Phys 3310, HW #8, Due in class Wed Mar 12

Q1. BOUND CHARGES
Consider a long teflon rod, (a dielectric cylinder), radius a. Imagine that we could set up a permanent polarization **P**(s,θ,z) = k**s** (= $ks\hat{\mathbf{s}}$), where **s** is the usual cylindrical radial vector from the z-axis, and k is a constant). *Neglect end effects, the cylinder is long.*
a) Calculate the bound charges σ_b and ρ_b (on the surface, and interior of the rod respectively). What are the units of "k"?
b) Next, use these bound charges (along with Gauss' law) to find the electric field inside and outside the cylinder. (Direction and magnitude)
c) Find the electric displacement field **(D)** inside and outside the cylinder, and verify that Griffiths' Eq 4.23 (p. 176) works. Explain briefly in words why your answer might be what it is.

Q2. BOUND CHARGES II
Now let's hollow out that teflon rod, so it has inner radius b, and outer radius is (still) a.
Just to make things a little different here, suppose I now set up a different polarization within the teflon material, namely **P**(s,θ,z) $= \frac{k}{s^2}\hat{\mathbf{s}}$ for b<s<a.
a) We have vacuum for s<b and s>a. What does that tell you about **P** in those regions? Find the bound charges σ_b and ρ_b (σ_b on the inner AND outer surfaces of the hollow rod, and ρ_b everywhere else. Use these bound charges, along with Gauss' law, to find the electric field everywhere in space. (Direction and magnitude)
b) Use Griffiths' Eq 4.23 (p. 176) to find **D** everywhere in space. (This should be quick - are there any *free* charges in this problem?) Use this (simple) result for **D** (along with Griffiths basic definition/relation of **E** to **D**, Eq 4.21) to find **E** everywhere in space. *(This should serve as a check for part a, and shows why sometimes thinking about **D** fields is easier and faster!)*

Q3. FIELD PATTERNS FROM BOUND CHARGES
Let's go back to the "solid" teflon rod (radius a), but let's change the polarization once again - now assume it is uniform and parallel to the z-axis, i.e. $\mathbf{P} = k\hat{\mathbf{z}}$. This time, let's also keep it finite, and consider realistic "end effects"
a) Sketch the electric field lines, inside and outside, in two different limits: First, when the cylinder is very LONG compared to its diameter, and second, when it is very SHORT compared to its diameter. Explain in words and formulas all your reasoning behind the sketches.
b) In both of the above two limiting cases, sketch (and explain in words) **P** everywhere in space, and also **D** everywhere (inside *and* outside the rods)
c) In both of the above limits, explain/show how Griffiths' "boundary condition" (page 178) Eq 4.26 and 4.28 work out nicely right near the top end of the rod.
Also (just briefly, *very* qualitatively, e.g. just in terms of signs/directions of these fields) explain how boundary conditions 4.27 and 4.29 makes sense/agree with your sketches in part b.
(Here, just consider the outer edge of the middle of the cylinder at z=0 i.e. at (s=a, z=0) if the rod runs from z=-L to z=+L)

Such an object is referred to as a "bar electret", it is an electrical analogue of a permanent bar magnet. Do you see why, from your sketches? Griffiths points out that this is quite unusual, most materials cannot maintain such a permanent electric polarization in the absence of an external field, although he claims they do exist. But if you did have one, it would attract charged ions from the air which would negate the interesting fields you found above.

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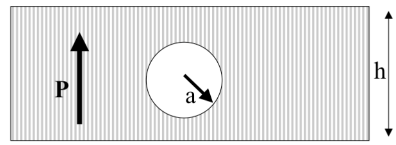
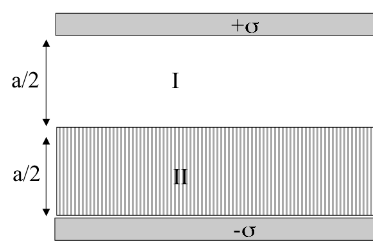
Q4. MATH OF BOUND CHARGES
Using Griffiths' formal definition of bound charge distributions, eqns 4.11 and 4.12 (p. 167-8) prove (mathematically) that if you have a dielectric which starts off neutral, and then it gets polarized, that the total bound charge is still exactly zero. Explain how this formal mathematical result makes (simple) physical sense. Look back at all previous questions, does the total charge in those problems satisfy what you found here? *(This problem need not be too long!)*

Q5. SUPERPOSITION
A large slab (infinite in the x-y directions) of dielectric material has thickness h, and has uniform polarization $\mathbf{P} = k\hat{\mathbf{z}}$. The polarization is perpendicular to the surfaces of the slab. At the midplane of the sheet is a spherical bubble, of radius a. The inside of the bubble contains vacuum: it has zero polarization.



Hint: Carving out a cavity is the same as superimposing an object of the same shape but opposite *polarization!*
a) Assume there are no free charges anywhere (so this polarization **P** is just spontaneous), find the net electric field everywhere *inside* the bubble.
b) Assume there are still no free charges anywhere. Find the bound surface charge on the surface of the bubble. Then, find (or at least describe) the electric field *outside the slab.*
c) Now assume there was an external field **E**ext which *caused* that uniform polarization **P** in the first place. What is the net electric field everywhere *inside* the bubble, in terms of **Eext** and **P**?
Hints: Griffiths' Example 4.2 will prove useful in part a and b. (Ex 3.9 of Griffiths could also *be useful for b, at least as a check). Part c can be quick, if you think about it the right way.*

Q6. LINEAR DIELECTRIC, RECTANGULAR BOUNDARY
You have a large parallel-plate conductor (two big metal sheets, the upper one has free charge density $+\sigma$, the lower one has $-\sigma$) They are a distance a apart.
The space between the sheets is half filled with a (linear) dielectric oil.
Region I, (filling the top half of the volume) is vacuum.
The lower half, region II, has dielectric constant ε_r=3.0



a) Find the electric displacement ("**D** field") (direction and magnitude) in regions I and II, the electric field **E** in both regions, and the polarization **P** in both regions
b) Find the location and amount of bound charge (surface and volume) everywhere. Given this, go back and compute the E field in the two regions to check your answer for **E** in part a.
c) Find the voltage between the plates. If the plates are large but not infinite (area A), compare the capacitance before and after you added the dielectric oil. How does it compare with what you would get if you filled the *entire* region with that dielectric oil?

Q7. LINEAR DIELECTRIC, SPHERE
Do Griffiths problem 4.20, on page 185.
Does your answer come out *larger* or *smaller* than what you would have found for a simple sphere of charge with uniform free charge density ρ (i.e, if you had neglected the effect of the dielectric constant? In this limit, what is ε_r?) Does this result make good physical sense to you? What do you get in the limit of *infinite* dielectric constant? What *physical situation* does that limit remind you of?

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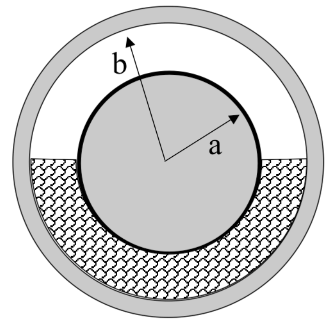
EXTRA CREDIT: (tricky but worth thinking about, you can learn a lot from this one!)

Consider two concentric conducting spherical shells of radii a and b as shown in the figure. The space between them is filled with a liquid having a dielectric constant ε_r.



A total charge of "+Q" is placed in the inner conducting shell and "-Q" in the outer shell.

a) Find the magnitude and direction of the fields E, D, P everywhere; namely
for $r > b$, $a < r < b$, and $r < a$.

b) Determine expressions for the free surface charge on each conducting shell everywhere (namely, on the dielectric side, and the air side).

c) Determine expressions for the bound charge at $r = a$ and $r = b$.

d) Determine the potential everywhere.

Be careful to satisfy the boundary conditions everywhere, including the boundary between the dielectric and the air (consider it vacuum) in between the spherical shells. If you think a bit about satisfying the boundary conditions you will notice that the solution for the "E" field is very simple, and makes a good starting point for the problem! (Once you know E somewhere, D is very simple to find, it's just ε E, with $\varepsilon = \varepsilon_r \varepsilon_o$ = a given constant!)

It would not be safe to assume that the charge "Q" on the inner metal sphere will distribute itself uniformly - after all, there will be (opposite) bound charges forming next to it in the bottom half, but not on the top half. So, where do you expect to see more charge congregate, the top half or the bottom half of that sphere?