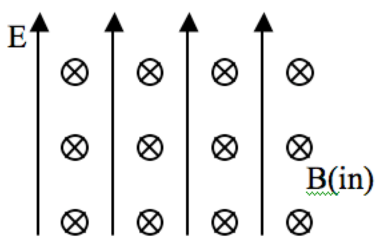
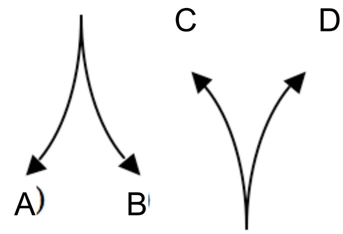
Chapter 5 - Magnetostatics

LORENTZ FORCE

A proton (q=+e) is released from rest in a uniform E field and a uniform B field. The E field points up and the B field points into the page. Which of the paths will the proton follow?

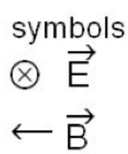


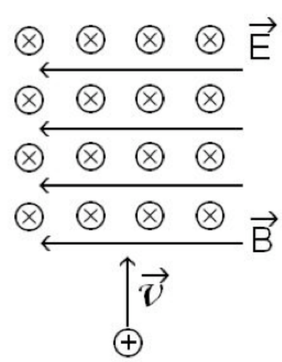


E. It will remain stationary

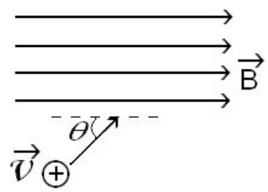
A positively charged particle moving upward with a speed v enters a region with a uniform B field that points to the left and a uniform E field into the page. What is the direction of the net force on the particle at the instant it enters the region?

A.To the left
B.Into the page
C.Out of the page
D.No net force
E.Not enough information





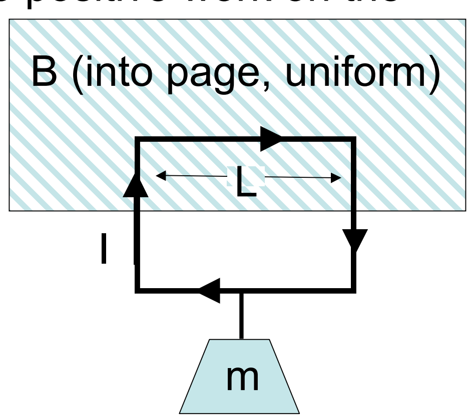
A proton with speed v enters a region with a uniform magnetic field B. The velocity of the proton makes an angle theta with the B field. Which choice best describes the path of the proton after it enters the region?



A. Helical motion
B. Continues along straight line
C. Circular motion perpendicular to the plane of the page. The plane of the circle is perpendicular to B.
D. Circular motion perpendicular to the plane of the page. The plane of the circle is at an angle theta to B
E. This situation is impossible. The velocity of the proton should always be perpendicular to B

5.4

A wire loop in a B field has a current I. The mass is "levitated" by the magnetic force F(mag)=ILb. If you increase the current , does the magnetic force do positive work on the mass?



A) Yes
B) No

CURRENTS & CHARGE CONTINUITY

A student argues that the current through a wire flows throughout its volume, you:

A) Agree, resistance is inversely proportional to cross sectional area, not circumference
B) Disagree, it must flow only on the surface of the wire because the negative charges repel each other
C) Agree for different reasons
D) Disagree for different reasons

5.5

Positive ions flow right through a liquid, negative ions flow left.
The spatial density and speed of both ions types are identical.
Is there a net current through the liquid?

A) Yes, to the right
B) Yes, to the left
C) No
D) Not enough information given

5.7

Current I flows down a wire (length L) with a square cross section (side *a)*
If it is uniformly distributed over the entire wire area, what is the magnitude of the volume current density *J*?

A) $J = I/a^2$ B) $J = I/a$

C) $K = J/(4a)$ D) $J = I/(a^2L)$

E) None of the above

5.6 Current I flows down a wire (length L) with a square cross section (side *a)*
If it is uniformly distributed over the outer surfaces only, what is the magnitude of the surface current density *K*?

A) $K = I/a^2$

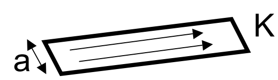
B) $K = I/a$

C) $K = I/(4a)$

D) $K = I/(a^2L)$

E) None of the above

5.8 A "ribbon" (width a) of surface current flows (with surface current density K)
Right next to it is a second identical ribbon of current.
Viewed collectively, what is the new total surface current density?



5.8 A "ribbon" (width a) of surface current flows (with surface current density K)

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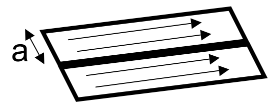


A) K

B) 2K

C) K/2

D) Something else

5.10 Which of the following is a statement of charge conservation?

A) $\frac{\partial \rho}{\partial t} = -\int \vec{\mathbf{J}} \bullet d\vec{\mathbf{l}}$

B) $\frac{\partial \rho}{\partial t} = -\iint \vec{\mathbf{J}} \bullet d\vec{\mathbf{A}}$

C) $\frac{\partial \rho}{\partial t} = -\iiint (\nabla \bullet \vec{\mathbf{J}})\, d\tau$

D) $\frac{\partial \rho}{\partial t} = -\nabla \bullet \vec{\mathbf{J}}$

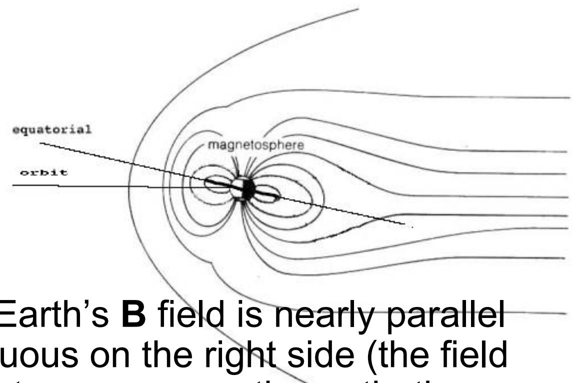
E) Not sure/can't remember

Discussion

- Why does B follow the right hand rule? Is it contained in Ampere's Law?
- When you find the B field for a point in space near a long current carrying wire, what *could* B depend on? Given the form of Biot-Savart law, what would you GUESS?

BIOT SAVART LAW

Earth’s Magnetosphere

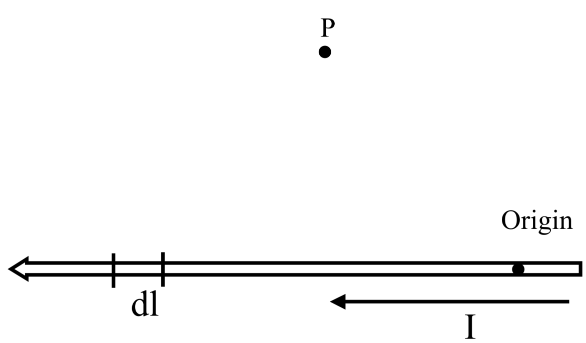


Notice that the Earth’s **B** field is nearly parallel and discontinuous on the right side (the field lines from the top are magnetic south, the bottom ones are magnetic north). Is the Earth’s magnetic field driving a sheet of current in outer space? If so where does it go?

5.11 To find the magnetic field B at P due to a current-carrying wire we use the Biot-Savart law,

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$$

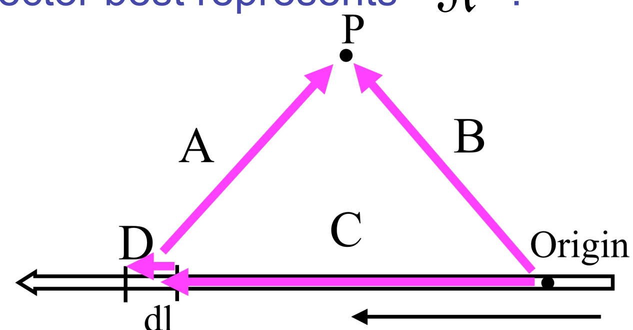
In the figure, with “dl” shown, what is $\vec{\mathfrak{R}}$?



5.11 To find the magnetic field B at P due to a current-carrying wire we use the Biot-Savart law,

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$$

In the figure, with "dl" shown, which purple vector best represents $\mathfrak{R}$?



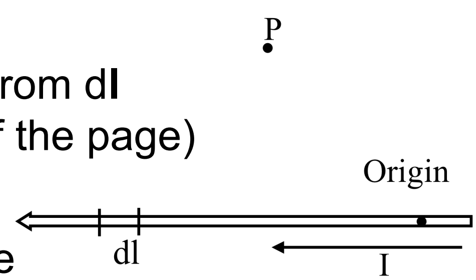
E) None of these is close!

5.12 To find the magnetic field B at P due to a current-carrying wire we use the Biot-Savart law,

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$$

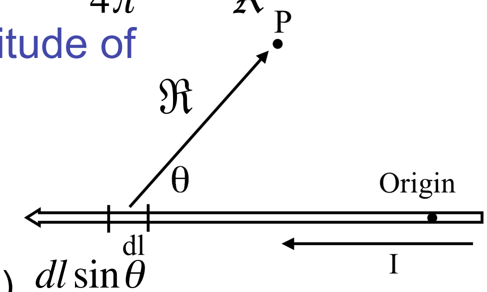
What is the *direction* of the infinitesimal contribution d**B**(P) created by current in d**l**?

A) Up the page
B) Directly away from d**l** (in the plane of the page)
C) Into the page
D) Out of the page
E) Some other direction



5.13 To find the magnetic field B due to a current-carrying wire, below, we use the Biot-Savart law, $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$

What is the magnitude of $\frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$?



a) $\frac{dl \sin\theta}{\mathfrak{R}^2}$

b) $\frac{dl \sin\theta}{\mathfrak{R}^3}$

c) $\frac{dl \cos\theta}{\mathfrak{R}^2}$

d) $\frac{dl \cos\theta}{\mathfrak{R}^3}$

e) $\frac{dl}{\mathfrak{R}^2}$

(And, what's $\mathfrak{R}$ here?)

5.14

What is B at the point shown?

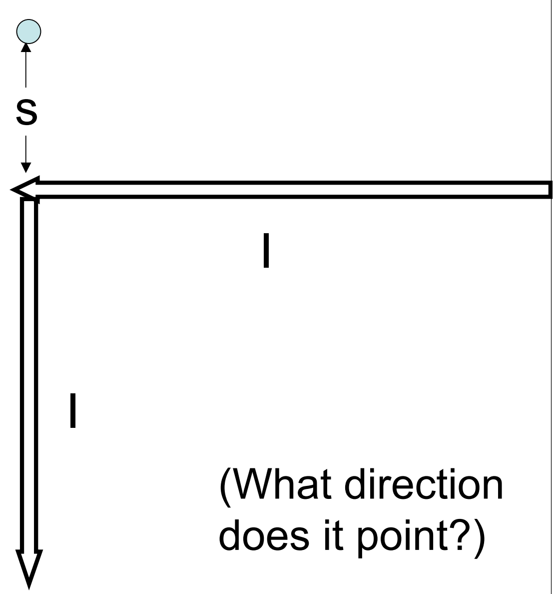
A) $\frac{\mu_0}{\pi s} I$

B) $\frac{\mu_0}{2\pi s} I$

C) $\frac{\mu_0}{4\pi s} I$

D) $\frac{\mu_0}{8\pi s} I$

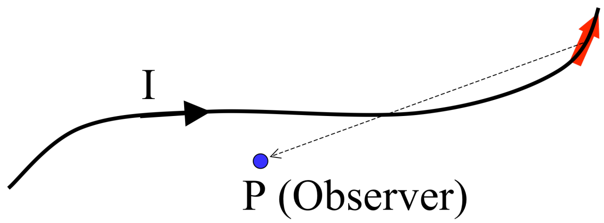
E) None of these



(What direction does it point?)

5.16 com

What do you expect for direction of **B**(P)? How about direction of d**B**(P) generated JUST by the segment of current d**l** in red?



A) **B**(p) in plane of page, ditto for d**B**(P, by red)

B) **B**(p) into page, d**B**(P, by red) into page

C) **B**(p) into page, d**B**(P, by red) out of page

D) **B**(p) complicated - has mult component (*not* ⊥ or || to page), ditto for d**B**(P, by red)

E) Something else!!

5.15

To find the magnetic field B due to a current-carrying loop, we use the Biot-Savart law, $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$

What is the magnitude of $\frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$?

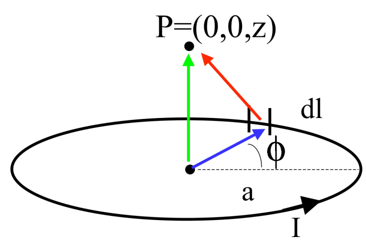
A) $\frac{dl \sin\phi}{z^2}$

B) $\frac{dl}{z^2}$

C) $\frac{dl \sin\phi}{(z^2 + a^2)}$

D) $\frac{dl}{(z^2 + a^2)}$

E) Something quite different!



(Which colored arrow is $\mathfrak{R}$? **r**? **r'**?)

5.15 b

To find the magnetic field B due to a current-carrying loop, we use the Biot-Savart law, $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$

What is the d$\mathbf{B}_z$ (the contribution to the vertical component of **B** from this dl segment?)

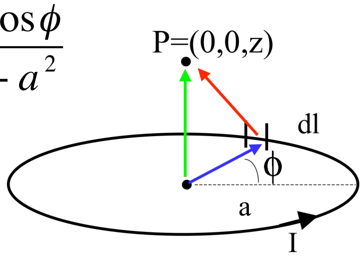
A) $\frac{dl}{z^2 + a^2} \frac{a}{\sqrt{z^2 + a^2}}$

B) $\frac{dl}{z^2 + a^2}$

C) $\frac{dl}{z^2 + a^2} \frac{z}{\sqrt{z^2 + a^2}}$

D) $\frac{dl \cos\phi}{z^2 + a^2}$

E) Something quite different!



Biot-Savart: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}') \times \Re(hat)}{\Re 2} d\tau'$

What does **J**(**r**') x **R**(hat) even mean?

A) It is the "swirling" of the **B** field at **r**'

B) It is the "swirling" of the **B** field at **r** due to the current at **r**'

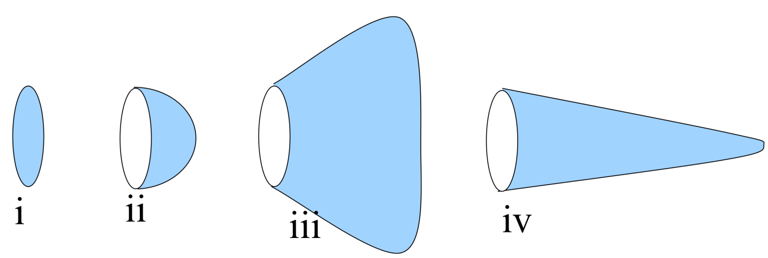
C) It is the direction and magnitude of the **B** field at **r**'

D) It is the direction and magnitude of the **B** field at **r** due to the current at **r**'

DIVERGENCE & CURL OF B; STOKES THEOREM

5.16

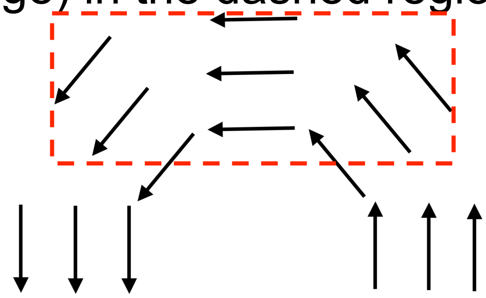
Rank order $\left|\iint \vec{\mathbf{J}} \bullet d\vec{\mathbf{A}}\right|$ (over blue surfaces) where **J** is uniform, going left to right:



A) iii > iv > ii > i
B) iii > i > ii > iv
C) i > ii > iii > iv
D) Something else!!
E) Not enough info given!!

5.17 a

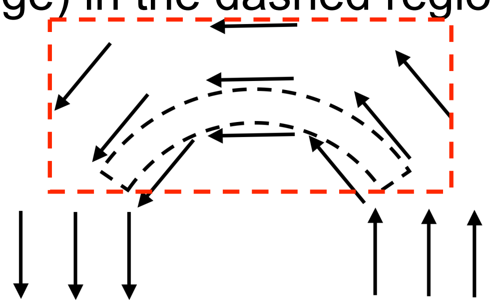
If the arrows represent a B field (note that |B| is the same everywhere), is there a nonzero **J** (perpendicular to the page) in the dashed region?



A. Yes
B. No
C. Need more information to decide

5.17 b

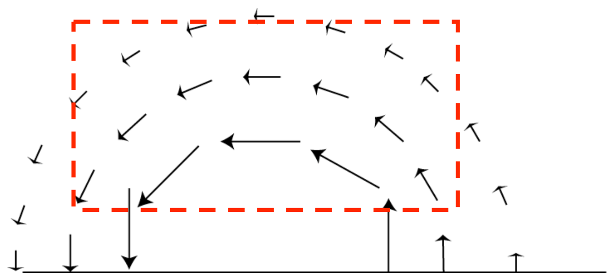
If the arrows represent a B field (note that |B| is the same everywhere), is there a nonzero **J** (perpendicular to the page) in the dashed region?



A. Yes
B. No
C. Need more information to decide

5.17
c

If the arrows represent a B field, is there a **J** (perpendicular to the page) in the dashed region?

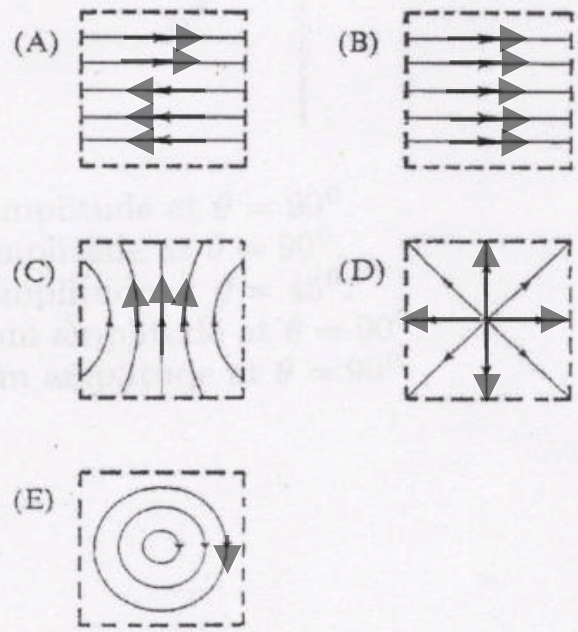


A. Yes
B. No
C. Need more information to decide

AMPERE'S LAW

5.18

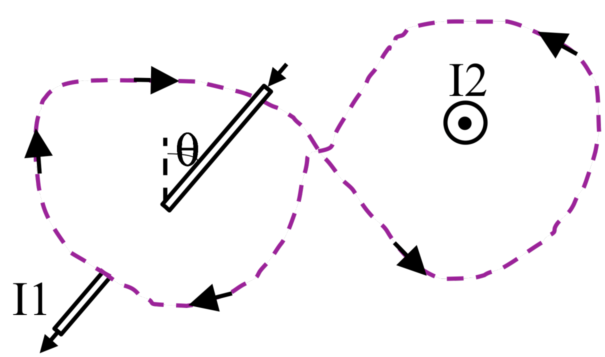
Pick a sketch showing B field lines that *violate* one of Maxwell's equations within the region bounded by dashed lines.



(What currents would be needed to generate the others?)

5.22

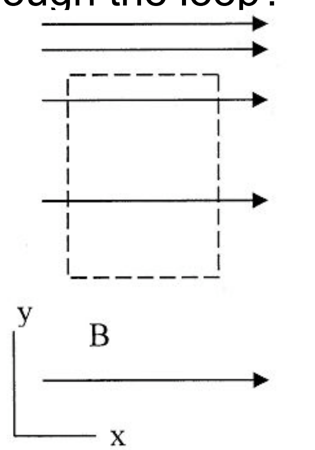
What is $\oint \vec{\mathbf{B}} \bullet \mathbf{d}\vec{\mathbf{l}}$ around this purple (dashed) Amperian loop?



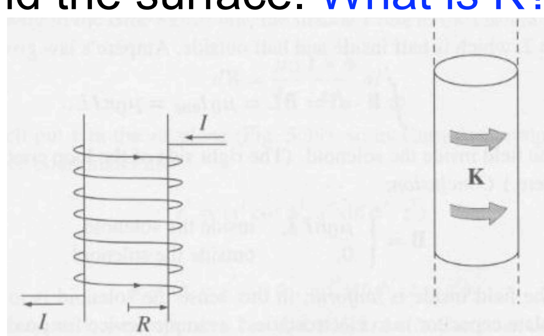
A) $\mu_0 (|I_2| + |I_1|)$
B) $\mu_0 (|I_2| - |I_1|)$
C) $\mu_0 (| I_2 | + | I_1 | \sin\theta)$
D) $\mu_0 (| I_2 | - | I_1 | \sin\theta)$
E) Something else!

5.19 The magnetic field in a certain region is given by $B(x,y) = Ay\hat{x}$ (A is a positive constant) Consider the imaginary loop shown. What can you say about the electric current passing through the loop?

A. must be zero
B. must be nonzero
C. Not enough info



5.20 A solenoid has a total of N windings over a distance of L meters. We "idealize" by treating this as a surface current running around the surface. What is K?

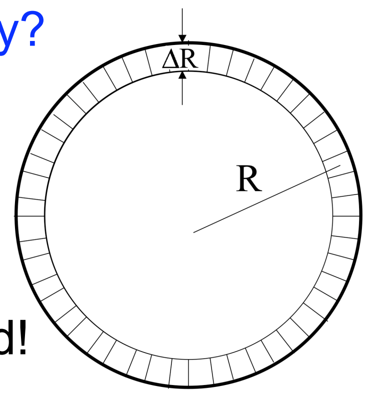


A) I B) NI C) I/L D) I N/L
E) Something else...

5.21
a

A thin toroid has (average) radius R and a total of N windings with current I. We "idealize" this as a surface current running around the surface.

What is K, approximately?



A) I/R B) $I/(2\pi R)$

C) NI/R D) $NI/(2\pi R)$

E) Something totally different, ΔR is involved!

5.21
b

What direction do you expect the B field to point?

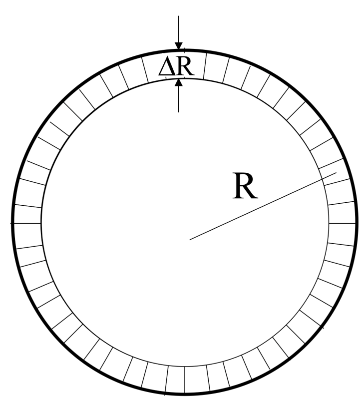
A) Azimuthally

B) Radially

C) In the z direction (perp to page)

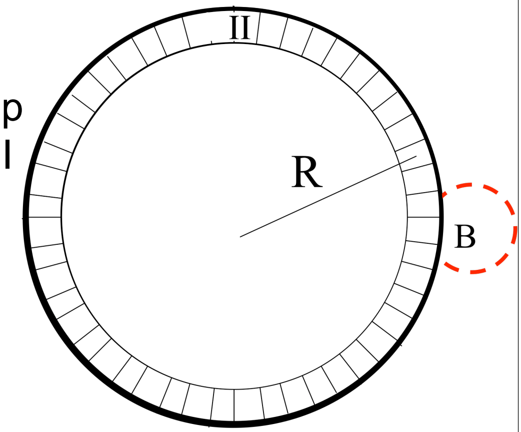
D) Mix of the above/depends on where...

E) zero everywhere



5.21
c

What Amperian loop would you draw to find B "inside" the Torus (region II)



A) Large "azimuthal" loop
B) Small loop in region II
C) Smallish loop from region II to outside (where B=0)
D) Like A, but perp to page
E) Something entirely different

5.21
d

Two long coaxial solenoids each carry current I but in opposite directions.

The inner solenoid (radius a) has n1 turns per unit length, and the outer one (radius b) has n2.

Find B (i) inside the solenoid, (ii) between them, and (iii) outside both.

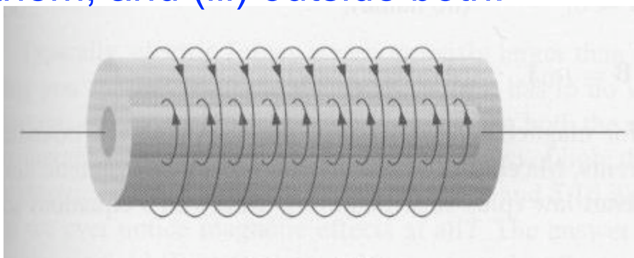
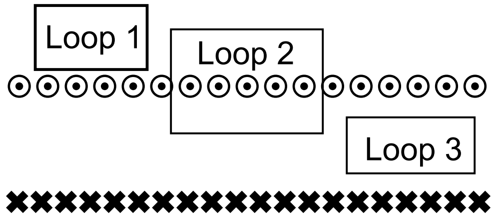


Figure 5.42

5.23

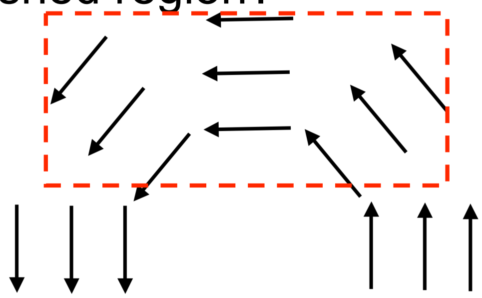


In the case of the infinite solenoid we used loop 1 to argue that the **B**-field outside is zero. Then we used loop 2 to find the **B**-field inside. What would loop 3 show?

a) The **B**-field inside is zero
b) It does not tell us anything about the **B**-field
c) Something else

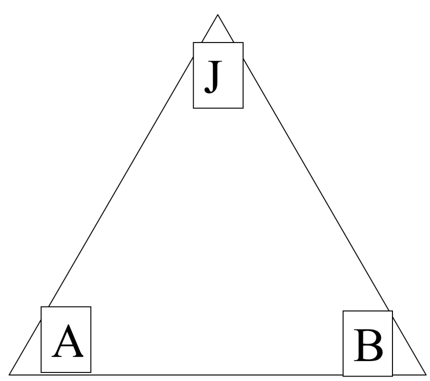
MAGNETIC VECTOR POTENTIAL

5.24 If the arrows represent the vector potential **A** (note that |A| is the same everywhere), is there a nonzero **B** in the dashed region?



A. Yes
B. No
C. Need more information to decide

Compare the magnetostatic triangle (p.240) with the electrostatic triangle (pg. 87). How is the potential similar/different to the vector potential?



5.25

$$\nabla^2 \vec{\mathbf{A}} = -\mu_0 \vec{\mathbf{J}}$$

In Cartesian coordinates, this means:

$$\nabla^2 \mathrm{A}_x = -\mu_0 \mathrm{J}_x \quad , \text{ etc.}$$

Does it also mean, in spherical coordinates, that $\nabla^2 \mathrm{A}_r = -\mu_0 \mathrm{J}_r$?

A) Yes
B) No

5.25
b

$$\vec{\mathbf{A}}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint \frac{\vec{\mathbf{J}}(r')}{\Re} d\tau'$$

Can you calculate that integral using spherical coordinates?

A) Yes, no problem
B) Yes, r' can be in spherical, but **J** still needs to be in Cartesian components
C) No.

5.26

What is $\oint \vec{\mathbf{A}}(\vec{r}) \bullet d\vec{\mathbf{l}}$

A) The current density **J**
B) The magnetic field **B**
C) The magnetic flux Φ_B
D) It's none of the above, but is something simple and concrete
E) It has no particular physical interpretation at all

5.27

Suppose A is azimuthal, given by

$$\vec{\mathbf{A}} = \frac{c}{s}\hat{\varphi}$$

What can you say about curl(**A**)?

A) curl**(A)**=0 everywhere

B) curl(**A**) = 0 everywhere except at s=0.

C) curl(**A**) is nonzero everywhere

D) ???

On paper (don’t forget your name!) in your own words (by yourself):

What is the idea behind the magnetic vector potential?
What does it accomplish?
In what ways is it like (or NOT like!) the electric potential?

BOUNDARY CONDITIONS

5.28

Choose all of the following statements that are implied by $\oiint \vec{B} \cdot d\vec{a} = 0$ (for any closed surface you like)

(I) $\vec{\nabla} \cdot \vec{B} = 0$

(II) $B^{//}_{above} = B^{//}_{below}$

(III) $B^{\perp}_{above} = B^{\perp}_{below}$

A) (II) only

B) (III) only

C) (I) and (II) only

D) (I) and (III) only

E) All of the above

5.28 b

In general, which of the following are continuous as you move past a boundary?

A) **A**

B) Not all of **A**, just A_{perp}

C) Not all of **A**, just $A_{//}$

D) Nothing is guaranteed to be continuous regarding **A**

DIPOLES, MULTIPOLES

Griffiths derives a B field:

$$\vec{\mathbf{B}} = \frac{c}{r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

Sketch this, what does it look like?

This is the formula for an ideal magnetic dipole:

$$\vec{\mathbf{B}} = \frac{c}{r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

What is different in a sketch of a *real* (physical) magnetic dipole (like, a small current loop)?

5.29 This is the formula from Griffiths for a magnetic dipole:

$$\vec{\mathbf{A}}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi}$$

Is this the *exact* vector potential for a flat ring of current with m=Ia, or is it approximate?

A) It's exact

B) It's exact if |r| > radius of the ring

C) It's approximate, valid for large r

D) It's approximate, valid for small r

5.30

The leading term in the vector potential multipole expansion involves $\oint d\vec{\mathbf{l}}'$

What is the magnitude of this integral?

A) R

B) 2 π R

C) 0

D) Something entirely different/it depends!

(See Chapter 6 concept tests for force and torque on dipole questions)