3220 Exam 2 Cribsheet

Spin ½: $|\pm\rangle_x = \frac{1}{\sqrt{2}}(|+\rangle \pm |-\rangle) \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm 1\end{pmatrix}$, $|\pm\rangle_y = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle) \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm i\end{pmatrix}$

$S_x \doteq \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, S_y \doteq \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, S_z \doteq \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, S_n \doteq \frac{\hbar}{2}\begin{pmatrix}\cos\theta & \sin\theta e^{-i\phi}\\ \sin\theta\, e^{i\phi} & -\cos\theta\end{pmatrix}$,

$S^2 \doteq \frac{3\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$. Eigenvectors of S_n are $|+\rangle_n \doteq \begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}e^{i\phi}\end{pmatrix}$, $|-\rangle_n \doteq \begin{pmatrix}\sin\frac{\theta}{2}\\ -\cos\frac{\theta}{2}e^{i\phi}\end{pmatrix}$.

Spin 1 systems:

$|1\rangle_x = \frac{1}{2}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle$, $\quad |1\rangle_y = \frac{1}{2}|1\rangle + \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-1\rangle$

$|0\rangle_x = \frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|-1\rangle$, $\quad |0\rangle_y = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|-1\rangle$,

$|-1\rangle_x = \frac{1}{2}|1\rangle - \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle$, $\quad |-1\rangle_y = \frac{1}{2}|1\rangle - \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-1\rangle$

$\hat{S}_x \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$, $\quad \hat{S}_y \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{pmatrix}$, $\quad \hat{S}_z \doteq \hbar\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$

Postulate #4: If you make a quantum measurement on state $|\psi\rangle$ of an operator A the probability of getting one of its eigenvalues a_n as your outcome is $|\langle a_n|\psi\rangle|^2$

Postulate #5: If you make a quantum measurement on state $|\psi\rangle$ of an operator A and get one of its eigenvalues a_n as your outcome, then you "collapse" into state:

$|\psi_{new}\rangle = \frac{P_{a_n}|\psi\rangle}{\sqrt{\langle\psi|P_{a_n}|\psi\rangle}}$, where P_{a_n} (the "projection operator into state a_n") is defined as:

$P_{a_n} = |a_n\rangle\langle a_n|$

Completeness (or "closure") says $\sum_n P_{a_n} = \mathbf{1}$: , or in continuous basis $\int |a\rangle\langle a|\, da = \mathbf{1}$

Quantum expectation value $\langle A\rangle = \langle\psi|A|\psi\rangle = \sum_n a_n\,\mathrm{Prob}(a_n)$

Quantum uncertainty: $\Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2}$

QM uncertainty principle: $\Delta A\,\Delta B \geq \frac{1}{2}|\langle[A,B]\rangle|$

Postulate #6: $i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle$ (The time-dependent Schrödinger Eqn)

Solution if H is time independent: $|\psi(t)\rangle = \sum_n c_n e^{-\frac{iE_n t}{\hbar}}|E_n\rangle$

Spin commutation relations: $[S_x, S_y] = i\hbar S_z, \quad [S_y, S_z] = i\hbar S_x, \quad [S_z, S_x] = i\hbar S_y$

Energy of a particle in a magnetic field: $\boldsymbol{H} = -\boldsymbol{\mu} \cdot \boldsymbol{B} = -\frac{\boldsymbol{gq}}{\boldsymbol{2m}} \boldsymbol{S} \cdot \boldsymbol{B}$
(where g= gyromagnetic constant, it's about 2 for electrons with q=-e,
and **S** is the spin operator…)

Rabi's Formula: For a particle spin up in a B-field oriented at angle θ from the z-axis in the x-z plane, i.e. $\vec{B} = B_0\hat{z} + B_1\hat{x}$, we define $\omega_{0(1)} = \frac{e}{m} B_{0(1)}$
and find energies $E_{\pm} = \pm\frac{\hbar}{2}\sqrt{\omega_0^2 + \omega_1^2}$

$$\text{Prob(flipping + to −)} = \frac{\omega_1^2}{\omega_0^2+\omega_1^2}\sin^2\left(\frac{\sqrt{\omega_0^2+\omega_1^2}}{2}t\right) = \sin^2\theta\ \sin^2\left(\frac{E_+ - E_-}{2\hbar}t\right)$$

EPR maximally entangled wavefunction is $|\psi\rangle = \frac{1}{\sqrt{2}}(\ |+\rangle_1 |-\rangle_2 \quad - \quad |-\rangle_1 |+\rangle_2\ )$,
which you showed in homework is also $|\psi\rangle = \frac{1}{\sqrt{2}}(\ |+\rangle_{1n} |-\rangle_{2n} \quad - \quad |-\rangle_{1n} |+\rangle_{2n}\ )$
(I.e. this state is anticorrelated in any spin direction $\hat{n}$.)

Spatial wavefunctions and the position representation:

States: $|\psi\rangle \doteq \psi(x) = \langle x|\psi\rangle$
Bras: $\langle\psi| \doteq \psi^*(x) = \langle\psi|x\rangle$
Operators: $\hat{\boldsymbol{x}} \doteq x,\ \hat{\boldsymbol{p}} \doteq -i\hbar\frac{\partial}{\partial x}$
Brackets: $\langle\varphi|\psi\rangle = \int_{-\infty}^{\infty} \varphi^*(x)\,\psi(x)dx$
Probability of finding a particle (a<x<b) = $\int_a^b |\psi(x)|^2 dx$
Expectation values in a state ψ: $\langle A\rangle = \int_{-\infty}^{\infty} \psi^*(x)\hat{A}\,\psi(x)dx$
Schrödinger's time-independent equation: $\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\varphi_n(x) = E_n\,\varphi_n(x)$

Infinite Square well from 0 to a: Allowed energies $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$,
with corresponding normalized energy eigenfunctions $\varphi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$