Physics 3310: Math Diagnostic and First Homework.

YOUR NAME (neatly!)_____

IMPORTANT, PLEASE READ THIS FIRST!!

I'd like you to do this homework twice.

This first version (GREEN COPY) IS A PRETEST, due tomorrow (Tues) in my mailbox (left of the physics office). This is not for a grade!

Please answer these questions *without looking anything up*.
Please, no talking to friends, not even opening a book, browser or notes!!!
Don't stress about it, just show me what you can do entirely by yourself.
Again, you will not be graded on this version.

The OTHER VERSION (WHITE COPY) is a regular homework (due Wed, start of class) For that version, use any books, notes, talk to friends, come to office hours...

NOTE: No need to redo any questions you are confident about - just write down "*see green copy*" on any problems you want!

(If you run out of room, just attach some extra pages.)

Please do not "back fill" *anything* onto this (green version) once you've gotten any help!!

Please *show your work* or explain your reasoning whenever possible.
(In general, we will grade homeworks for clarity of explanation as much as we do for mere "correctness of final answer"!)

1.) Given a triangle (NOT necessarily a "right triangle") with sides a, b, and c, and an angle (θ) opposite side c. Suppose I tell you a, b, and θ, and ask "what is c?"
-> do you know a formula? (what is it?)

Remember, *please* do not look ANYTHING up (no calculators that integrate). If you don't know how to do an integral without a table, just say so. If you KNOW you could easily do it by using a resource, let us know, but don't do so (until the "second pass", white version, of this homework!!)

2.) $\int \frac{4x}{\left(x^2+a^2\right)^{3/2}}\,dx$ (where a is a known constant. Note that it is an indefinite integral)

3.) $\frac{d}{dx}\int_1^x f(y)dy$ (where f(y) is some given, known (well behaved) function of y)

4.) $\frac{d}{dx}\int_0^1 (y+x)\,dy$

Make a quick sketch, in the x-y plane, of the following vector functions.
Plot enough different vectors to get a feeling for what this field looks like in the x-y plane.

5.)(a) $y\,\hat{\mathbf{x}}$

(b) $r\,\hat{\mathbf{r}}$ (The symbol **r** here refers to the usual **r** in spherical coordinates)

(c) $\frac{x}{\sqrt{x^2+y^2}}\hat{\mathbf{x}}+\frac{y}{\sqrt{x^2+y^2}}\hat{\mathbf{y}}$

Also just for this one (part c) - can you explain in words what this plot is showing?

6.) Given the scalar function T(x,y,z) (e.g. the temperature at any point in the room)
Which of the three operations (div, grad, or curl) can be sensibly operated on T?
For each which can:
a) give a formula for the result
b) explain in words how you would interpret the result.
c) is the result a vector or scalar?

7.) Given an arbitrary vector function **V**(x,y,z) (e.g. the velocity of a flowing liquid)
Which of the three operations (div, grad, or curl) can be sensibly operated on **V**?
For each which can:
a) give a formula for the result
b) explain in words how you would interpret the result.
c) is the result a vector or scalar?

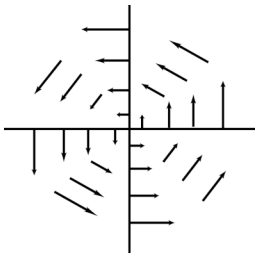
8.) For each of the four vector fields sketched below....

Which of them have nonzero *divergence* somewhere? _____
(If the divergence is nonzero *only* at isolated points, which point(s) would that be?)

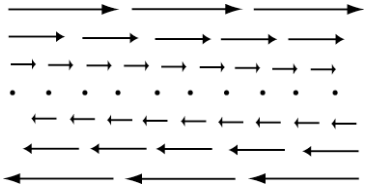
Which of the following fields have nonzero *curl* somewhere? _____
(If the curl is nonzero *only* at isolated points, which point(s) would that be?)

(A brief explanation of your answers below each figure would be welcome)

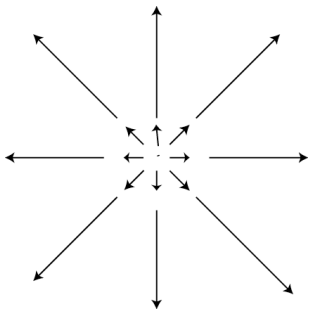
A.



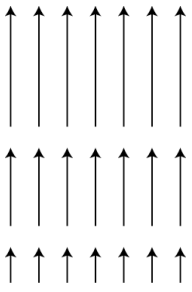
B



C.



D.



9.) Given vectors **A** and **B**. ("Given" means you know the components, or alternatively, the length and angle of the vectors)
Define the dot product mathematically in two very different looking ways.
(Hint: one way should involve the components, the other, the length/angles)

Give a brief physical interpretation of what the dot product means or tells you (you can give a concrete example if you like)

10.) Define the vector cross product mathematically in two very different looking ways.
(Hint: one way should involve the components, the other, the length/angles)

Give a brief physical interpretation of what the cross product means or tells you (you can give a concrete example if you like)

11.) Can you compute the gradient of the following two scalar fields?

a) $e^x \cos(y)$

b) $\cos(x^2 + y^2 + z^2)$

12.) Compute the divergence and curl of $\hat{\imath}(x^2 + yz) + \hat{\jmath}(y^2 + zx) + \hat{k}(z^2 + xy)$.

All remaining questions are just for my information, you will learn about them in this class, so you do NOT have to "re do" them for the homework version if you don't already know them. (Though, if you DO know them, but would just need to look something up quickly to answer them, please let me know below!)

13.) In Phys 1120, one of the early chapters was on Gauss' law, one of the most fundamental laws of electricity. It looks like this: $\oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\ dA = q(\text{enclosed})/\varepsilon_0$,

where **E** is the electric field, S is a closed surface, **n** is a unit vector which points everywhere *outward* from the surface.

a) Is this formula familiar to you? (You just have to answer yes, no, barely...)

b) Explain briefly in words what this formula says, and how/when the formula might be used.

c) Suppose I evenly fill a cube (length L on a side) with electric charges. I then imagine a larger, closed cubical surface neatly surrounding this cube (length 2L on a side)

Is Gauss' law TRUE in this situation? (Briefly, why or why not?)

Can one *use* Gauss' law (written above) to simply compute the value of the electric field at arbitrary points outside the charged cube (Don't try, just tell me if you *could,* and why/why not?)

d) In the previous question, what exactly is **E**, the electric field? (Define it, and explain how you think about it, first mathematically *and* then in words? Please define any new technical words you introduce into your definition.)

Again - you do NOT have to "re do" these last questions for the homework version if you don't already know them. (Though, if you DO know them, but would just need to look something up quickly to answer them, please let me know.)

14.) Have you ever learned about Legendre Polynomials (in either a math or a physics class)? If yes, tell me briefly what you remember - do you know where they come from? What they look like? Anything interesting about them?
NOTE: this is not necessary to redo for the homework!! First pass only, if you don't know it, don't worry about it yet!

15.) Have you ever learned about orthogonality of functions in a math or physics class?
If so, tell me briefly what you remember about the orthogonality of sin and/or cos.
NOTE: this is not necessary to redo for the homework!! First pass only, if you don't know it, don't worry about it yet!

16.) Have you ever learned about the delta function (also called the "Dirac delta function", $\delta(x)$, in either a math or a physics class)? If yes, tell me briefly what you remember.

Can you evaluate $\int_{-\infty}^{\infty} (x-6)\delta(x-6)dx$

NOTE: this is not necessary to redo for the homework!! First pass only, if you don't know it, don't worry about it yet!