# Chapter 3

concept questions

## LAPLACE'S EQUATION AND UNIQUENESS

Poisson's equation tells us that

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

If the charge density throughout some volume is zero, what else *must* be true throughout that volume:

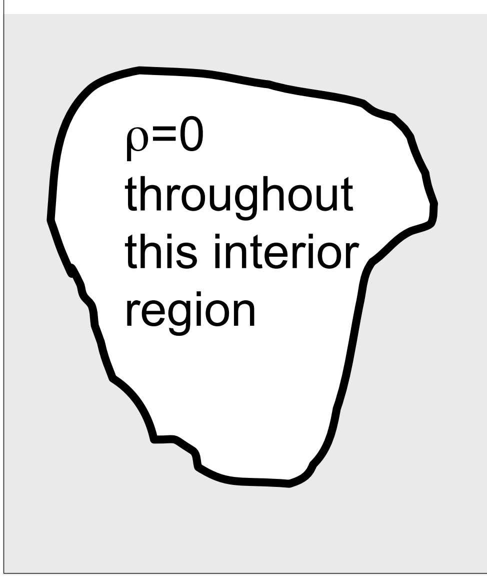
A) V=0

B) E=0

C) Both V and E must be zero

D) None of the above is necessarily true

3.1 A region of space contains no charges.
What can I say about V in the interior?



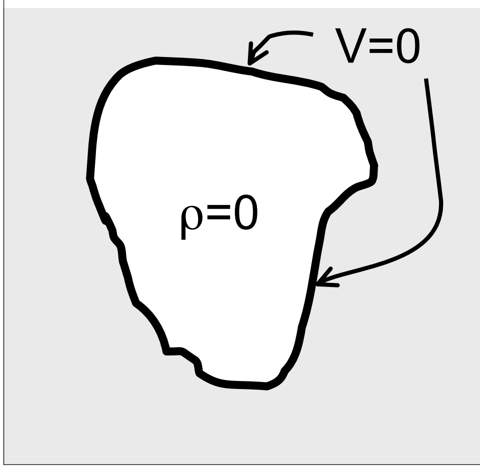

A) Not much, there are lots of possibilities for V(r) in there

B) V(r)=0 everywhere in the interior.

C) V(r)=constant everywhere in the interior

3.2 A region of space contains no charges. The *boundary* has V=0 everywhere.

What can I say about V in the interior?



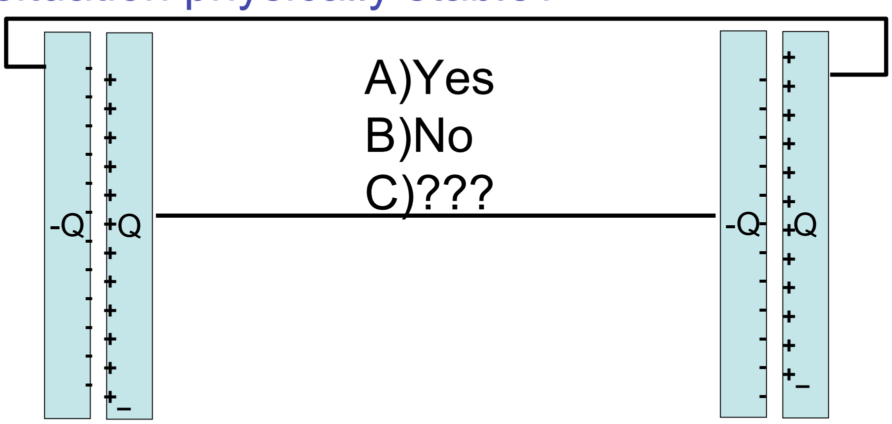

A) Not much, there are lots of possibilities for V(r) in there

B) V(r)=0 everywhere in the interior.

C) V(r)=constant everywhere in the interior

3.3 Two very strong (big C) ideal capacitors are well separated.

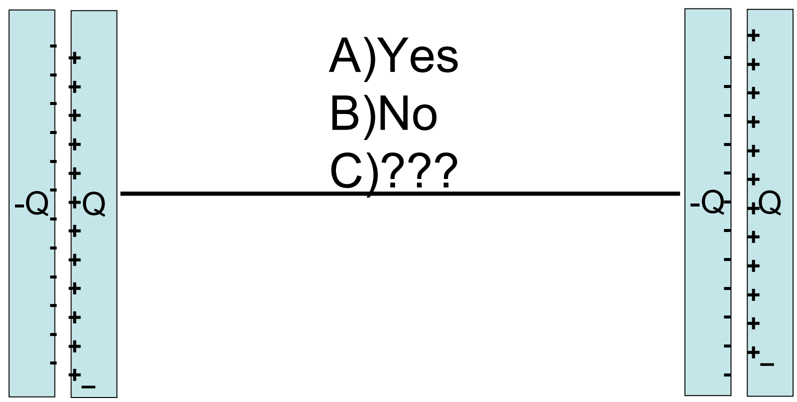
If they are connected by 2 thin conducting wires, as shown, is this electrostatic situation physically stable?




A)Yes
B)No
C)???

3.4 Two very strong (big C) ideal capacitors are well separated.

What if they are connected by one thin conducting wire, is this electrostatic situation physically stable?

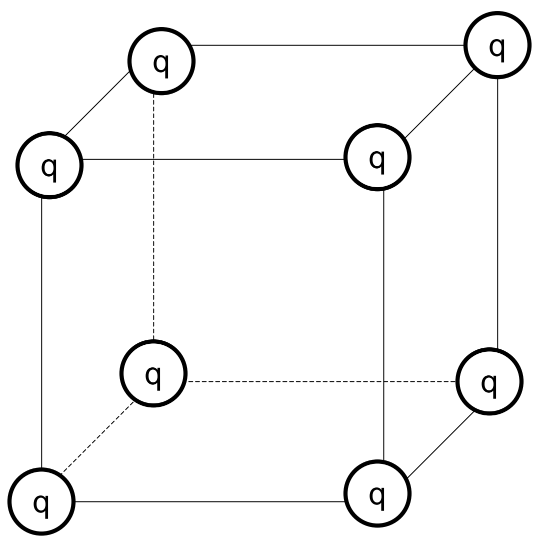



A)Yes
B)No
C)???

3.5 If you put a + test charge at the *center* of this cube of charges, could it be in stable equilibrium?

A) Yes
B) No
C) ???

**Earnshaw's Theorem**



3.6

What must be true for you to know that you've found the potential in a region?

a) It satisfies $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$

b) It satisfies the boundary conditions

c) a and b

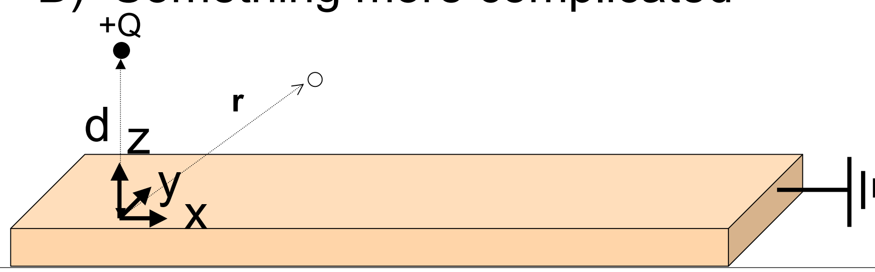
# METHOD OF IMAGES

3.7 A point charge +Q sits above a *very large grounded* conducting slab.
What is **E**(**r**) for points *above* the slab?

A) Simple Coulomb's law:

$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0}\frac{\vec{\mathfrak{R}}}{\mathfrak{R}^3} \qquad \text{with } \vec{\mathfrak{R}} = (\vec{r} - d\,\hat{z})$$

B) Something more complicated

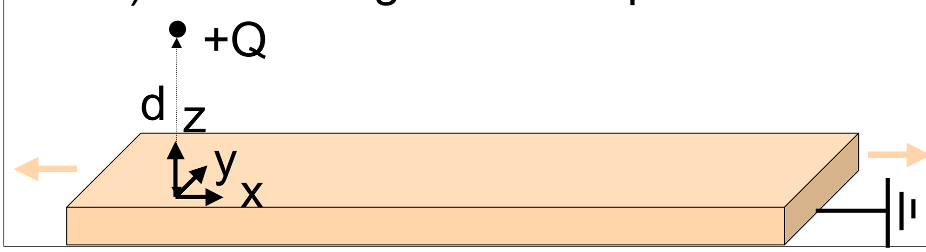



3.8 A point charge +Q sits above a *very large grounded* conducting slab.
What's the electric force on +Q?

A) 0 B) $\dfrac{Q^2}{4\pi\varepsilon_0(2d)^2}$ downwards

C) $\dfrac{Q^2}{4\pi\varepsilon_0 d^2}$ downwards

D) Something more complicated

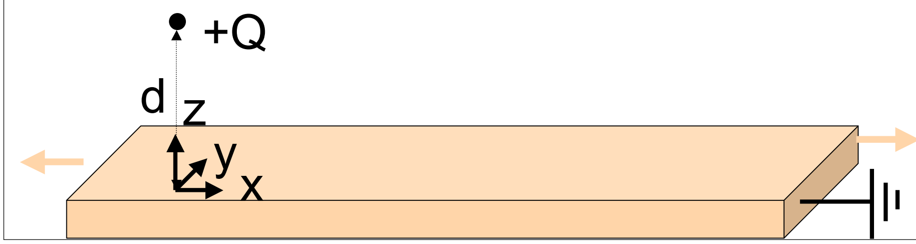


3.8b A point charge +Q sits above a *very large grounded* conducting slab.

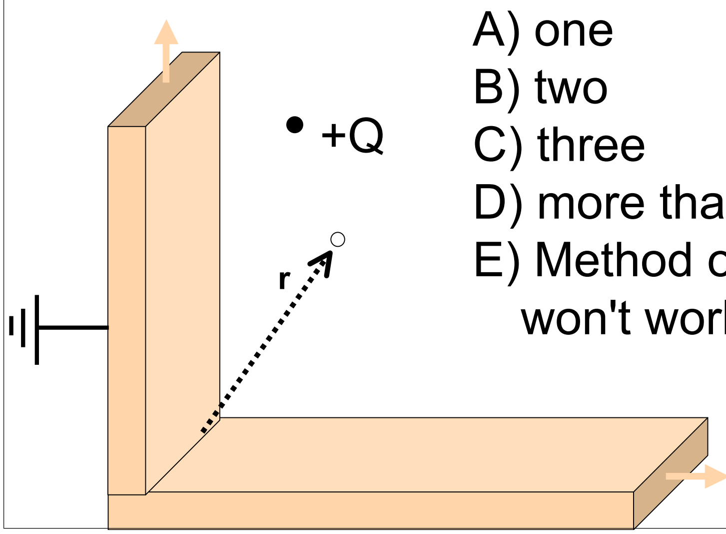
What's the energy of this system?

A) $\dfrac{-Q^2}{4\pi\varepsilon_0(2d)}$

B) Something else.




3.9 Two ∞ grounded conducting slabs meet at right angles. How many image charges are needed to solve for V(**r**)?




A) one
B) two
C) three
D) more than three
E) Method of images won't work here

On paper (don’t forget your name!) in your own words (by yourself):

What is the idea behind the method of images? What does it accomplish?

What is its relation to the uniqueness theorem?

# SEPARATION OF VARIABLES: CARTESIAN

3.10 Suppose $V_1(\mathbf{r})$ and $V_2(\mathbf{r})$ are linearly independent functions which *both* solve Laplace's equation, $\nabla^2 V = 0$

Does $aV_1(\mathbf{r})+bV_2(\mathbf{r})$ also solve it (with a and b constants)?

A) Yes. The Laplacian is a linear operator
B) No. The *uniqueness theorem* says this scenario is impossible, there are never two independent solutions!
C) It is a definite yes or no, but the *reasons* given above just aren't right!
D) It depends...

3.11 Given the two diff. eq's:

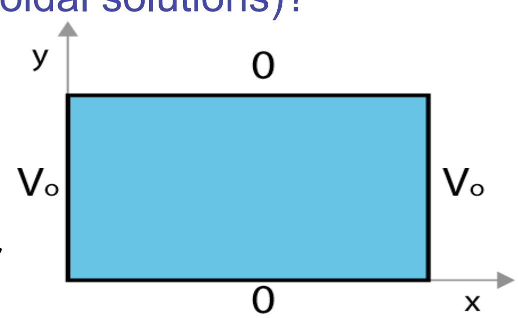
$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

where $C_1+C_2 = 0$. Given the boundary conditions in the figure, which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

A) x B) y

C) $C_1= C_2=0$ here

D) It doesn't matter



3.11 b

Given the two diff. eq's:

$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

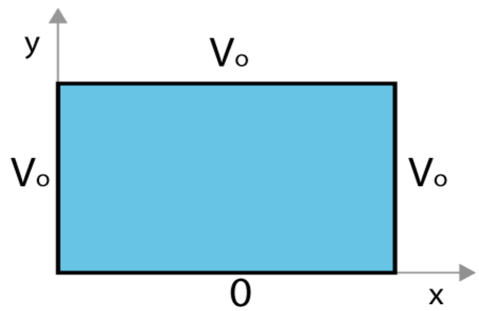
where $C_1+C_2 = 0$. Which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

A) x

B) y

C) $C_1 = C_2 = 0$ here

D) It doesn't matter




3.11 h

The X(x) equation in this problem involves the "positive constant" solutions:
A sinh(kx) + B cosh(kx)

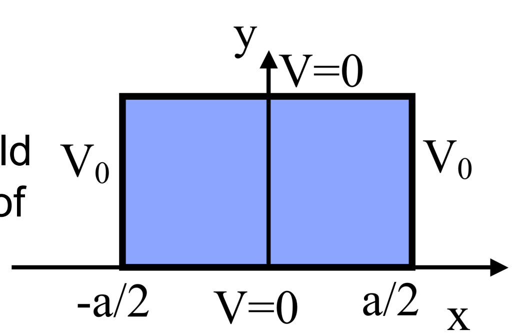
What do the boundary conditions say about A and B?

A) A=0 (pure cosh)

B) B=0 (pure sinh)

C) Neither: you should rewrite this in terms of $A\,e^{kx} + B\,e^{-kx}$ !

D) Other/not sure?



3.11
c

Given the two diff. eq's:

$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

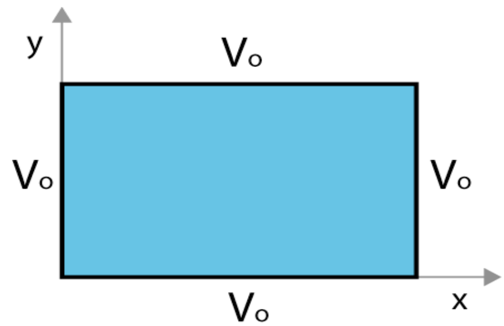
where $C_1+C_2 = 0$. Which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

A) x

B) y

C) $C_1 = C_2 = 0$ here

D) It doesn't matter




3.12

What is the value of

$$\int_0^{2\pi} \sin(2x)\sin(3x)dx \ ?$$

A) Zero

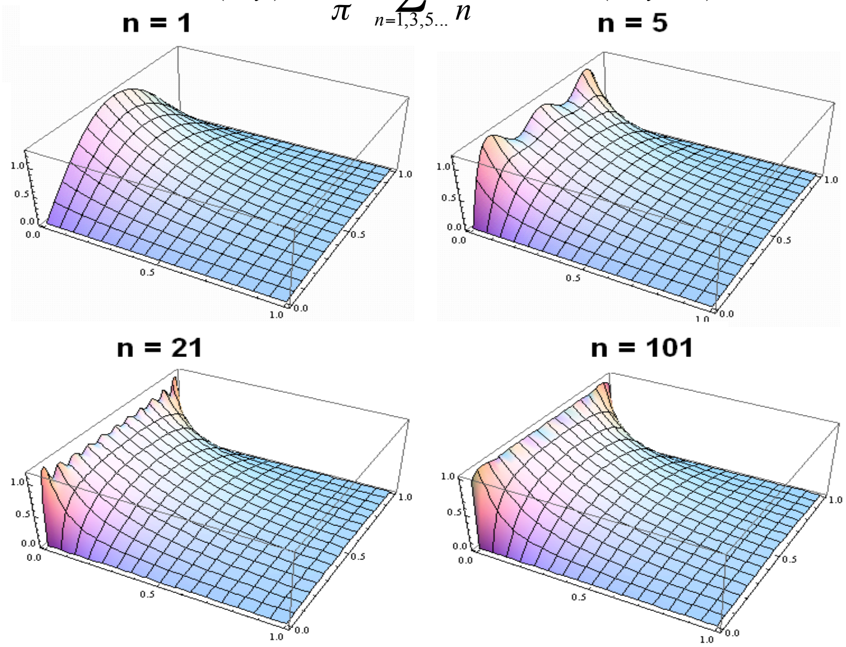
B) $\pi$

C) $2\pi$

D) $\pi/2$

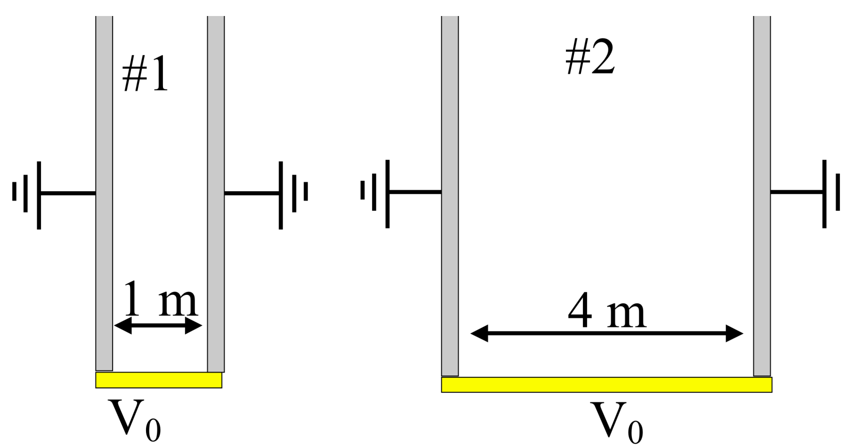
E) Something else/how could I possibly know this?

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5...}^{\infty} \frac{1}{n} e^{-n\pi x/a} \sin(n\pi y/a)$$




3.14

2 troughs (∞ in z, i.e. out of page) have grounded sidewalls.
The base of each is held at V0.



3.14 $$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5...}^{\infty} \frac{1}{n} \sin(n\pi x / a) e^{-n\pi y/a}$$

How does V(x,y) compare, 4 m above the middle of the base in the two troughs?

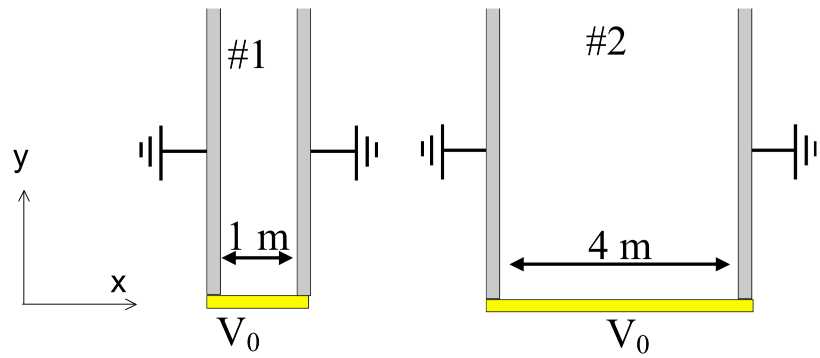
A) Same in each

B) 4x bigger in #1

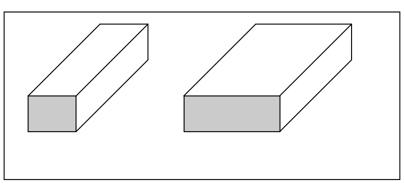
C) 4x bigger in #2

D) *much* bigger in #1

E) *much* bigger in #2




Discussion question:
Rectangular Pipe 2 is twice as wide as rectangular Pipe 1 and the shaded face is at a potential V(x,y):



Pipe 1 Pipe 2

How will the solutions to Laplace's Equation be qualitatively different or similar?

**SEP OF VAR: LEGENDRE POLYNOMIALS**

3.15 Given $\nabla^2 V = 0$ in Cartesian coords, we separated V(x,y,z) = X(x)Y(y)Z(z). Will this approach work in spherical coordinates, i.e. can we separate V(r,θ,φ) = R(r)P(θ)F(φ)?

A) Sure.

B) Not quite - the angular components cannot be isolated, e.g. f(r,θ,φ) = R(r)Y(θ,φ)

C) It won't work at all because the spherical form of Laplace's Equation has cross terms in it (see the front cover of Griffiths)

3.16 The Rodrigues formula for generating the Legendre Polynomials is

$$P_l(x) = \frac{1}{2^l l!}\left(\frac{dy}{dx}\right)^l (x^2-1)^l$$

If the Legendre polynomials are orthogonal, are the leading coefficients $\frac{1}{2^l l!}$ necessary to maintain orthogonality?

A) Yes, $f_m(x)$ must be properly scaled for it to be orthogonal to $f_n(x)$.

B) No, the constants will only rescale the integral

3.17 Given $V(\theta) = \sum_{l=0}^{\infty} C_l P_l(\cos\theta)$

(The $P_l$'s are Legendre polynomials.)

If we want to isolate/determine the coefficients

$C_l$ in that series, first multiply both sides by:

A) $P_m(\theta)$

B) $P_m(\cos\theta)$

C) $P_m(\theta)\sin\theta$

D) $P_m(\cos\theta)\sin\theta$

E) something entirely different

$$\int_0^\pi P_l(\cos\theta) P_m(\cos\theta) \sin\theta d\theta = \begin{cases} \dfrac{2}{2l+1} & \text{if } l = m \\ 0 & \text{if } l \neq m \end{cases}$$

$$\int_{-1}^{1} P_l(x) P_m(x) dx = \begin{cases} \dfrac{2}{2l+1} & \text{if } l = m \\ 0 & \text{if } l \neq m \end{cases}$$

3.18

$$V(r,\theta) = \sum_{l=0}^{\infty} \left( A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Suppose V on a spherical shell is constant, i.e. $V(R, \theta) = V_0$.
Which terms do you expect to appear when finding V(outside) ?

A) Many $A_l$ terms (but no $B_l$'s)
B) Many $B_l$ terms (but no $A_l$'s)
C) Just $A_0$
D) Just $B_0$
E) Something else!!

3.18 b

$$V(r,\theta) = \sum_{l=0}^{\infty} \left( A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Suppose V on a spherical shell is constant, i.e. $V(R, \theta) = V_0$.
Which terms do you expect to appear when finding V(inside) ?

A) Many $A_l$ terms (but no $B_l$'s)
B) Many $B_l$ terms (but no $A_l$'s)
C) Just $A_0$
D) Just $B_0$
E) Something else!

3.19a

$$P_0(\cos\theta) = 1, \qquad P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}, \qquad P_3(\cos\theta) = \frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta$$

Can you write the function $V_0(1+\cos^2\theta)$ as a sum of Legendre Polynomials?

$$V_0(1+\cos^2\theta) \overset{???}{=} \sum_{l=0}^{\infty} C_l P_l(\cos\theta)$$

A) No, it cannot be done
B) It would require an infinite sum of terms
C) It would only involve $P_2$
D) It would involve all three of $P_0$, $P_1$ AND $P_2$
E) Something else/none of the above

3.19

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is

$$V(R,\theta) = V_0(1+\cos^2\theta)$$

Which terms do you expect to appear when finding V(inside) ?

A) Many $A_l$ terms (but no $B_l$'s)
B) Many $B_l$ terms (but no $A_l$'s)
C) Just $A_0$ and $A_2$
D) Just $B_0$ and $B_2$
E) Something else!

3.19 b

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is

$$V(R,\theta) = V_0(1+\cos^2\theta)$$

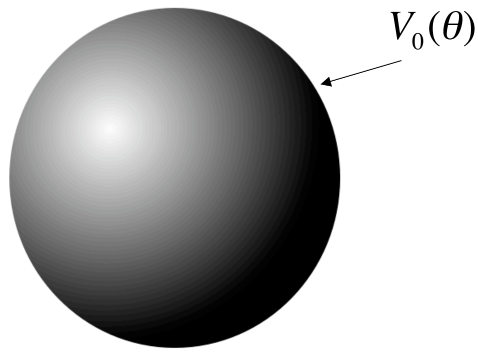
Which terms do you expect to appear when finding V(outside) ?

A) Many $A_l$ terms (but no $B_l$'s)
B) Many $B_l$ terms (but no $A_l$'s)
C) Just $A_0$ and $A_2$
D) Just $B_0$ and $B_2$
E) Something else!

3.20

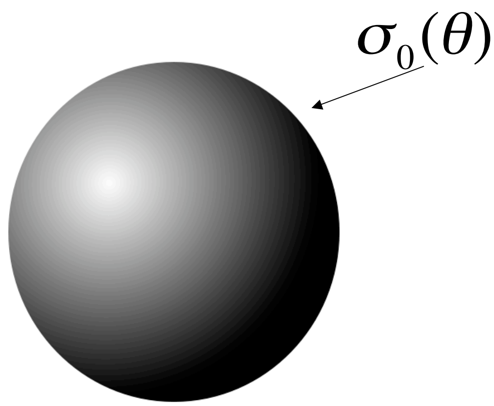
How many boundary conditions (on the potential V) do you use to find V inside the spherical plastic shell?

A) 1
B) 2
C) 3
D) 4
E) It depends on $V_0(\theta)$




3.21

How many boundary conditions (on the potential V) do you use to find V inside the thin plastic spherical shell?




A) 1
B) 2
C) 3
D) 4
E) depends on $\sigma_0$

3.21
b

Does the previous answer change at all if you're asked for V *outside* the sphere?

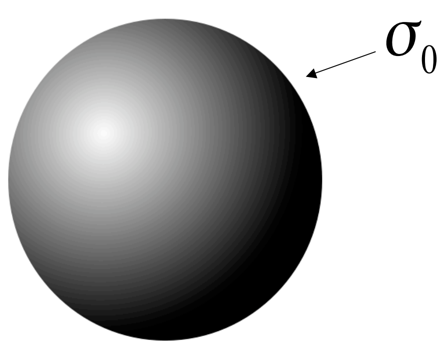
a) yes

b) no

Since the electric field is zero inside this conducting sphere, and $V = -\int \vec{E} \cdot d\vec{l}$ , is V=0 inside as well?

a) Yes

b) No



MULTIPOLE EXPANSION

3.22
a

A small dipole (dipole moment p=qd) points in the z direction.

We have derived $V(\vec{r}) \approx \frac{1}{4\pi\varepsilon_0}\frac{qd\,z}{r^3}$

Which of the following is correct (and "coordinate free")?

A) $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$

B) $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^3}$

C) $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^2}$

D) $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\times\hat{r}}{r^2}$

E) None of these

3.22
b

An ideal dipole (tiny dipole moment p=qd) points in the z direction.

We have derived $\vec{\mathbf{E}}(\vec{r}) = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\vec{\theta}\right)$

Sketch this E field...
(What would change if the dipole separation d was *not* so tiny?)

3.22
c

You have a physical dipole, +q and -q a finite distance d apart.
When can you use the expression:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

A) This is an exact expression everywhere.

B) It's valid for large r

C) It's valid for small r

D) ?

3.22
d

You have a physical dipole, +q and -q, a finite distance d apart.
When can you use the expression

$$V(r) = \frac{1}{4\pi\varepsilon_0} \sum \frac{q_i}{\mathfrak{R}_i}$$

A) This is an exact expression everywhere.

B) It's valid for large r

C) It's valid for small r

D) ?

3.23

Griffiths argues that the force *on* a dipole in an E field is: $\vec{\mathbf{F}} = (\vec{\mathbf{p}} \bullet \vec{\nabla})\vec{\mathbf{E}}$

If the dipole **p** points in the z direction, what direction is the force?

A) Also in the z direction
B) perpendicular to z
C) it could point in any direction

3.24

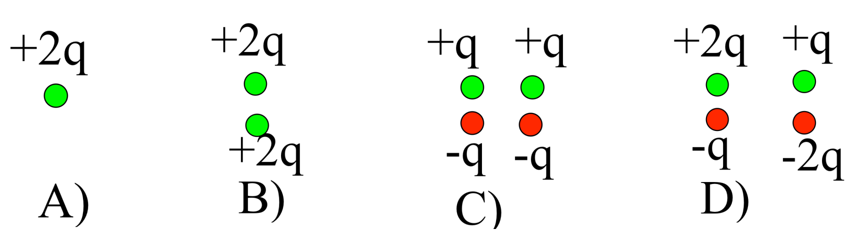
Griffiths argues that the force *on* a dipole in an E field is: $\vec{\mathbf{F}} = (\vec{\mathbf{p}} \bullet \vec{\nabla})\vec{\mathbf{E}}$

If the dipole **p** points in the z direction, what can you say about **E** if I tell you the force is in the x direction?

A) **E** simply points in the x direction
B) Ez must depend on x
C) Ez must depend on z
D) Ex must depend on x
E) Ex must depend on z

3.25

Which charge distributions below produce a potential which looks like $C/r^2$ when you are far away?

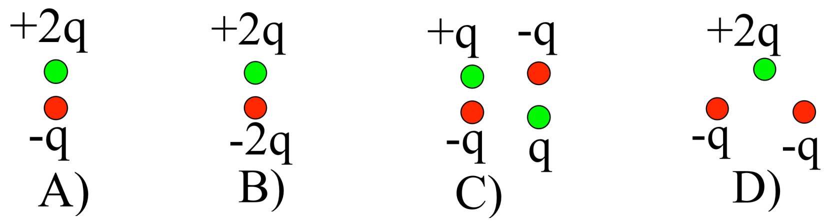



E) None of these, or *more* than one of these!

(Note: for any which you did not select, how DO they behave at large r?)

3.26

Which charge distributions below produce a potential which looks like $C/r^2$ when you are far away?

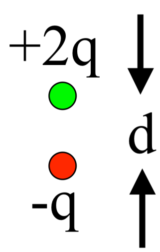



E) None of these, or *more* than one of these!

(Note: for any which you did not select, how DO they behave at large r?)

3.27

What is the magnitude of the dipole moment of this charge distribution?

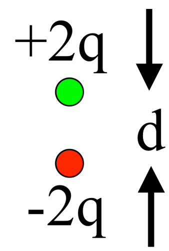



A) qd
B) 2qd
C) 3qd
D) 4qd
E) It's not determined

(To think about:  How does V(r) behave as r gets large?)

3.27

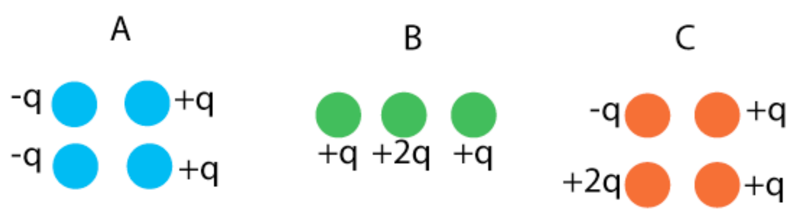
What is the magnitude of the dipole moment of this charge distribution?



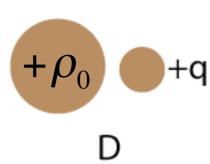

A) qd
B) 2qd
C) 3qd
D) 4qd
E) It's not determined

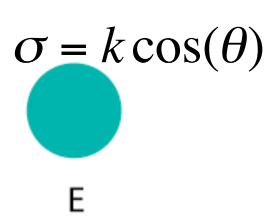
(To think about: How does V(r) behave as r gets large?)

3.28 In which situation is the dipole term the leading non-zero contribution to the potential?

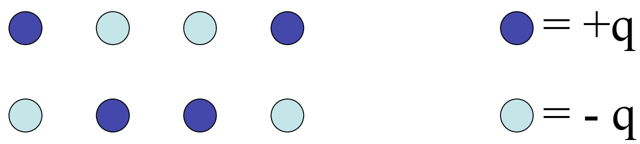



A) A and C
B) B and D
C) only E
D) A and E
E) Some other combo



3.29 In terms of the multipole expansion
V(r) = V(mono) + V(dip) + V(quad) + …
the following charge distribution has the form:




A) V(r) = V(mono) + V(dip) + higher order terms

B) V(r) = V(dip) + higher order terms

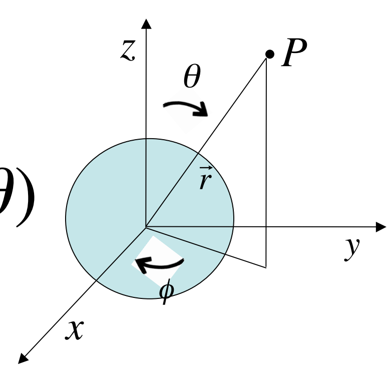
C) V(r) = V(dip)

D) V(r)=only higher order terms than dipole

E) No higher terms, V(r)=0 for this one.

3.30 What is the direction of the dipole moment of the blue sphere?

$$\sigma = k\sin(\theta)$$




a) $\hat{\theta}$
b) $\hat{r}$
c) $\hat{z}$
d) $\hat{\phi}$
e) the dipole moment is zero (or is ill defined)