Phys 3310, HW #7, Due in class Wed Mar 5

**Q1.** SEPARATION OF VARIABLES - SPHERICAL SIGMA
The surface charge density on a sphere (radius R) is a constant, $\sigma_0$
(As usual, assume $V(r=\infty)=0$, and there is no charge anywhere inside or outside, it's ALL on the surface!)
i) *Using the methods of section 3.3.2* (i.e. explicitly using separation of variables in spherical coordinates), find the electrical potential inside and outside this sphere.
ii) Discuss your answer, explain how you might have just "written it down" without doing all that work! (Be explicit - what about all the specific coefficients you got in i?) Also - can you think of a fairly simple (realistic) physical/experimental setup that might yield a situation like this?
iii) Now, suppose the surface charge density is $+\sigma_0$ on the entire northern hemisphere, but $-\sigma_0$ on the entire southern hemisphere. Again, find voltage inside and outside. (This time, you will in principle need an infinite sum of terms - but for this problem, just work out explicitly what the first *two* nonzero terms are. (In both cases, for $V(r<R)$, and $V(r>R)$)
Note: some terms you might have expected to be present will vanish. Explain physically or mathematically why the first "zero" term really *should* be zero.
*Griffiths solves a generic example problem, for which part i above is a simple special case (and for that matter, so is part iii). But, please work through the details on your own - you're welcome to use Griffiths to guide you if/whenever you need it, but in the end, solve the whole problem yourself and show your work!*

**Q2.** SEPARATION OF VARIABLES - CONCENTRIC SPHERES
Two concentric spherical surfaces have radii of a and b. If the potential on the inner surface, at r=a, is just a nonzero *constant* (call it $V_{in}$) and the potential on the outer surface is given by $V(b,\theta)=V_{out}P_1(\cos\theta) \quad (i.e.=V_{out}\cos\theta)$, find the potential in the region *between* the two surfaces ($a < r < b$).

**Q3.** SEPARATION OF VARIABLES - DISK
A disk of radius R has a uniform surface charge density $\sigma_0$. Way back on Set #2 you found the E-field along the axis of the disk (and on the midterm, you again solved a very similar (but *harder*) version of this where $\sigma$ was not uniform). You can check for yourself by direct integration, (but don't have to): I claim that along the z axis, (i.e. $\theta=0$),

$$V(r,\theta=0)=\frac{\sigma_0}{2\varepsilon_0}\left(\sqrt{r^2+R^2}-r\right)$$

i) Find the potential *away* from the axis (i.e nonzero $\theta$) , for distances $r > R$, by using the result above and fiddling with the Legendre formula, Griffiths' 3.72 on page 140. You will in principle need an infinite sum of terms here - but for this problem, just work out explicitly what the first *two* *non-zero* terms are.
*(It might help to remember that $P_l(1)$ is always equal to 1, and you will have to think mathematically about how the formula above behaves for $r>>R$)*

ii) Griffiths Chapter 3.4 talks about the "multipole expansion". Look at your answer to part i, and compare it to what Griffiths says it *should* look like (generically) on page 148. Discuss - does your answer make some physical sense? Note that there is a "missing term" - why is that? Can you think of some physical situation that might look a little like this problem?

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**Q4.** MULTIPOLES - point charges
You have four point charges. Their location and charges in Cartesian coordinates are:
A charge -q located at (a,0,0), another charge -q located at (-a,0,0), a third charge +3q located at (0,0,b), and finally a fourth charge -q located at (0,0,-b)
i) What is the total charge, and dipole moment, of this distribution of charges? Use the methods of "the multipole expansion" (Griffiths p. 148) to find a simple approximate formula for V(r,θ) (in spherical coordinates!) valid at points far from the origin. ("Simple" means only the *first non-zero term* is needed!)
ii) Find a simple approximate expression for the electric field valid at points far from the origin. (Again, express your answer in spherical coordinates, so we want $\vec{\mathbf{E}}(r,\theta) = E_r(r,\theta)\hat{\mathbf{r}} + E_\theta(r,\theta)\hat{\theta}$, and you should figure out what $E_r$ and $E_\theta$ are.)
Sketch this (approximate) E field. (Don't worry about what happens near the origin, I just want a sketch of the simple approximation)

**Q5.** MULTIPOLES - spherical shell charge distribution
Griffiths derives (on page 142-144) the exact potential V(r,θ) everywhere outside a spherical shell of radius R which has a surface charge distribution σ(R,θ)= kcosθ
i) Calculate the dipole moment of this object, and also the total charge on this object.
ii) Use the methods of "the multipole expansion" (Griffiths p. 148) to find an *approximate* form for the potential far from the sphere. You can stop with the leading nonzero term. Now compare with Griffiths *exact formula* from the earlier example (on p.144). What does this tell you about the quadrupole moment (and higher moments) of this surface charge distribution?

**Q6.** REAL DIPOLE.
Griffiths 4.1 (p. 163) Only, instead of just using/assuming the Bohr radius (like Griffiths suggests) please *estimate* the radius of hydrogen, wherever you may need it, by using the experimental atomic polarizability of hydrogen given in Table 4.1 (p. 161) of Grifffiths. (Griffiths' Example 4.1 should tell you how this would give you a quick estimate of the hydrogen atom radius. How does it compare with the Bohr radius?)

**Extra Credit:**
Go back through all your returned old homeworks (but preferably one of the more recent ones), and/or the midterm, and find a problem (or two) that you got some serious points taken off. i)clearly identify the set # and question # you are correcting, ii) state what was wrong with your original wrong answer (assuming it wasn't just blank) iii) explain where your original reasoning was incorrect (if it wasn't blank) or what you were missing (if it was blank), and then outline the correct *reasoning* for the problem, and how it leads to the right answer. Of course, solutions are posted, so we're not interested in having you just copy my solutions! What we're looking for is more your reflections on where you went wrong, and what *you understand* about the problem now.