CHAPTER 4

Electrostatics in Matter:

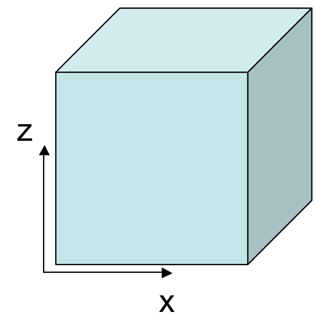
Polarization/dielectrics
Field of polarized object/bound charge
Electric displacement
Linear dielectrics

POLARIZATION

4.1

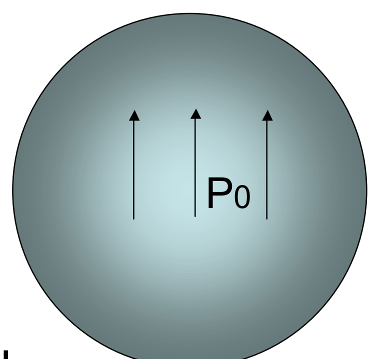
The cube below (side a) has uniform polarization $\mathbf{P}_0$
(which points in the z direction.)
What is the total dipole moment of this cube?

A) zero
B) a^3 P_0
C) P_0
D) P_0/a^3
E) $2\ P_0\ a^2$



4.1 alt

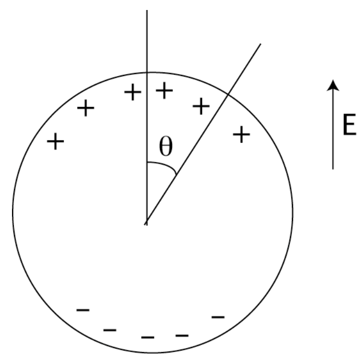
The sphere below (radius a) has uniform polarization $\mathbf{P}_0$ (which points in the z direction.)
What is the total dipole moment of this sphere?



A) zero
B) $P_0\ a^3$
C) $4\pi a^3\ P_0/3$
D) P_0
E) None of these/must be more complicated

4.2
a

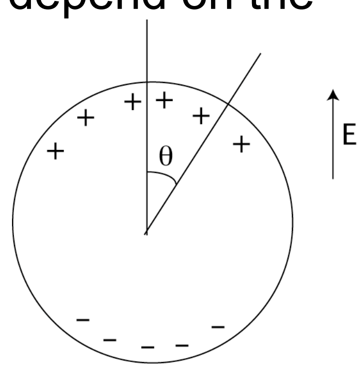
You put a conducting sphere in a uniform E-field. How do you expect the surface charge to depend on the polar angle (θ)?



a) Constant
b) $\cos(\theta)$
c) $\sin(\theta)$
d) Nothing simple, it will have to be an infinite series of sin's and cos's with coefficients.

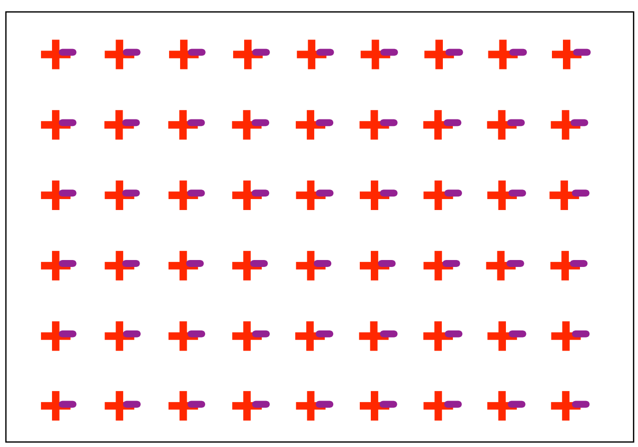
4.2
b

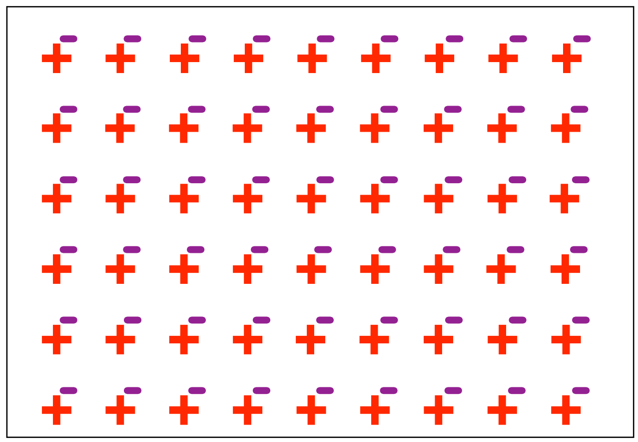
Now what if the sphere is a dielectric? How do you expect the bound surface charge to depend on the polar angle (θ)?

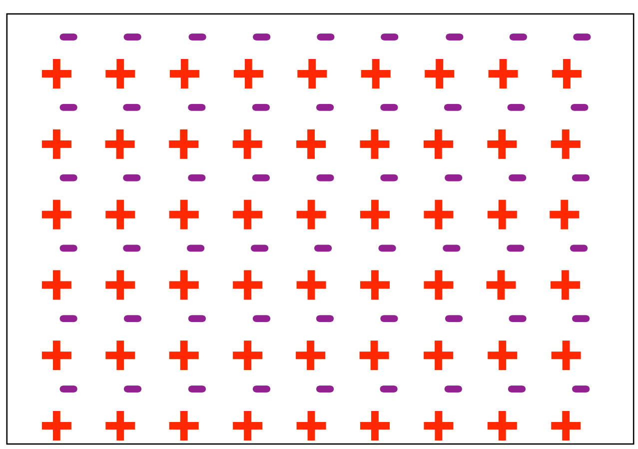


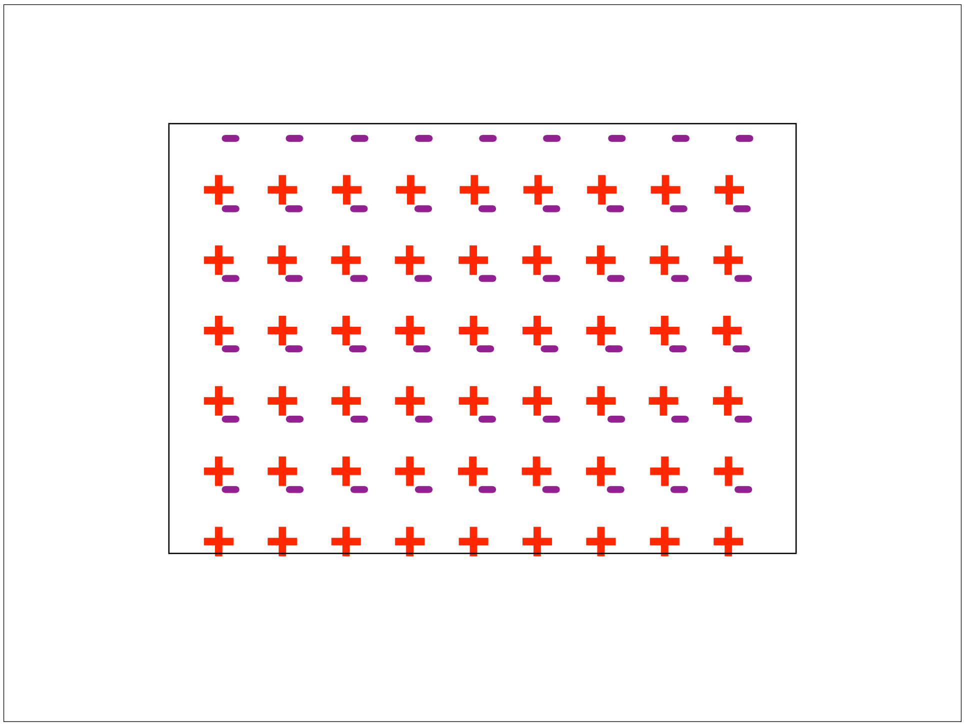
a) Constant
b) $\cos(\theta)$
c) $\sin(\theta)$
d) Nothing simple, it will have to be an infinite series of sin's and cos's with coefficients.

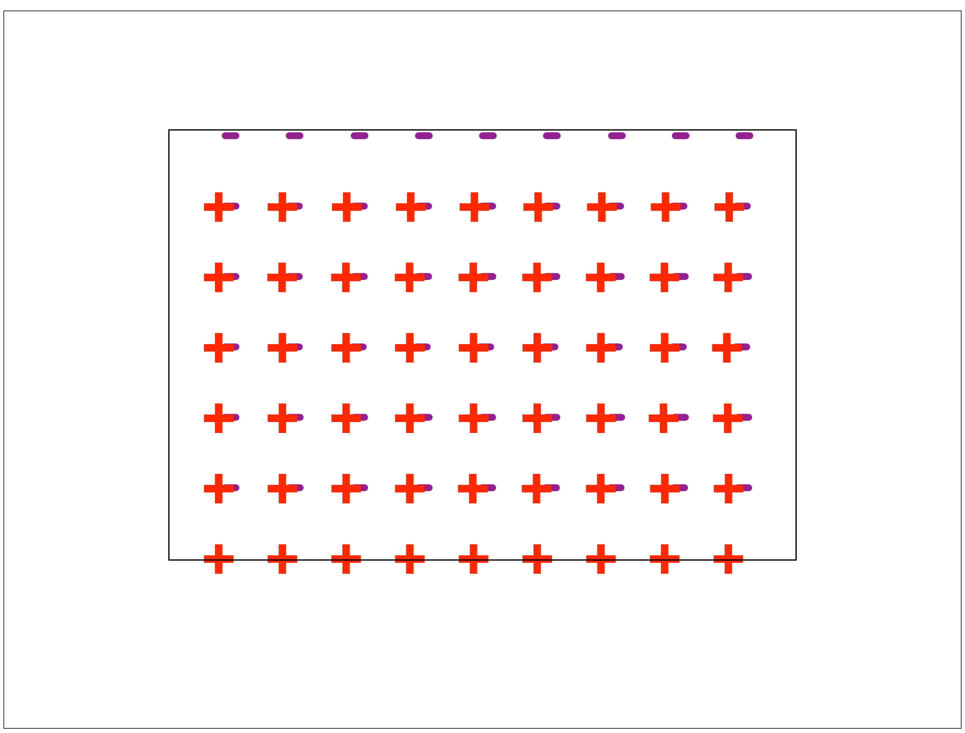
FIELD OF POLARIZED OBJECT

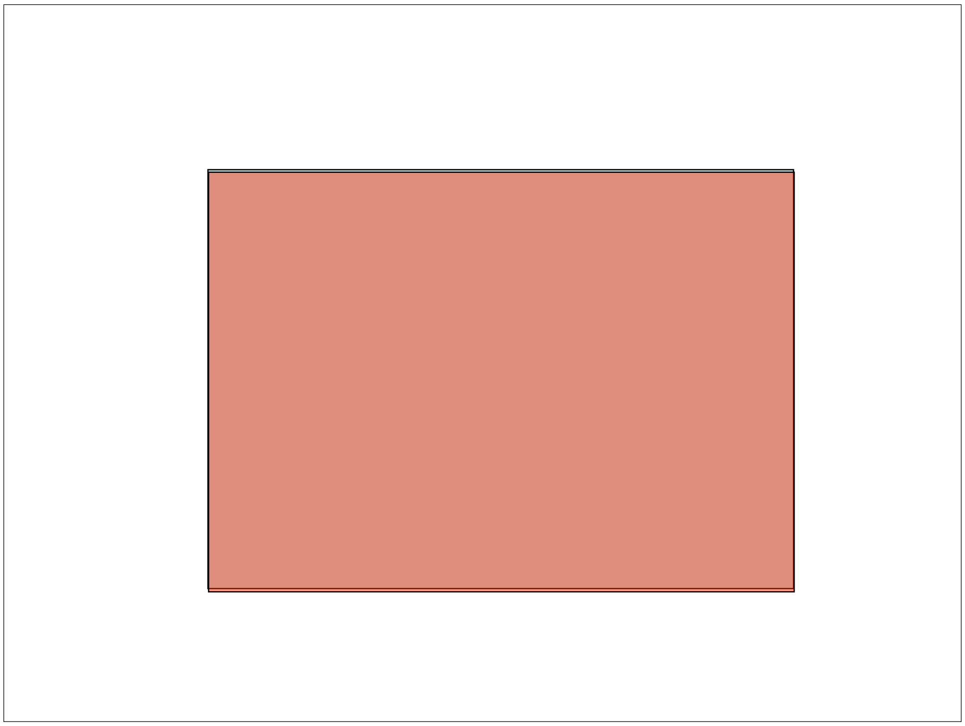


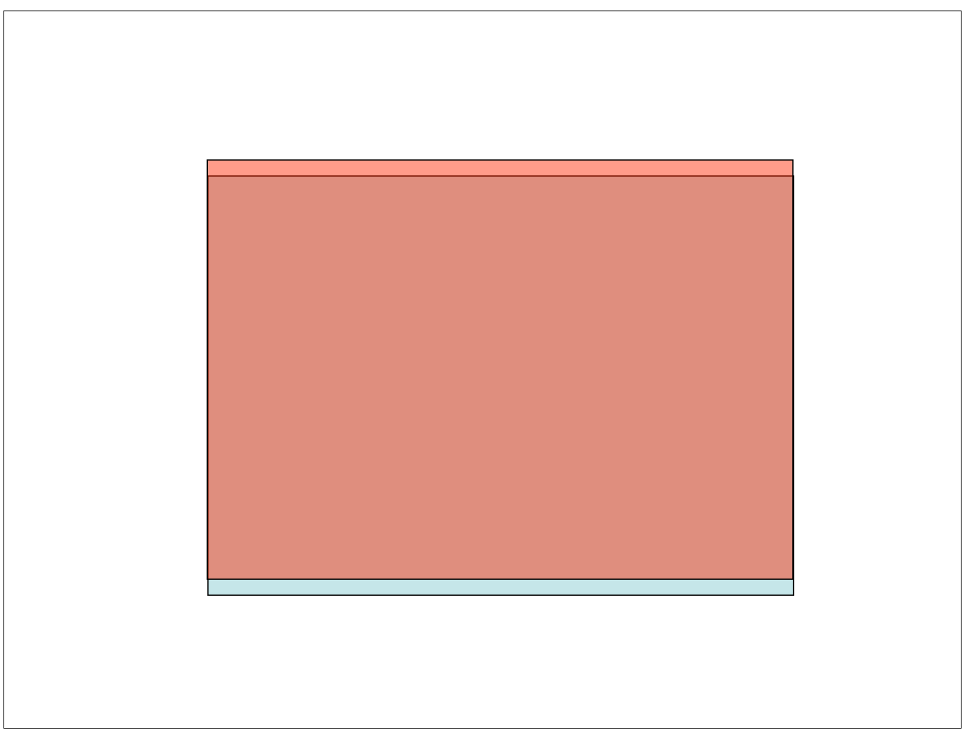






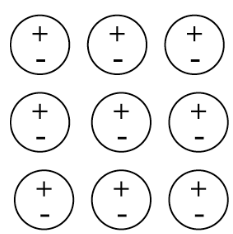




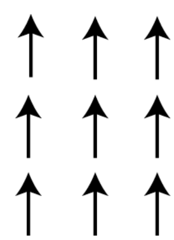


4.3

In the following case, is the bound surface and volume charge zero or nonzero?

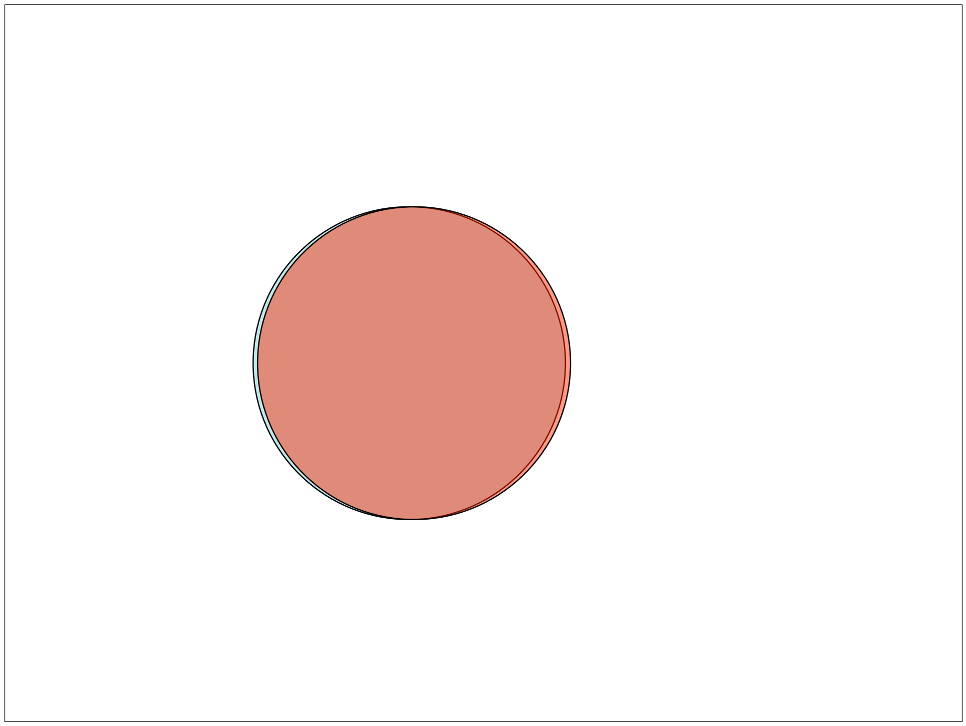


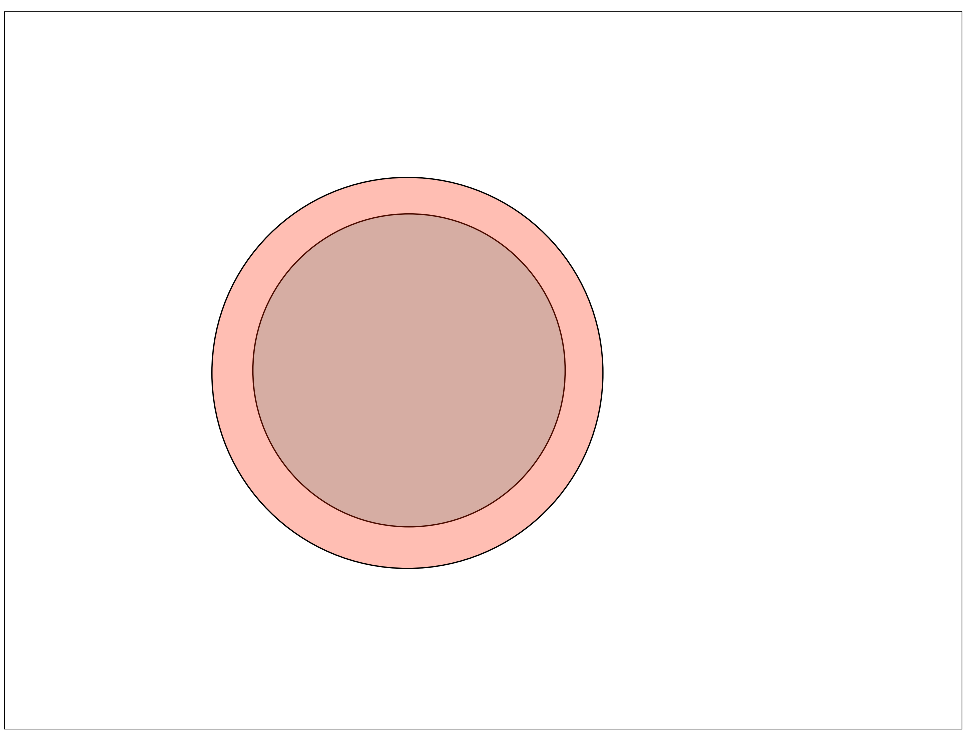
Physical dipoles



idealized dipoles

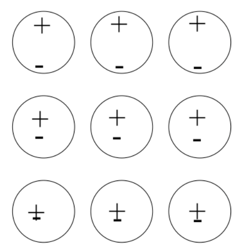
A. $\sigma_b = 0$, $\rho_b \neq 0$
B. $\sigma_b \neq 0$, $\rho_b \neq 0$
C. $\sigma_b = 0$, $\rho_b = 0$
D. $\sigma_b \neq 0$, $\rho_b = 0$

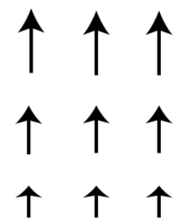




4.3
b

In the following case, is the bound surface and volume charge zero or nonzero?


Physical dipoles

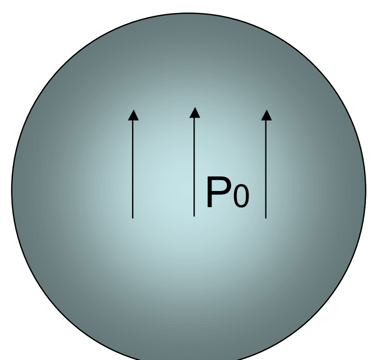

idealized dipoles

A. $\sigma_b = 0$, $\rho_b \neq 0$
B. $\sigma_b \neq 0$, $\rho_b \neq 0$
C. $\sigma_b = 0$, $\rho_b = 0$
D. $\sigma_b \neq 0$, $\rho_b = 0$

4.1
alt

The sphere below (radius a) has uniform polarization $\mathbf{P}_0$ (which points in the z direction.)

What is the total dipole moment of this sphere?

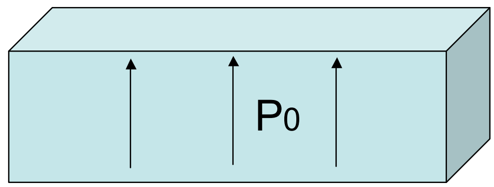


A) zero
B) $P_0 a^3$
C) $4\pi a^3 P_0/3$
D) P_0
E) None of these/must be more complicated

4.4

A dielectric slab (top area A, height h) has been polarized, with **P=**P_0 (in the +z direction)

What is the surface charge density, σ_b, on the bottom surface?



A) 0

B) $-P_0$

C) P_0

D) $P_0 A h$

E) $P_0 A$

In your own words, define what we mean by "free charge", and "bound charge"

ELECTRIC DISPLACEMENT

4.5

We introduced "Electric Displacement" or "D" field: $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$.

If you put a dielectric in an external field E_{ext}, it polarizes, adding a new field, $E_{induced}$ (from the bound charges).

These superpose, making a total field E_{tot}.

Which of these three E fields is the "E" in the formula for D above?

A) E_{ext} B) $E_{induced}$ C) E_{tot}

4.7 c-alt We define **D** $=\varepsilon_0$**E** + **P**, with

$$\oiint \vec{\mathbf{D}} \bullet d\vec{\mathbf{A}} = \mathrm{Q(free, enclosed)}$$

A point charge +q is placed at the center of a dielectric sphere (radius R).
There are no other free charges anywhere.
What is |D(r)|?

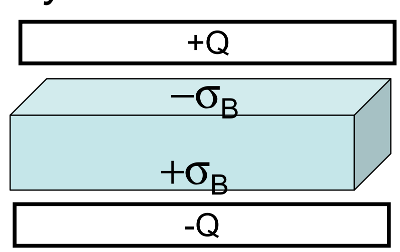
A) $q/(4\pi r^2)$ everywhere
B) $q/(4\pi\varepsilon_0 r^2)$ everywhere
C) Answer A (above) for r<R, but answer B (above) for r>R
D) None of the above, it's more complicated
E) We need more info to answer!

4.6 An ideal (large) capacitor has charge Q. A neutral dielectric is inserted into the gap (and of course, it will polarize)
We want to find **D** *everywhere*

(i) $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$

(ii) $\oiint \vec{\mathbf{D}} \bullet \mathbf{d}\vec{\mathbf{A}} = Q_{\text{free}}$

(iii) $\oiint \vec{\mathbf{E}} \bullet \mathbf{d}\vec{\mathbf{A}} = Q/\varepsilon_0$



Which equation would *you* go to first?
A) i B) ii C) iii
D) Your call: *more* than 1 of these would work!
E) *Can't* solve, unless know the dielectric is linear!

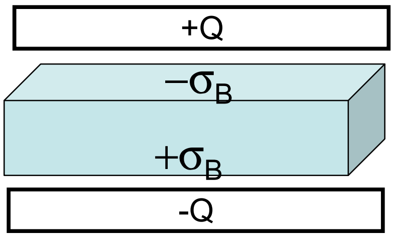
4.6 b

An ideal (large) capacitor has charge Q. A neutral dielectric is inserted into the gap (and of course, it will polarize) We want to find **E** *everywhere*

(i) $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$

(ii) $\oiint \vec{\mathbf{D}} \bullet d\vec{\mathbf{A}} = Q_{free}$

(iii) $\oiint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = Q/\varepsilon_0$



Which equation would *you* go to first?

A) i B) ii C) iii

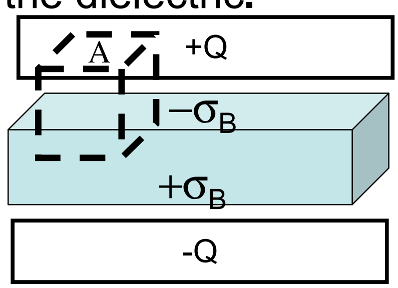
D) Your call: *more* than 1 of these would work!

E) *Can't* solve, unless know the dielectric is linear!

4.6 c

An ideal (large) capacitor has charge Q. A neutral *linear* dielectric is inserted into the gap (with given dielectric constant) We want to find **D** in the dielectric.

$$\oiint \vec{\mathbf{D}} \bullet d\vec{\mathbf{A}} = Q_{free}$$



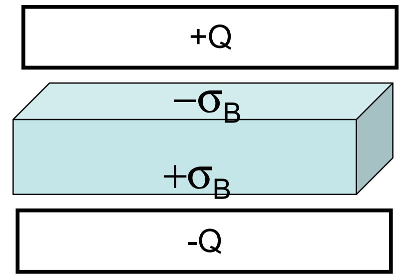
When drawing the Gaussian pillbox shown, what is Q_{free}(enclosed)?

A) σA B) $-\sigma_B A$ C) $\sigma A - \sigma_B A$

D) $\sigma A + \sigma_B A$ E) Something else

4.6 d

An ideal (large) capacitor has charge Q. A neutral *linear* dielectric is inserted into the gap (with given dielectric constant) Now that we have **D** in the dielectric:

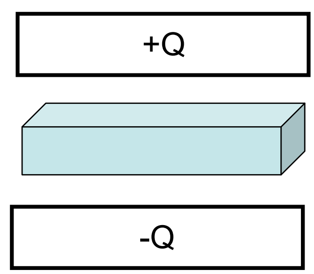


What is **E** inside the dielectric ?

A) **E** = **D** ε
B) **E** = **D**/ε
C) **E** = **D** ε_0
D) **E** = **D**/ε_0
E) Not so simple! Need another method

4.6 e

An ideal (large) capacitor has charge Q. A neutral *linear* dielectric is inserted into the gap (with given dielectric constant) Now that we have **D** in the dielectric:



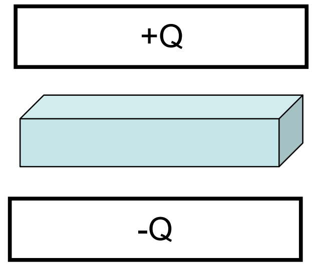
What is **E** in that small gap *above* the dielectric ?

A) **E** = **D** ε
B) **E** = **D**/ε
C) **E** = **D** ε_0
D) **E** = **D**/ε_0
E) Not so simple! Need another method

4.6 f

An ideal (large) capacitor has charge Q. A neutral *linear* dielectric is inserted into the gap (with given dielectric constant)

Where is E discontinuous?



i) near the free charges on the plates

ii) near the bound charges on the dielectric surface

A) i only

B) ii only

C) both i and ii (but nowhere else)

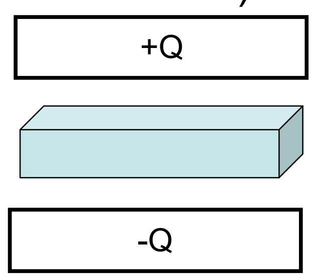
D) both i and ii but also other places

E) none of these/other/???

4.6 g

An ideal (large) capacitor has charge Q. A neutral *linear* dielectric is inserted into the gap (with given dielectric constant)

Where is D discontinuous?



i) near the free charges on the plates

ii) near the bound charges on the dielectric surface

A) i only

B) ii only

C) both i and ii (but nowhere else)

D) both i and ii but also other places

E) none of these/other/???

4.7 A point charge +q is placed at the center of a dielectric sphere (radius R). There are no other free charges anywhere, but the sphere polarizes, producing bound charge in the sphere.

Which charges contribute to the total E field inside the dielectric?

A) +q only
B) +q and the bound charges
C) bound charges only
D) It depends on whether the material is a linear dielectric or not.

4.7b A point charge +q is placed at the center of a dielectric sphere (radius R). There are no other free charges anywhere, but the sphere polarizes, producing bound charge in the sphere.

Which charges contribute to calculating **D** inside the dielectric?

A) +q only
B) +q and the bound charges
C) bound charges only
D) It depends on whether the material is a linear dielectric or not.

4.7
c

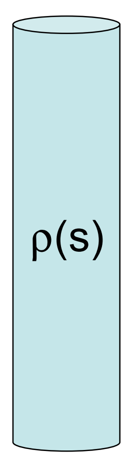
A point charge +q is placed at the center of a dielectric sphere (radius R).
There are no other free charges anywhere.
What is $|D(r)|$?

A) $q/(4 \pi r^2)$ everywhere
B) $q/(4 \pi \varepsilon_0 r^2)$ everywhere
C) It's answer A (above) for $r<R$,
but answer B (above) for $r>R$
D) None of the above, it's more complicated
E) We need more info to answer!
(e.g., is it a linear dielectric?

4.8

An infinitely long, solid non-conducting dielectric rod has been injected ("doped") with a fixed, known charge distribution $\rho(s)$.
(The material responds, polarizing internally)

When computing D in the rod, do you treat this $\rho(s)$ as the "free charges" or "bound charges" or neither?



A) "free charge"
B) "bound charge"
C) Neither of these - a given $\rho(s)$ is already some combination...
D) ???

LINEAR DIELECTRICS

4.9

Can a dielectric material have *free* charges inside it and/or on its surface?

A) No, any charges in or on a dielectric are by definition *bound* charges
B) Yes, but they can ONLY be inside, not on the surface
C) Yes, but they can ONLY be on the surface, not inside
D) Yes.

4.9 b

Is the polarization of a dielectric always proportional to the Electric field, E?

A) Yes, if by "E" you mean "$E_{external}$"
B) Yes, if by "E" you mean "E_{total}"
C) Yes, if by "E" you mean "$E_{induced}$"
D) No, it doesn't have to be proportional.

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

What is ε_r in a vacuum?

(a) 1
(b) 0
(c) infinity
(d) not defined
(e) None of the above

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

What is ε_r in copper?

(a) 1
(b) 0
(c) infinity (or extremely large)
(d) not defined
(e) None of the above

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

What is ε_r in water?

(a) 1
(b) 0
(c) infinity
(d) not defined
(e) None of the above

BOUNDARY VALUE PROBLEMS WITH DIELECTRICS

4.10

You have a straight boundary between two linear dielectric materials (ε_r has one value above, another below, the boundary) There are no free charges in the regions considered.

What is continuous across the boundary?

i) E(parallel) ii) E(perpendicular)

iii) D(parallel) iv) D(perpendicular)

A) i and iii B) ii and iv

C) i and ii D) iii and iv

E) Some other combination!

4.10 You have a boundary between two linear dielectric materials (ε_r has one value above, another below, the boundary) There are no free charges in the regions considered. Which formula will voltage satisfy at the boundary?

A) $\left.\frac{\partial V}{\partial n}\right|_{out} - \left.\frac{\partial V}{\partial n}\right|_{in} = \frac{-\sigma_{free}}{\varepsilon_0}$

B) $\left.\frac{\partial V}{\partial n}\right|_{out} - \left.\frac{\partial V}{\partial n}\right|_{in} = \frac{-\sigma_{bound}}{\varepsilon_0}$

C) $\varepsilon_{out}\left.\frac{\partial V}{\partial n}\right|_{out} - \varepsilon_{in}\left.\frac{\partial V}{\partial n}\right|_{in} = \frac{-\sigma_{free}}{\varepsilon_0}$

D) $\varepsilon_{out}\left.\frac{\partial V}{\partial n}\right|_{out} - \varepsilon_{in}\left.\frac{\partial V}{\partial n}\right|_{in} = 0$

E) None of these, or MORE than one...

4.10 b You have a boundary between two linear dielectric materials (ε_r has one value above, another below, the boundary) There are no free charges in the regions considered. Which formula will voltage satisfy at the boundary?

A) $\left.V\right|_{out} - \left.V\right|_{in} = 0$

B) $\left.V\right|_{out} - \left.V\right|_{in} = \frac{-\sigma_{tot}}{\varepsilon_0}$

C) $\varepsilon_{out}\left.V\right|_{out} - \varepsilon_{in}\left.V\right|_{in} = 0$

D) $\varepsilon_{out}\left.V\right|_{out} - \varepsilon_{in}\left.V\right|_{in} = -\frac{\sigma_{tot}}{\varepsilon_0}$

E) None of these, or MORE than one...

Boundary Value Exercise

- A multi-step question:
- For a dielectric sphere in an electric field E0
- 1 – draw sketch, including E and charge sigma-bound
- 2 – write down boundary conditions
- 3 – What equation will you use to solve E inside (ie, what solution to Laplace)
- 4 – Tell me the steps you will go through without doing calculation
- 5 – How would you expect E inside to depend on E0

FORCE AND ENERGY

4.11 We argued that C goes UP by a factor of ε_r if you fill a capacitor with dielectric. What happens to the stored energy of a capacitor if it's filled with a dielectric?

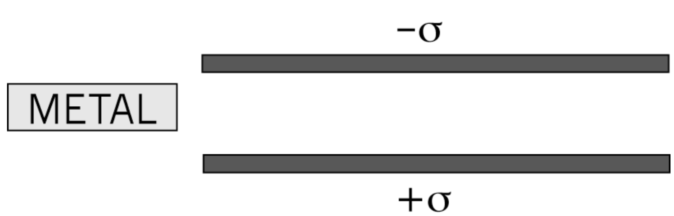
A) It goes up

B) It goes down

C) It is unchanged

D) The answer depends on what else is "held fixed" (V? Q?)

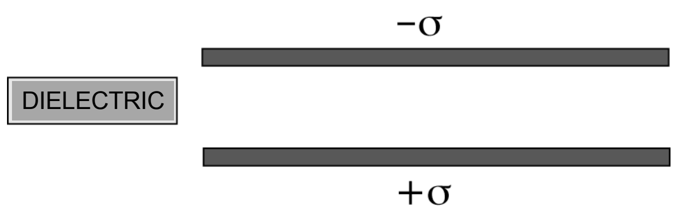
4.12 If we push this conductor inside the *isolated* capacitor, will it be drawn into the capacitor or repelled?



A. It gets sucked into the capacitor
B. It gets pushed out from the capacitor
C. I just don't know.

4.12 b

If we push this conductor inside the *isolated* capacitor, will it be drawn into the capacitor or repelled?



A. It gets sucked into the capacitor
B. It gets pushed out from the capacitor
C. I just don’t know.