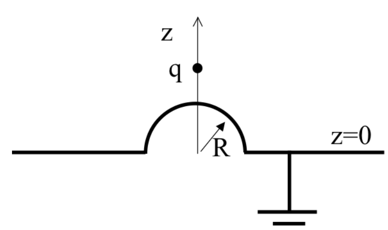
Phys 3310, HW #6, Due in class Wed Feb 27.

Q1. UNIQUENESS THEOREM
Griffiths section 3.1.6 is a discussion and proof of the "Second uniqueness theorem". Read through that theorem (and his proof), make sense of it! Then for this homework problem, prove it yourself, using a slightly *different* method than what Griffiths does (though you may find some common "pieces" are involved!) Do it like this:
Go back to Green's Identity (stated in Problem 1.60c, p. 56, you may recognize this from our HW#2) This identity is true for ANY choice of T and U, so let the functions T and U in that identity both be the SAME function: specifically, you should set them both equal to $V_3=V_1-V_2$. Then, Green's Identity (along with some arguments about what happens at the boundaries, rather like Griffith's uses in *his* proof) should let you quickly show that E_3 (which is defined to be the negative gradient of V_3, as usual) must vanish everywhere throughout the volume. QED.
Understand the game. We are checking if there are two different potential functions, V_1 and V_2, each of which satisfies Laplace's equation throughout the region we're considering. You construct (define) V_3 to be the difference of these, and you prove that V_3 (or in this case, $E_3 = -\nabla V_3$) must vanish everywhere in the region. Which means there really is only one unique E-field throughout the region after all! This is another one of those "formal manipulation" problems, giving you a chance to practice with the divergence theorem and think about boundary conditions...

Q2. METHOD OF IMAGES - spherical
Take a look at Griffiths' Fig 3.12, which shows a grounded metal sphere with a charge q outside it. He argues (leading up to Eq. 3.17) that there is a simple "method of images" trick available here - you just have to put the right charge (q') at the right spot (b, inside the radius of the sphere). Your task:
i) Solve Griffiths' problem 3.7a (p. 126)
(which shows WHY this particular "image trick" works for a spherical conductor)
ii) Solve Griffiths' problem 3.7b
iii) Now let's apply this result to a novel situation:
Imagine a grounded infinite conducting plane in the x-y plane, that has a (conducting) hemispherical bump (radius R) in it, centered at the origin, as shown.
A charge q sits a distance "a" above the plane, i.e. at the point (0,0,a)
I claim that you can find the potential V anywhere in the plane above the conductor using the method of images, with *three* image charges.



Where should they be? (Explain your reasoning- you need to ensure the boundary condition V=0 on the entire conductor.) Is it now easy for you to construct a formula for V at any point above the plane?

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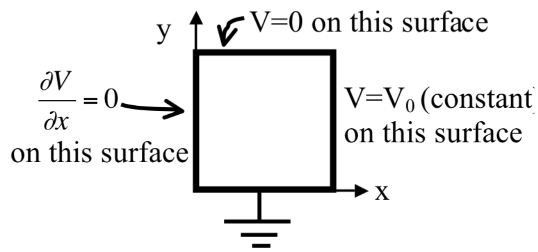
Q3. SEPARATION OF VARIABLES - CARTESIAN 2-D

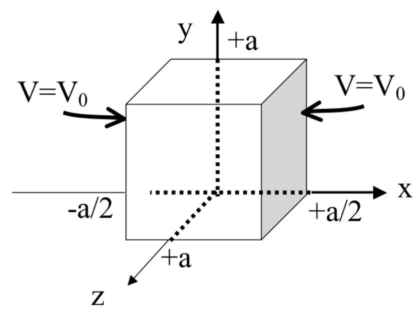
A square rectangular pipe (sides of length a) runs parallel to the z-axis (from $-\infty$ to $+\infty$)
The 4 sides are maintained with boundary conditions given in the figure. (Each of the 4 sides is insulated from the others at the corners)



i) Find the potential V(x,y,z) at all points in this pipe.
ii) Sketch the E-field lines and equipotential contours inside the pipe. (Also, state in words what the boundary condition on the left wall means - what does it tell you? Is the left wall a conductor?)
iii) Find the charge density $\sigma(x,y=0,z)$ everywhere on the bottom conducting wall (y=0).

Q4. SEPARATION OF VARIABLES - CARTESIAN 3-D

You have a cubical box (sides all of length a) made of 6 metal plates which are insulated from each other.
The left wall is located at x=-a/2,the right wall is at x=+a/2.
Both left and right walls are held at constant potential $V=V_0$.
All four other walls are grounded.
(Note that I've set up the geometry so the cube runs from y=0 to y=a, and from z=0 to z=a, but from x=-a/2 to x=+a/2
This should actually make the math work out a little easier!)



Find the potential V(x,y,z) everywhere inside the box.
(Also, is V=0 at the center of this cube? Is E=0 there? Why, or why not?)

Q5. SEPARATION OF VARIABLES - SPHERICAL

The potential on the surface of a sphere (radius R) is given by $V=V_0 \cos(2\theta)$.
(Assume $V(r=\infty)=0$, as usual. Also, assume there is no charge inside or outside, it's ALL on the surface!)
i) Find the potential inside and outside this sphere.
(Hint: Can you express $\cos(2\theta)$ as a simple linear combination of some Legendre polynomials?)
ii) Find the charge density $\sigma(\theta)$ on the sphere.

Extra Credit: CHARGED METAL SPHERE

You have a conducting metal sphere (radius R), with a net charge +Q on it.
It is placed into a pre-existing uniform external field **E**0 which points in the z direction.
(So, this is *exactly like* Griffiths Example 3.8, except the sphere is not neutral to start with.) Find the potential everywhere inside and outside this sphere. Please explain clearly *where* you are setting the zero of your potential. Do you have any freedom in this matter? Briefly, explain.