\rangle + \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-1\rangle$

$|0\rangle_x = \frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|-1\rangle$, $\quad |0\rangle_y = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|-1\rangle$,

$|-1\rangle_x = \frac{1}{2}|1\rangle - \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle$, $\quad |-1\rangle_y = \frac{1}{2}|1\rangle - \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-1\rangle$

$\hat{S}_x \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$, $\quad \hat{S}_y \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{pmatrix}$, $\quad \hat{S}_z \doteq \hbar\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$