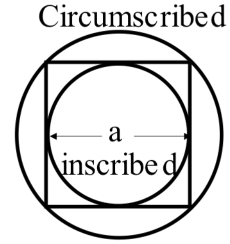
Phys 3310, HW #10, Due in class Wed Apr 9

Q1. BIOT-SAVART - SQUARE LOOP

A) Find the magnetic field at the exact center of a square current loop (current I running around a wire bent in the shape of a square of side a)

B) If I had such a loop in my lab and wanted the B field at the center, I might do the above calculation, but if I was planning an experiment and just wanted a *rough estimate* of the B-field, I might "assume a spherical cow": assume the square was really a circle. We've *done* that problem (**B** at center of a circular loop - it's much simpler than the square! You don't have to rederive it, but do think back to how we got that result, and why it turned out to be a relatively easy application of Biot-Savart) But what radius circle would you use, to estimate B? You might consider finding the B field for the "inscribed" and "circumscribed" circles and then average. How good an approximation does that turn out to be? (Can you think of a better way?)



Q2. FORMAL MANIPULATIONS

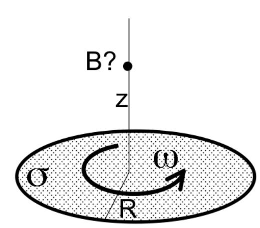
A) Griffiths (p. 223 - 224) shows that, *given* Biot-Savart, we can arrive at Ampere's law. Go through that derivation and try to recreate it/make sense of it. Don't just copy it down - do the steps yourself. There are a few "gaps" in his derivation that you should be explicit about - e.g., Eq 5.50 is missing a couple of terms, what happened to them? Are you convinced of the minus sign shenanigans leading to 5.52? Convince us you understand them! Do you understand the "ending" of the proof, why did the contribution given by Eq. 5.53 "go away"?

B) Let's use these same kinds of math gymnastics to *derive* Eq. 5.63, i.e. to show that $\vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}}$, where **A** is given by Equation 5.63, in a different way than Griffiths does (though you should convince yourself you can see it *his* way too!)

Start with the Biot-Savart law, written in the form of Eq 5.45 (on p. 222), and use the handy identity we've seen several times this term: $\vec{\nabla}\frac{1}{\mathfrak{R}} = -\frac{1}{\mathfrak{R}^2}\hat{\mathfrak{R}}$ (Do you know where this relation comes from, can you show it?) *You'll also need Griffiths' "product rule #7 (front flyleaf)". At some point you will need to pull the curl past the integral sign - be sure to justify why this is a perfectly legitimate thing to do.*

Q3. B FIELD FROM ROTATING DISK

Last week we had a problem with a CD (radius R) with a fixed, constant, uniform surface electric charge density σ everywhere on its top surface. It was spinning at angular velocity $\omega\hat{\mathbf{z}}$ about its center (the origin). You found the current density **K** at a distance r from the center. Use that result (solutions are posted on CULearn if you didn't get it!) to find the magnetic field **B**(0,0,z) at any distance z directly above the origin.



Does your answer seem reasonable? Please check it, with units, and some limiting behaviours (e.g. what do you expect if $R\to 0$? $z\to\infty$? $\omega\to 0$? Slightly less "obvious", but also worth checking/thinking about, what about $z\to 0$?)

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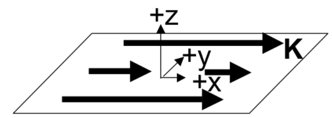
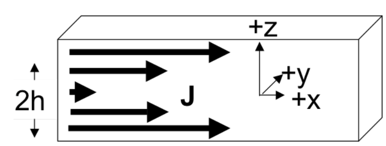


Q4. AMPERES LAW - THEMES AND VARIATIONS
Consider a thin sheet with uniform surface current density $K_0\ \hat{\mathbf{x}}$
A) Use the Biot-Savart law (Eq 5.39 on page 219) to find **B**(x,y,z) both above and below the sheet, by integration.
<u>Note:</u> The integral is slightly nasty. Before you start asking Mathematica for help - simplify as much as possible! Set up the integral, be explicit about what curly R is, what da' is, etc, what your integration limits are, etc. Then, make clear mathematical and/or physical arguments based on symmetry to convince yourself of the *direction* of the **B** field (both above and below the sheet), and to argue how **B**(x,y,z) depends (or doesn't) on x and y. (If you know it doesn't depend on x or y, you could e.g. set them to 0... But first you must convince us that's legit!)
B) Now solve the above problem using Ampere's law. (Much easier than part a, isn't it?) Please be explicit about what Amperian loop(s) you are drawing and why. What assumptions (or results from part a) are you making/using?
(Griffiths solves this problem, so don't just copy him, work it out for yourself!)
C) Now let's add a second parallel sheet at z=+a with a current running the other way. (Formally, this means $\mathbf{J}' = -K_0\,\delta(z-a)\,\hat{\mathbf{x}}$. Do you understand this notation?)
Use the superposition principle (do NOT start from scratch or use Ampere's law again, this part should be relatively quick) to find **B** *between* the two sheets, and also *outside* (above or below) both sheets. *Does this remind you of a familiar electrostatics problem at all? How?*
D) Griffiths derives a formula for the B field from a solenoid (pp. 227-228) If you view the previous part (with the two opposing sheets) from the +x direction, it looks vaguely solenoid-like (I'm picturing a solenoid running down the y-axis, can you see it? At least when viewed in "cross-section": there would be current coming towards you at the bottom, and heading away from you at the top, a distance "a" higher.) Use Griffiths' solenoid result to find the B field in the interior region (direction and magnitude), expressing your answer in terms of K (rather than how Griffiths writes it, which is in terms of I) and briefly compare with part C. Does it make some sense? Why might physicists like to use solenoids in the lab?

Q5. AMPERE'S LAW-II
Now the *sheet* of current has become a thick SLAB of current.
The slab is *infinite in (both) x and y*, but finite in z.
So we must think about the volume current density **J**, rather than **K**.
The slab has thickness 2h (It runs from z=-h to z=+h)
Let's assume that the current is still flowing in the +x direction, and is uniform in the x and y dimensions, but now **J** depends on height linearly, i.e. $\vec{\mathbf{J}} = J_0\,|\,z\,|\,\hat{\mathbf{x}}$ inside the slab (but is 0 above or below the slab). Find the **B** field (magnitude and direction) everywhere in space (above, below, and also, most interesting, *inside* the slab!)



Q6. UNIFICATION
A) Griffiths 5.12.
B) Along with the necessary addition made by Maxwell, Ampere's law in full glory reads
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \dfrac{1}{c^2}\dfrac{\partial \mathbf{E}}{\partial t}$ (We haven't talked much about the second term yet, since we've been focused on statics this term, but it's there!)
Take the divergence of this equation, and show that electric charge is conserved globally.
I think both *these results are pretty cool, and carry very deep messages about the nature and unification of electricity and magnetism,* and *their connection to special relativity!*
EXTRA CREDIT is on the next page: (Check it out, give it a try this week - at least get started!)

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EXTRA CREDIT: MAGNETIC MONOPOLES

Suppose magnetic monopoles DO exist. (They *might*! We just haven't found direct experimental evidence for them yet)

First, start off by just writing down Maxwell's equations in electro- and magneto-statics and also write down the usual Lorentz force law for a charge in E and B fields.

Now *modify these (5) equations* to allow for the possibility of magnetic "charges", or magnetic monopoles, existing in nature. (Don't forget to think about moving magnetic charges, "magnetic currents"!)

Think carefully about the *units* of any/all new constants you introduce. (Try to express things in terms of SI base units, I think that helps a lot) In particular - what would be a good guess for the SI units for your new "magnetic charge?"
(The "force law" you write down may be quite helpful in this respect).

Based just on units, can you make some guesses for the new constants? (Can you make the full set of equations look especially symmetrical and lovely?) Can you perhaps see why physicists have looked so long and hard for magnetic monopoles?

As a result of these equations, write down a "Coulomb's law" for the force between static magnetic monopoles.

You may decide that there is some ambiguity in your answers to many of these questions - what experiments can you propose to settle any questions?