Phys 3310, HW #5, Due in class Wed Feb 13.

Q1. ENERGY OF POINT CHARGE DISTRIBUTION
Imagine a small square (side "a") with four point charges +q, one on each corner. Calculate the total stored energy of this system (i.e. the amount of work required to assemble it).
Then, calculate how much work it takes to "neutralize" these charges by bringing in *one* more point charge (-4q) from far away and placing it right at the *center* of this square.
(When studying crystal structures, it is sometimes convenient to model them as rectangular grids of charged ions, this problem forms the starting point for such a model)

Q2. ENERGY OF A CONTINUOUS CHARGE DISTRIBUTION
a) Find the total electrostatic energy stored in a uniformly charge sphere of radius R and charge q. (Note: this is uniform throughout the whole volume, it's not a shell) Express your answer in terms of q, R, and constants of nature. There are many different ways to do this, I want you to use two *different methods* so you can check yourself. The most obvious are:
i) figure out E(r) and then use Griffiths Eq. 2.45 (being careful to integrate over *all space*, not just where the charge is! Think about that for yourself, it's quite important)
ii) figure out V(r), use Griff. 2.43 (what region do you need to integrate this over?)
There are still other ways if you prefer - e.g. Griffiths Eq 2.44. Or you could build up the charge "shell by shell" and explicitly integrate the work to build each thin shell, to find the total work done building the complete sphere. But two methods is enough for me!

b-i) According to Einstein, a static electron has a rest mass energy $E = mc^2$. It is tempting (though wrong!) to imagine that the electron is NOT really a point charge, but instead is a tiny sphere, with uniform charge density, whose total electrostatic energy (found above) EQUALS this mass energy of the electron. (In other words, the idea would be that the rest energy of the electron is purely electromagnetic).
What radius would an electron have to have, for this to work out?
You will need to look up the charge and mass of the electron. Don't take your result too literally, we are not taking quantum mechanics into account at all here!

ii) Here's another application of part a: compute the electrostatic energy of an atomic nucleus with Z protons and a total of A nucleons, using an approximation for the nuclear radius as $R = (1.2 \times 10^{-15}\text{ m})\, A^{1/3}$. Convert your answer (which is likely in SI units of Joules) to the energy unit of MeV, which is Mega eV. (Give your result in units of MeV times $Z^2/A^{1/3}$) Now use this result to estimate the *change* of electrostatic energy when a uranium nucleus undergoes fission into two roughly equal-sized nuclei. Go online and check what the experimental answer is - what does this tell you about the source of the energy released in a nuclear explosion - is it mostly electrostatic? (You might be surprised by your answer!)

Q3. GAUSS' LAW AND CONDUCTORS: Griffiths 2.35 (p. 101)

(cont)

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Q4. GAUSS' LAW AND CAVITIES: Griffiths 2.36 (p. 101)
Please be sure to explain your reasoning on all parts, and add to this the problem the following:
In part a, be sure to *sketch* the charge distribution.
In part c, *sketch* the E fields (everywhere in the problem, i.e. in the cavities and also outside the big sphere)
f) Lastly - if someone moved q_a a little off to one side, so it was no longer at the *center* of its little cavity, which of your answers would change? Please explain.
I really like this problem, lots of good physics in it! Try to think it through carefully, make physical sense of all your answers, don't just take someone else's word for it!

Q5. COAX CAPACITORS
In HW 3 you found the E field everywhere in and around a coaxial cable. We can slightly modify that problem, making it more realistic by letting the inner cylinder be a conductor (a wire!) So you have an inner conducting cylinder (radius "a") and an outer conducting cylindrical shell (inner radius "b"). It is physically easy to then set up any fixed potential difference ΔV between the inner and outer conductors that you like.
a) Assuming these are infinite cylinders, find the energy stored per unit length inside this capacitor. Once again, let's do it two ways so we can check:
i) Integrate the energy density stored in the E field (like method i, in Q2 above)
ii) Find the capacitance C of this system, and then use stored energy = $\frac{1}{2} C(\Delta V)^2$
Based on your answers above, *where* in space would you say this energy is physically stored?

b) This is also an excellent model for "axons", which are long cylindrical cells (basically coax cables!) carrying nerve impulses in your body and brain. Using your result from part ii above, estimate the *capacitance* (in SI metric units, which are Farads) of your sciatic nerve.
Assumptions - the sciatic nerve is the longest in your body, it has a diameter of roughly 1 micron, and a length of perhaps 1 m. Note that axons generally have a value of b which is very close to a (i.e. the gap is extremely tiny, b-a is about 1 nanometer.) so you can simplify your expression using $\ln(1+\varepsilon)\approx\varepsilon$

Extra Credit follows on next page.

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Extra credit:

Griffiths finds the capacitance of two concentric spherical metal shells. By taking the limit as radius of the outer shell $\rightarrow \infty$, you get a perfectly finite result, which is referred to as "the Capacitance of the sphere". (It tells you the amount of charge you'd need to put on the sphere to get the surface up to a voltage V with respect to infinity).

· Estimate the capacitance of i) an isolated human body and ii) planet earth. (in Farads)
(Here I would "assume a spherical person" :-)
These estimates should give you a sense of how BIG a one Farad capacitance is!
Now let's use the results:

Case i: By shuffling over a carpet on a dry winter day, you can easily charge yourself up to a high voltage. *Estimate* this voltage by thinking about the length of the spark when your hand comes close to a grounded conductor. Now use your capacitance to estimate the *energy* dissipated when you touch a doorknob (or your little brother/sister) and spark.

Case ii: Planet earth has a static E field at sea level of roughly 100 V/m. Use this to estimate the net charge on the earth, and thus the total stored energy our "planetary capacitor" holds. If we could tap this, would it solve our energy problems? (Turns out that the role of lightning in this story may surprise you, lightning is not DISCHARGING the giant earth capacitor... it's the charging mechanism!)