Notes 4-1

If I have a spin-$\frac{1}{2}$ particle, in, say, $|\psi\rangle = |+\rangle$, I say "the z-component of spin is known". I can predict future outcomes of S_z S-G exp'ts perfectly. But, S_x- and S_y exp'ts are (in this case) not predictable. I do NOT "know" the x + y components of spin.

What if you objected "well, you don't know. But nature does." After all, a measurement of S_x will give one (+ only 1!) outcome. Can't we say the particle knows? (like a hidden coin "knows" if it is heads or tails even if I haven't looked at it yet)

This would be a "hidden variable" theory.

Einstein thought this was reasonable! He felt QM is an incomplete theory - it shows human ignorance of "the elements of reality" (like the value of S_x + S_y) that he thought must be there.

But Einstein was wrong! His theory leads to exp'tal predictions which have been falsified. To see this, we need more complex setups than mere "chained S-G's". Those alone can neither prove nor disprove Einstein. (So, till the 1960's, most people thought this was all just a philosophical debate!)

To show the problem with "local hidden variables", we need to add one new notational element to describe quantum states of two particles. We do this with a "tensor product"

$$\underbrace{|\psi\rangle}_{\text{state of 2-particle system}} = \underbrace{|\alpha\rangle_1}_{\text{particle \#1 is in state } |\alpha\rangle} \otimes \underbrace{|\beta\rangle_2}_{\text{particle 2 is in state } |\beta\rangle}$$

TENSOR PRODUCT

Some people (including McIntyre) drop the $\otimes$ and write

$$|\psi\rangle = |\alpha\rangle_1 \; |\beta\rangle_2.$$

It's just notation, I'll use whichever seems convenient...

Example: Suppose particle #1 is "spin up in z", $|+\rangle$ while #2 has been prepared "spin down", $|-\rangle$.

I write $|\psi\rangle = |+\rangle_1 \; |-\rangle_2$. These are not added, they are tensor multiplied

We are in a larger Hilbert space, denoted $\mathcal{H}_1 \otimes \mathcal{H}_2$.
This example is 4-D, there must be 4 basis states.

- Adding is done within a space.

Tensor multiplication grows you to a larger space.

Notes 4-5

Back to the example, $|\psi\rangle = |+\rangle_1 |-\rangle_2$.

If I measure S_z for particle #1, the opperator is $\hat{S}_{z1}$

Since I'm in an eigenstate of S_z for particle #1 there is a <u>definite</u> outcome

$$\hat{S}_{z_1} |\psi\rangle = \hat{S}_{z_1} |+\rangle_1 \otimes |-\rangle_2 = +\frac{\hbar}{2} |+\rangle_1 \otimes |-\rangle_2 = +\frac{\hbar}{2} |\psi\rangle$$

so $|\psi\rangle$ is an eigenstate of S_{z1}, I <u>know</u> the z spin of #1 is up!

Similarly,

$\hat{S}_{z_2} |\psi\rangle = -\frac{\hbar}{2} |\psi\rangle$, <u>this</u> 2 particle <u>state</u> is an eigenstate of $\hat{S}_{z1}$ and $\hat{S}_{z2}$.

We might <u>further</u> simplify our notation here, e.g

Some people say $|\psi\rangle = |+,-\rangle$ or $|\uparrow\downarrow\rangle$

$+$: state of #1; $-$: state of #2.

In "<u>quantum computing</u>", spin up is denoted $|0\rangle$
spin down $|1\rangle$, so
this state would be $|\psi\rangle = |01\rangle$

(<u>Could</u> be spin, or could be <u>any</u> 2-valued system, like photon polarization, energy, etc.)

$\boxed{\text{Inner products}}$ behave as you expect:

$$\langle\psi|\psi\rangle = {}_1\langle +|\,{}_2\langle -|\,\big|\,|+\rangle_1 |-\rangle_2\rangle = ({}_1\langle +|+\rangle_1)({}_2\langle -|-\rangle_2)$$

$$= 1 \cdot 1 = 1 \checkmark$$

Some basic tensor product rules, just to be formal about it:

If $\mathcal{H}_1$ and $\mathcal{H}_2$ are Hilbert spaces with dimensions n, m

then $\mathcal{H}_1 \otimes \mathcal{H}_2$ is a combined " " " " $n \times m$.

<u>Basis</u> states are $|b_i\rangle \otimes |c_j\rangle$ with $|b_i\rangle$ any basis in $\mathcal{H}_1$
$|c_j\rangle$ " " " $\mathcal{H}_2$.

<u>Constants</u>: $c\left(|\psi_1\rangle \otimes |\psi_2\rangle\right) = \left(c\,|\psi_1\rangle\right) \otimes |\psi_2\rangle$
$= |\psi_1\rangle \otimes \left(c\,|\psi_2\rangle\right)$

<u>Superpositions</u>: if $|\psi_1\rangle, |\varphi_1\rangle$ live in $\mathcal{H}_1$
$|\psi_2\rangle, |\varphi_2\rangle$ " " $\mathcal{H}_2$, then

$$\underbrace{\left(|\psi_1\rangle + |\varphi_1\rangle\right)}_{\text{still in } \mathcal{H}_1!} \otimes \left(|\psi_2\rangle\right) = |\psi_1\rangle \otimes |\psi_2\rangle + |\varphi_1\rangle \otimes |\psi_2\rangle$$

and $|\psi_1\rangle \otimes \left(|\psi_2\rangle + |\varphi_2\rangle\right) = |\psi_1\rangle \otimes |\psi_2\rangle + |\psi_1\rangle \otimes |\varphi_2\rangle$

It makes no sense to write $|\psi_1\rangle + |\psi_2\rangle$, they live in different spaces!

$|\psi_1\rangle|\varphi_1\rangle + |\psi_2\rangle|\varphi_2\rangle$ is <u>fine</u>, <u>each</u> lives in $\mathcal{H}_1 \otimes \mathcal{H}_2$!

Matrices:

$$\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a \begin{pmatrix} c \\ d \end{pmatrix} \\ b \begin{pmatrix} c \\ d \end{pmatrix} \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$$

2-d $\otimes$ 2-d = (4-d!)

Here's the cool part. There are 2-particle states that can <u>not</u> be written as $|\psi\rangle_1 \otimes |\psi\rangle_2$. They are "<u>entangled</u>"

<u>Example</u>: Consider $|\psi\rangle = \frac{1}{\sqrt{3}}\left(|-\rangle_1|-\rangle_2 + |-\rangle_1|+\rangle_2 + |+\rangle_1|-\rangle_2\right)$

$$= \frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right)$$

Just to simplify / clean up notation.

$$= \frac{1}{\sqrt{3}}\left(|-,-\rangle + |-,+\rangle + |+,-\rangle\right)$$

or in quantum computing $= \frac{1}{\sqrt{3}}\left(|11\rangle + |10\rangle + |01\rangle\right)$

Q1) What is the probability in state $|\psi\rangle$ that #1 is <u>spin up</u>?

A1) It's $\frac{1}{3}$. (<u>only</u> the last term contributes)

Q2) What's the prob. that #2 is spin up?

A2) Also $\frac{1}{3}$ (only the middle term contributes)

Q3) Prob #1 is $\downarrow$ <u>and</u> #2 is $\uparrow$? A3) $\frac{1}{3}$

Q4) Prob <u>both</u> are $\uparrow$? A4) 0

Q5) Prob they are <u>same spin</u> state? A5) $\frac{1}{3}$

Q6) " " " opposite? A6) $\frac{2}{3}$.

To think about: What is the state of #1?

It's not easy; it's <u>not</u> $\sqrt{\frac{1}{3}}\,|\uparrow\rangle_1 + \sqrt{\frac{2}{3}}\,|\downarrow\rangle_1$!!

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right)$$

<u>If</u> you thought #1 is in state $\frac{1}{\sqrt{3}}|\uparrow\rangle + \sqrt{\frac{2}{3}}|\downarrow\rangle$

then you'd surely think #2 is $\frac{1}{\sqrt{3}}|\uparrow\rangle + \sqrt{\frac{2}{3}}|\downarrow\rangle$ <u>also</u>

then $|1\rangle \otimes |2\rangle \overset{?}{=} \left(\frac{1}{\sqrt{3}}|\uparrow\rangle_1 + \sqrt{\frac{2}{3}}|\downarrow\rangle_1\right) \otimes \left(\frac{1}{\sqrt{3}}|\uparrow\rangle_2 + \sqrt{\frac{2}{3}}|\downarrow\rangle_2\right)$

$$= \left(\frac{1}{3}|\uparrow\uparrow\right) + \sqrt{\frac{2}{9}}|\downarrow\uparrow\rangle + \sqrt{\frac{2}{9}}|\uparrow\downarrow\rangle + \frac{2}{3}|\downarrow\downarrow\rangle$$

Nope! That's not right!

Here's <u>another</u> counter argument:

If you think the above "single particle" states are right,

then you'd think Prob(#1 is up <u>and</u> #2 is down) $= \left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{2}{9}$

(But look @ Q3 on the previous page, it's $1/3$!)

The 2 objects are <u>entangled</u>. The states are connected,

they cannot be considered separate independent particles.

Entangled states are highly corre<u>lated</u>, in ways that

are fundamentally <u>NOT</u> classical, as we shall see next!

– you <u>can</u> <u>not</u> write the state of #1 as a single "ket" in $\mathcal{H}_1$.

Consider $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

This is a "maximally entangled" 2-state system, a "Bell-state"

Other MAXIMALLY entangled states are $\frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$

$\frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle$

and $\frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$

Our $|\psi\rangle$ can be produced in many ways. E.g.

spin 0 particle, at rest

$\pi^0 \longrightarrow e^+ e^-$. By conservation of $\vec{p}$, these fly off back to back.

Conservation of $\vec{L}$ says total spin $=0$, so must have $|\uparrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle$ only. The relative $-$ sign is not obvious, (if it was a $+$ sign, we'd have $\vec{L}_{tot} = 1$) but is provable after ch. 6...

Given a pair $|\psi\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$

Q1) If you measure $\hat{S}_{z_1}$, what's Prob (up)? A1) $1/2$

Does that mean #1 is $\frac{1}{\sqrt{2}}|\uparrow\rangle_1 - \frac{1}{\sqrt{2}}|\downarrow\rangle_1$? No !

If it were, then so is #2, + so I would expect by chance to get #1 up and #2 up one quarter of the time.

But no, our state is entangled, Prob (both up) $= 0$!

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

If #1 measures $\uparrow$, then I guarantee #2 measures $\downarrow$.

Apparently, measurement of #1 alone collapses this superposition state, fixing the spin of #2. This is "spooky action at a distance"! #1 & #2 could be very far apart when the measurements are made! But this collapse is apparently instantaneous, because if #1 is up, #2 is 100% guaranteed down + vice versa!

Note: I would not say observer #1 "causes" result #2. observer #1 cannot make result #2 be what she wants. #1 sees a random outcome. Only after the fact when #1 + 2 get together to compare results do they notice the perfect anti-correlation.

("FTL")
There is no "Faster than Light causation" going on.
There is no mechanism for #1 to send a message (instantly) to #2.

Still, there is FTL correlation, or collapse, going on.

Einstein thought this up in 1935, + published with Podolsky + Rosen

Einstein's "EPR" paper basically said "See! I was right!
This anticorrelation proves that #1 + #2 must each "know" in advance what their S_z measurement will be. They must each contain a secret "hidden S_z variable" to tell them what result they will have, like a coin .... otherwise, this instantaneous collapse/correlation would happen (FTL... no way!?)

He rejected spooky action at a distance, + said there must be hidden (local) elements of reality in the spins, made when they first formed + preserved till the measurement.

Others (like Bohr followers) rejected hidden elements of reality and accepted FTL correlations. (It's not causality, so it may be spooky but does not violated special relativity!)

Einstein believed in "locality": no experiment "far away" can have any relevance "over here".

For years, this seemed to be a philosophical debate....

Until 1964, when along comes Bell....

Let's investigate Einstein's stance a bit more, because what he's claiming really is hard to reconcile with a quantum perspective...

Given our $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$, we've seen S_z measurements are <u>perfectly</u> (100%) anti-correlated. What about $\underline{\underline{S_x}}$?

<u>Claim</u>: those are <u>also</u> 100% " - " !

<u>Proof</u>: $|\uparrow\rangle_x = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$ and $|\downarrow\rangle_x = \frac{1}{\sqrt{2}}|\uparrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\rangle$

This means $|\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_x + |\downarrow\rangle_x\right)$ and $|\downarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_x - |\downarrow\rangle_x\right)$

(convince yourself!) Let's write $|\psi\rangle$ out <u>carefully</u> now...

$$\text{so } |\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_{x1} + |\downarrow\rangle_{x1}\right) \otimes \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_{x2} - |\downarrow\rangle_{x2}\right) - \left(\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_{x1} - |\downarrow\rangle_{x1}\right) \otimes \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_{x2} + |\downarrow\rangle_{x2}\right)\right)\right)$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\left(|\uparrow_x\uparrow_x\rangle + |\downarrow_x\uparrow_x\rangle - |\uparrow_x\downarrow_x\rangle - |\downarrow_x\downarrow_x\rangle - \left(\underset{\text{cancels}}{|\uparrow_x\uparrow_x\rangle} - \underset{\text{adds}}{|\downarrow_x\uparrow_x\rangle} + \underset{\text{adds}}{|\uparrow_x\downarrow_x\rangle} - \underset{\text{cancels}}{|\downarrow_x\downarrow_x\rangle}\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left(|\downarrow_x\uparrow_x\rangle - |\uparrow_x\downarrow_x\rangle\right)$$

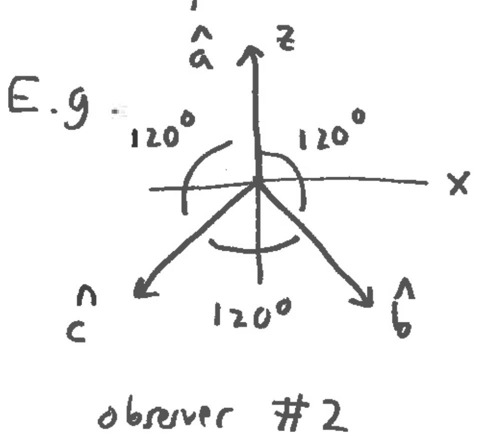
Ooh! I see, this still is 50/50 "up/down" anti-correlated in x. Makes <u>sense</u>, we started with spin = 0, so no matter <u>what</u> direction you measure spin, they <u>better</u> give opposite results.

So Einstein is forced to say that not only is S_z "outcome" hidden in each particle, but S_x + S_y as well!! How else can measuring S_x far away instantly collapse the outcome of S_x here? So Einstein says sure... particles have all elements of reality, there is a pre-determined value for S_x, S_y, + S_z measurements. Maybe we don't know all 3, + maybe there's a clever reason we can't know them... but they must be there, hidden, to avoid spooky action at a distance. And along comes Bell...

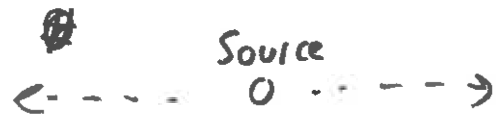
---

what if observer #1 measure $\hat{n}$, + #2 measures $\hat{n}'$.

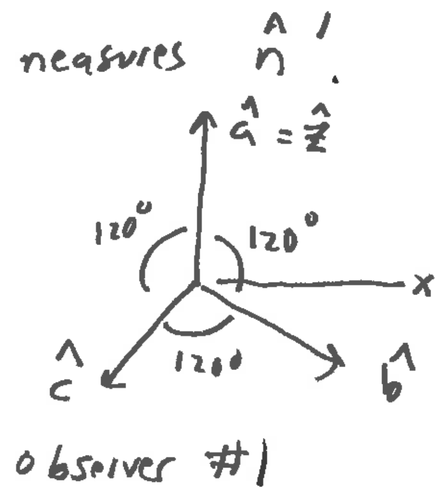
E.g.



observer #2



[we pick $\hat{n}$ or $\hat{n}'$ to all lie in the x-z plane, + choose one of those 3 axis]



observer #1

So each observer chooses for herself, at the last second, to measure S_a, or S_b, or S_c. Each individual result is of course $\pm \hbar/2$!

↑ this is S_z

↑ this is S (in the b direction)

So if #1 chooses ("whatever") + #2 chooses ("whatever")
they will later get together, + simply write down
+ $\equiv$ we got the same result (__both__ $+\hbar/2$, or __both__ $-\hbar/2$)
$-$ $\equiv$ " " opposite " s (__one__ got $+\hbar/2$, other $-\hbar/2$)

So we have a string of events, all + (same) or $-$ (different).
__If__ we had not switched up the axes, if e.g. __both__ always chose
axis a, then we know __all__ outcomes will be "$-$" $\equiv$ "opposite".
__True__ if both choose b, or both choose c. But if #1 picks
a + #2 pick b, then we __might__ start seeing some "+" = "sames"...

Question: After __many__ tries, what __fraction__ of these outcomes
will be "same"?

We will __compute__ this fraction! First assuming rules of QM,
but then assuming Einstein is right + there are "hidden variables"
telling what a, b, + c measurements __would__ give __in__ advance.
we will find the answers are __irreconcilable__, + QM is
__experimentally__ correct (these exp'ts have been done).
+ there __is__
spooky

Quantum analysis.

Recall $|+\rangle_n = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2} e^{i\varphi}|-\rangle$

we have $\theta = 0$, or $120^\circ \rightarrow$ with $\varphi = 0$ or π.

(Actually, $\theta = -120^\circ$ with $\varphi = 0$ is same state)

our starting state is $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

#1 measures in some direction, which I will use to define the z-axis

then #2 will randomly measure in that ($\theta=0$), or $\theta = \pm 120^\circ$....

Prob (#1 measures $+\hbar/2$ and #2 measures $+\hbar/2$ in their (θ) direction)

$$= \left|{}_1\langle +| \otimes {}_{2n}\langle +|\Psi\rangle\right|^2$$

$$= \left|{}_1\langle + \otimes \left({}_2\langle +|\cos\frac{\theta}{2} + {}_2\langle -|\sin\frac{\theta}{2}\right) \Big| \frac{1}{\sqrt{2}}\left(|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2\right)\right|^2$$

$$= \frac{1}{2}\left| {}_1\langle +|+\rangle_1 \left({}_2\langle +|\cos\frac{\theta}{2} + {}_2\langle -|\sin\frac{\theta}{2}\right)|-\rangle_2 - \underbrace{{}_1\langle +|-\rangle_1}_{\text{zero}} \underbrace{(\;\cdots\;)}_{\text{who cares}}\right|^2 \Rightarrow$$

$$= \frac{1}{2}\left|\sin\frac{\theta}{2}\right|^2$$ Simple enough!

$\boxed{\text{I need to recall } |-\rangle_n = \sin\frac{\theta}{2}|+\rangle - \cos\frac{\theta}{2}|-\rangle}$

Prob (#1 gets $-\hbar/2$ and #2 gets $-\hbar/2$)

$$= \left|{}_1\langle -|\,{}_{2n}\langle -|\Psi\rangle\right|^2 = \left|{}_1\langle -| \otimes \left({}_2\langle +|\sin\frac{\theta}{2} - {}_2\langle -|\cos\frac{\theta}{2}\right)\right| \frac{1}{\sqrt{2}}\left(|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2\right)\Big|^2$$

$$= \frac{1}{2}\left| \underbrace{{}_1\langle -|+\rangle_1}_{0!} \underbrace{(\;\cdots\;)}_{\text{who cares}} - {}_1\langle -|-\rangle_1 \left({}_2\langle +|\sin\frac{\theta}{2} - {}_2\langle -|\cos\frac{\theta}{2}\right)|+\rangle_2\right)\Big|^2$$

$$= \frac{1}{2}\left|\sin\frac{\theta}{2}\right|^2 = \frac{1}{2}\sin^2\frac{\theta}{2}$$ also.

so Prob (both are <u>same</u> outcome) $= \frac{1}{2}\sin^2\frac{\theta}{2} + \frac{1}{2}\sin^2\frac{\theta}{2}$.

$= \sin^2\frac{\theta}{2}$.

Now, 1/3 of the time, on average, $\theta = 0$ giving Prob(same) = 0

1/3 of the time $\theta = 120°$, $\sin^2 60 = 3/4$ = Prob (same)

1/3 " " " $\theta = -120°$, $\sin^2 -60 = 3/4$ = " "

So <u>total</u> Prob (same) $= \frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot 0 = \frac{1}{2}$

Ah! QM predicts that Prob (same) = Prob(opposite) = 1/2.

This is a crisp + unambiguous statistical prediction.

Now, <u>let's do the "Einstein / Hidden local variable" analysis</u>

I'm assuming when each pair is made, the outcome for any future measurement is a "secret instruction" to the particles.

E.g, particle #1 might be encoded with the hidden instructions $(a+, b+, c-)$ which means <u>If</u> we measure "$\hat{a}$" the answer will be $+\frac{\hbar}{2}$ <u>if</u> we measure "$\hat{b}$" we get $+\hbar/2$, <u>if</u> we measure "$\hat{c}$" we get $-\hbar/2$.

you and I may not know all 3 outcomes results simultaneously, (The particle doesn't <u>share</u> this instruction set with us, + we only

This is Einstein's position. It's the only way he knows to guarantee that #1 + #2 can be perfectly anti-correlated, 100% guaranteed, whenever both measurements happen to "match" (both a, or b, or c)

There are only 8 possible hidden instruction sets!

For particle #1 they are			For particle #2 they must match, to ensure anti-correlation
1st possibility:	$(a+, b+, c+)$	$\longleftrightarrow$	$(a-, b-, c-)$
2nd :	$(a+, b+, c-)$	$\longleftrightarrow$	$(a-, b-, c+)$
3rd :	$(a^+, b-, c+)$	$\longleftrightarrow$	$(a-, b+, c-)$
#4 :	$(a+, b-, c-)$	$\longleftrightarrow$	$(a-, b+, c+)$
#5 :	$(a-, b+, c+)$	$\longleftrightarrow$	⋮
#6 :	$(a-, b+, c-)$		
#7 :	$(a-, b-, c+)$		
#8 :	$(a-, b-, c-)$		

etc. Whatever #1's set is, it must be that #2's is opposite for each outcome.

If the above was not true, there's no way without *instantaneous, non-local communication, that I could guarantee that if #1 measures, say, $\hat{a}$ + #2 does also, that they must be opposite.

- These 8 possibilities may not be equally likely
- But, there are no other possibilities!

For set #1 $\overset{\#1}{(a+, b+, c+)} \longleftrightarrow \overset{\#2}{(a-, b-, c-)}$

an interesting thing happens. Whatever orientation #1 chooses to measure, and whatever orientation #2 choose, they get + and −, i.e. any combo of measurements gives "opposite" as the outcome.

Same for set #8, now #1 gets "−" + #2 gets "+", ⇒ opposite.

So for "#1" + "#8" situations, Prob (opp) = 1
Prob (same) = 0.

For set #2 $\overset{\#1}{(a+, b+, c-)}$ and $\overset{\#2}{(a-, b-, c+)}$

there are 9 equally likely (randomly selected) detector combinations.

	#1 measures	#2 measures	⇒ outcome
*	a,	a	⇒ + and − = opposite
	a,	b	⇒ + and − = opposite
	a,	c	⇒ + and + = same
	b,	a	⇒ + − = opposite
*	b,	b	⇒ +, − = opposite
	b,	c	⇒ +, + = same
	c,	a	⇒ −, − = same
	c,	b	⇒ −, − = same
*	c,	c	⇒ −, + = opposite

(the * cases are where both measure same thing, so of course they are opp)

of the 9 equally likely setups, 5 come out opposite, 4 are same

That's it. Orientation of S-G is not "particle property", it's our

Do this for "instruction sets" 3, 4, 5, + 6.
you will find All of them also give $\text{Prob (opp)} = 5/9$.
(List 'em out) Nobody says instruction sets are equally likely.
That's unknown physics. Still, there are exactly 8 instruction sets
If you run many times, let's say that N_1 times the instruction set was #1, N_2 times it was #2, etc.

Then Prob (Same) $= \dfrac{N_1 \cdot 0 + N_8 \cdot 0 + (N_2 + N_3 + \dots + N_7) \cdot \frac{4}{9}}{(N_1 + N_2 + N_3 + N_4 + \dots N_8)}$

$\leq 4/9$ No matter what $N_1 - N_8$ are!

and Prob (Diff) $= \dfrac{N_1 \cdot 1 + N_1 \cdot 1 + (N_2 + \dots + N_7) \cdot 5/9}{(N_1 + N_2 + \dots)}$

$\geq 5/9$.

This is a "Bell inequality". We do not need to know the "hidden physics", we don't need to know the various likelihoods the mere existence of instruction sets ("hidden variables") is all we needed, along with the requirement of perfect anticorrelation whenever measurement axes happen to align, that gives us Prob (same) $\leq 4/9$.

Aside:

Somehow this outcome is not weird, it makes sense.

Suppose e.g. that those instruction sets 1-8 were all equally likely.

then

① +++ / - - - opposite is always more likely than same.

② ++- / - - + the anticorrelation forces this on us,

③ +-+ / -+- there are always either 9/9 opposites or 5/9 opposites.

⋮

Note If we only measured two orientations, a and b,

++ / - - all are opposite

+- / -+ half the possible pairings are opposite !

-+ / +- " " " " " " !

- - / ++ all are opposite.

so if nature created instruction sets that weren't "all possible", but only those middle situations occurred... then we'd have no disagreement with the 50/50 prediction of QM

So we need this funky "3-options" measurement setup of Bell to show that there's NO possible distribution of instruction sets that yields anything except

Prob(same) $\leq 4/9$.

Bottom line: Running these exp'ts and computing Prob(same)

- Quantum predicts Prob(same) = .5000
- Hidden local variables/EPR predict Prob(Same) $\leq 4/9 = .4444\ldots$

- Exp'ts give ... drum roll ... .50.

This is not $\leq 4/9$, so local hidden variables are excluded.

Entangled states show that particles in QM do not "know" what the outcome of a future measurement will be (or least, not for more than one axis) It is not "hidden", the measurement is truly random + unpredictable. At the same time, perfect anti-correlation of matching measurements confirms that wave function collapse is instantaneous across arbitrary distances. Measuring #1 does "collapse" #2.

It is spooky action at a distance
It does not violate relativity - no information is sent from #1 to #2, no event @ location #2 is (or can be) caused by #1's actions.

- This hurts my brain, but it's how nature is.

Schrödinger's Cat:

we put a cat in a chamber + allow a quantum system with 50% chance of occurance to determine whether a container of cyanide is opened, after 1 hr.

$$|\Psi(1\,hr)\rangle = \frac{1}{\sqrt{2}}\left(|alive\rangle + |dead\rangle\right)$$

It is not "alive or dead", it is alive and dead.
This is not a mixed state, it's a superposition. (or so it appears)

If you measure the state, it chooses one or the other, 50/50 odds. (Did you kill it by looking, then?)

This seems to be pure nonsense. Macro systems in superpositions violate our sense of physics...

The resolution is that "looking" is not what consitutes a measurement! Interaction with the universe (air in the room bounces differently off live + dead cats, countless irreversable consequences to the env't when a cat dies) is what is "measuring" the state, continuously. The Buzzword here is "decoherence":
a quantum superposition state requires a delicate + definite

phase relationship between orthogonal states. The more interactions you have, the harder it is to maintain unambiguous phase coherence. In a macroscopic / thermal environment, all phase coherence is rapidly lost, + the system is indeed a MIXED state:
50% chance of finding a live cat, 50% chance it's dead, but there was a state (one or the other) before you opened the box.

What exactly constitutes "measurement", + how "macro" the system can get + still maintain coherence are open + interesting questions in modern research labs.