Magnetostatics

New topic. Until 1800's, different distinct force of nature!

Lodestones attract + repel. Is it electric? No.

→ Think of how you convince yourself Kitchen magnets are not electrical in nature!

→ Magnets don't attract or repel charges!

→ Magnets don't fade away

→ Charged rods never repel magnets

Compass needles are simple indicators of presence (+ direction, + even strength) of magnetic field. (1600 Gilbert: Earth is a big lodestone

1568 Mercator: Needles point terrestrial source, not North star

90 yrs before Newton realizes earth gravitates!

1820 Oersted observed that currents produces magnetic effects on compasses.

$$\vec{F}_{\text{on } q} (= m\vec{a}) = q\vec{E} + q\vec{v} \times \vec{B}$$

Purely from exp't. "Lorentz force Law"

Spent 8 weeks learning about this!

Magnetic field

Units: $1 \text{ Tesla} = \dfrac{1 \text{ N}}{\text{C} \cdot \text{m/s}}$

Big! $= 10^4$ gauss

(Earth field $\simeq \frac{1}{2}$ gauss)

Order of investigation:

1) General features + sources of $\vec{B}$ (still static/steady, no time dep. yet!)

2) Consequences of $\vec{F} = q\vec{v} \times \vec{B}$

3) Current = source of $\vec{B}$ fields
   - current
   - surface current
   - volume current

4) Biot-Savart: "Like Coulomb's law for magnetism"

   (How current creates $\vec{B}$).

5) Ampere's Law: "Like Gauss' Law: How currents $\Rightarrow$ $\vec{B}$ fields"

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1) $\vec{B}$ fields are both very different + also very similar/analogous to $\vec{E}$. Look for parallels + connections, some are very deep!

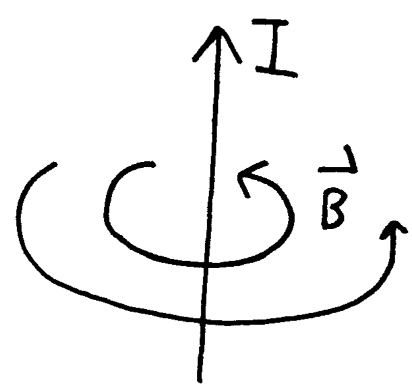
Statics:

$$\nabla \cdot E = \rho/\varepsilon_0$$ ← ρ is "source of E" which diverges

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$ ← $\vec{J}$ = current is "source of B" which curls.

$\underbrace{4\pi \cdot 10^{-7}}$ N/A^2 with 1 A = 1 C/sec.

(Exact! Defines the Amp $\Rightarrow$ the Coulomb!)



$$\nabla \cdot \vec{B} = 0$$ ← No magnetic monopoles

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2) Since $\vec{F} = q\vec{v}\times\vec{B}$, $\quad \vec{F}\cdot d\vec{\ell} = q\left(\vec{v}\times\vec{B}\right)\cdot(\underbrace{\vec{v}\,dt}_{d\vec{\ell}}) = 0.$

↑ $\perp$ to $\vec{v}$!

$\vec{B}$ fields do not work. (They "bend" trajectories) (we'll return to this)

<u>Cyclotrons</u>: $\vec{F} = \frac{d\vec{p}}{dt} = q\,\vec{v}\times\vec{B}$. Suppose $\vec{B} = B_0\,\hat{z}$

- so $F_z = 0$, any motion in z-direction is unaffected, "drifts"

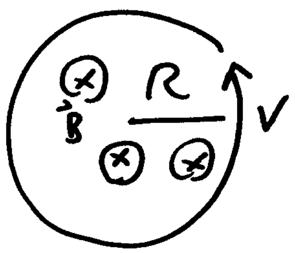
(so let's set $v_z = 0$.)

- No work $\Rightarrow$ $|\vec{p}|$ won't change. Force is $\perp$ to motion,

constant speed $\Rightarrow$ <u>uniform</u> circular motion.

$$\vec{F} = m\vec{a} \Rightarrow qvB = m\,v^2/R$$

Circular motion with



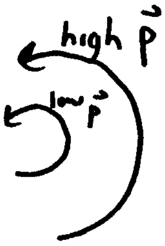
$$R = \frac{mv}{qB} = \frac{|p|}{qB}$$

$$\text{Frequency} = \frac{\text{cycles}}{\text{sec}} = 1/(\text{sec}/\text{revolution})$$

But $v\cdot T = 2\pi R$

$\Rightarrow T = 2\pi R/v$

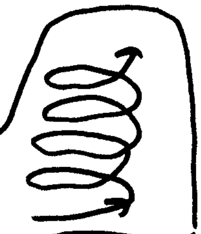
so $$f_{\text{cyclotron}} = \frac{v}{2\pi R} = \frac{qBR/m}{2\pi R} = \frac{qB}{2\pi m}$$

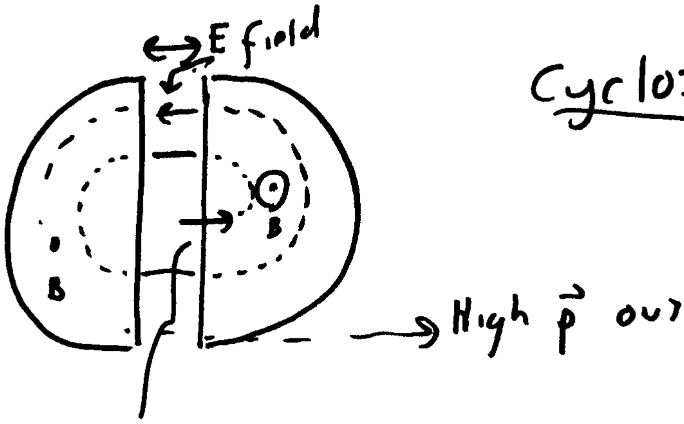


same freq!

<u>Indep.</u> of radius or initial speed!

If $v_z \neq 0$, superpose "z drift" $\Rightarrow$ <u>helix</u>





Cyclotron

inject low $\vec{p}$ electron, fixed time $T_{\frac{1}{2}} = \frac{1}{2f} = \frac{qB}{4\pi m}$ to go "half circle", then turn on $\vec{E}$ to accel across gap (capac!)

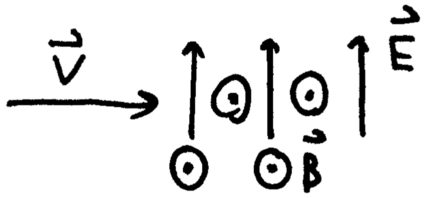
Capac switches sign every $\frac{qB}{4\pi m}$ sec, (steady freq) $\Rightarrow$ electrons accel each time in $\vec{E}$ field, can build up KE, $\vec{B}$ field "contains" the electrons.

Lots of uses, particle accelerators
nowadays, medical beams!

Works up to relativistic energies, then need other tricks

[E.g Fermilab has fixed R, increases $\vec{B}$ as speed up, gets protons up to TeV = 10^{12} eV
($R \simeq 2$ Km)]

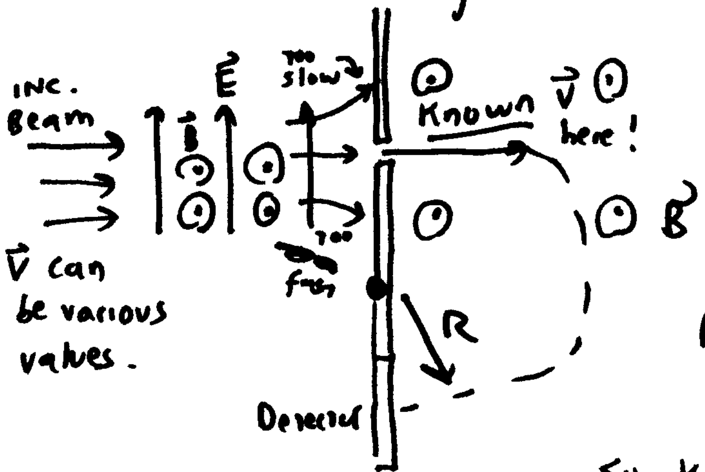
<u>Mass</u> spec<u>trometer</u>



velocity selector. $\vec{F} = qE$ up
qvB down

If $qE = qvB$, i.e. $v = E/B$, then

make it through, in straight line...



If know q (usually e, or $-e$)
Veloc. selector tells you $\vec{v}$

$R = \frac{|P|}{qB}$ tells you $m\vec{v}$,

so know $m \Rightarrow$ <u>identify</u> <u>atoms</u>.

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what if release q from <u>rest</u> in the "velocity selector".

Something different!

$\vec{E} = E\hat{z}$

$\vec{B} = B\hat{x}$

$\Rightarrow F_x = 0$ so $V_x = 0$

$\Rightarrow \vec{V} = (0, V_y, V_z)$

$$\vec{F} = m(0, \dot{v}_y, \dot{v}_z) = q(0,0,E) + q\vec{v}\times\vec{B}$$

$$V_y\hat{y}\times B\hat{x} + V_z\hat{z}\times B\hat{x} = V_y B(-\hat{z}) + V_z B\hat{y}$$

$$= (0, qV_zB, qE - qV_yB)$$

so $m\dot{V}_y = qV_zB \quad \Rightarrow m\ddot{V}_y = qB\dot{V}_z$ Plug this into next one:

$m\dot{V}_z = qE - qV_yB \Rightarrow m\left(m\ddot{V}_y/qB\right) = qE - qBV_y$

thus, $\dot{V}_y = \frac{q^2 B}{m^2}\left(E - BV_y\right)$

this is of the form $\ddot{x} = a - bx$ $\left(a = \frac{q^2}{m^2}BE, \quad b = \frac{q^2 B^2}{m^2} \right)$

The general sol'n of this ↑ is

$$x = \tilde{C}_1 \sin(\sqrt{b}\,t) + \tilde{C}_2 \cos(\sqrt{b}\,t) + a/b$$

(Proof: check it! :) It has 2 undetermined coeffs...)

But here $x = V_y = dy/dt$, so integrating gives

$$y = C_1 \cos(\sqrt{b}\,t) + C_2 \sin(\sqrt{b}\,t) + \frac{a}{b}t + C_3$$

with $\sqrt{b} = qB/m$ and $\frac{a}{b} = E/B$, see above!

~~then~~ $\cancel{\dot{V}_z = \frac{qE}{m} - \frac{qB}{m}V_y}$ ~~can be integrated, to get~~

$$\cancel{V_z = \frac{qE}{m}t - \frac{qB}{m}\left(\frac{C_1}{?}\cos\sqrt{b}\,t + C_2(\sqrt{b}\,t + \frac{E}{b}t + C_3\right)}$$

Easier! $V_z = \frac{m\dot{V}_y}{qB} = \frac{m}{qB}\left(-\frac{q^2B^2}{m^2}\left(C_1\cos\sqrt{b}\,t + C_2\sin\sqrt{b}\,t\right)\right)$

See prev page

$$= -\frac{qB}{m}\left(C_1\cos\sqrt{b}\,t + C_2\sin\sqrt{b}\,t\right)$$

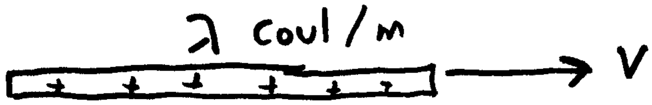
so $z = \int V_z \cdot dt = -C_1\sin\sqrt{b}\,t + C_2\cos\sqrt{b}\,t + C_4$

(See Griffiths for interp!)

Currents: current is the source of $\vec{B}$ fields, (+ also reacts to B fields), so it's very important to clearly define + understand it. It's a measure of the flow-rate of charge. Current counts "how many charges pass by each sec."

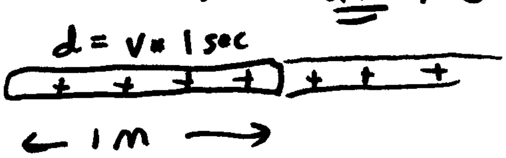
$$|I| = dQ/dt$$

Consider first a line charge (λ coulombs/meter) moving steadily with velocity $\vec{V}$



↑ How many coul move past this point in 1 sec?

• $V \cdot T = d$, so. all the charges in the chunk $V \cdot 1$ sec long "behind" the point will make it past!



That's $Q = \lambda \cdot d = \lambda \cdot V \cdot (1\,\text{sec})$

so charge in one sec $= \dfrac{Q}{T} = \lambda V$.

so $\vec{I} = \lambda \vec{V}$. ← Griffiths convention is to call $\vec{I}$ a vector, (not everyone does this)

Note: If λ is negative, current goes "other way".

So $\xleftarrow{V_-} \ominus$ is same current as $\oplus \xrightarrow{V_+}$, both are $\vec{I} \longrightarrow$

- I measured in $\frac{\text{Coul}}{\text{sec}}$ = Amperes.
- If the wire has $n_L \frac{\text{charge carriers}}{\text{length}}$, each of which carries q, then $\lambda \frac{\text{coul}}{m} = n_L \frac{\text{carriers}}{m} * q \frac{\text{Coul}}{\text{carrier}}$

so $\vec{I} = n_L q \vec{v}$

Since $F_{MAG} = q\,\vec{v}\times\vec{B}$ for individual charges, a current feels a force too (each individual charge feels a force, so the "current" feels the sum of those)

For a small piece of my wire (above), $d\ell$ long, there are $(n_L\, d\ell)$ charges, each feeling $q\,\vec{v}\times\vec{B}$, giving $d\vec{F}_{\text{on chunk}} = n_\ell (d\ell) q\, \vec{v}\times\vec{B}$

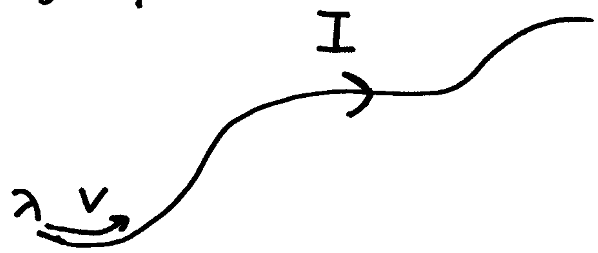
Since $\vec{v}$ is along the wire, and so is $d\vec{\ell}$,

I can write $d\ell\,\vec{v} = v\,d\vec{\ell}$ ← (No dots here!)

so $d\vec{F} = n_\ell q v\, d\vec{\ell}\times\vec{B} = I\,d\vec{\ell}\times\vec{B}$

(I is here mag of current, $d\vec{\ell}$ tells direction)

Summary:



$I = \lambda v$ along wire $= n_L q v$

$$\underset{d\vec{\ell}}{dF_{chunk}} = I\, d\vec{\ell} \times \vec{B}$$

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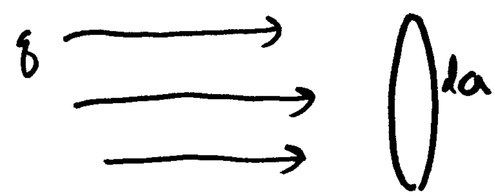
what if charges move throughout a volume, not just along lines?



we define the current passing our little da as usual,

$$\text{current} = \frac{\text{total charge}}{\text{sec}} = dq/dt$$

If da is tiny, we can think of dI, a tiny current passing through.

I will define $\vec{J}$ = volume current density $= \dfrac{d\vec{I}}{da}$

(and the direction of $\vec{J}$ will be the direction of $d\vec{I}$)

But to be careful, da is really $da_\perp$, here $\longrightarrow$ (see next p)

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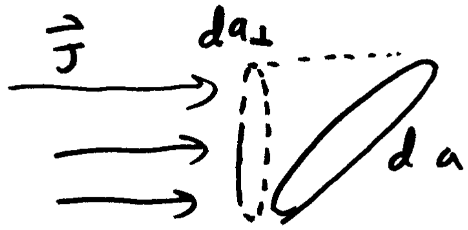


Given a steady $\vec{J}$ clearly current through either da or $da_\perp$ is same here, so to uniquely define $\frac{dI}{da}$ we need to pick $da_\perp$ as the area we mean...

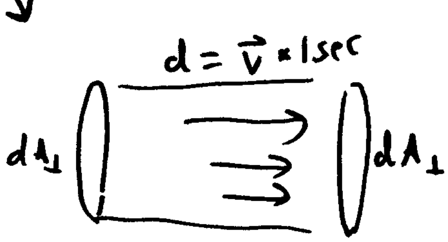
$$\text{so} \quad d\vec{I} = \vec{J}\, da_\perp = \underbrace{\vec{J} \cdot d\vec{a}}$$

This would also serve to define $\vec{J}$!

Just like w. line charges, you can see that in 1 sec, the total current passing through = total charge in a volume that extends back (along $\vec{J}$) by distance $\vec{V} \cdot (1\text{sec})$

If we have $\rho \; \frac{\text{charges}}{\text{Volume}}$ in that region, that means

$\rho \left[\vec{V} \cdot 1\text{sec}\right]\left[d a_\perp\right]$ charges will pass through, so



$$dI = \rho \frac{\vec{V} \cdot 1\text{sec} \cdot da_\perp}{1\,\text{sec}} = \vec{J}\, da_\perp$$

$$\text{so} \quad \boxed{\vec{J} = \rho \vec{V}}$$

as before, instead of ρ, we might use

$$\rho = N \frac{\text{charge carriers}}{\text{Volume}} \cdot q \frac{\text{coulombs}}{\text{carrier}}$$

so $\vec{J} = N q \vec{v}$ // Volume current density

↳ number density, per unit volume [Units are $\frac{A}{m^2}$ = current passing area].

so in 3-D situations, for a "chunk" of volume $d\tau$,

$$d\vec{F} = \underbrace{N\, d\tau\, q}\ \vec{v} \times \vec{B}$$

that's sum of $q\vec{v}\times\vec{B}$ for all $N\,d\tau$ charges !

Once again, $\vec{v}$ and $\vec{J}$ point in same direction locally,

so $Nq\vec{v} = \vec{J}$, and

$$\boxed{d\vec{F} = (\vec{J} \times \vec{B})\, d\tau}$$

We jumped for line currents to volume currents.

But many times charges live on surfaces, so we can also define a surface current density

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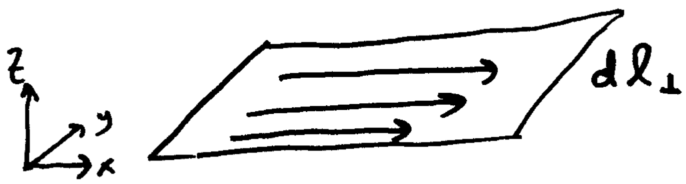
Surface current density $\vec{K}$:



Here the current passing a <u>line</u> segment $d\ell_\perp$ is defined

as $dI = \frac{dQ_{passing}}{dt}$ as usual

I will define $\vec{K}$ = sulface current density = $\frac{d\vec{I}}{d\ell_\perp}$

again, direction of $\vec{K}$ = direction of $d\vec{I}$.

(As before, for uniqueness I put $d\ell_\perp$ in the definition)

~~So I could also say~~

This one is slippery! It's a <u>ribbon</u> of current, + K tells how much current passes by a unit <u>length</u> perpendicul to flow!

Just as in prev 2 cases, we can quickly get

$$\vec{K} = \sigma\vec{V} \quad \text{with} \quad \sigma = \frac{\text{coulombs}}{m^2} = \text{surface charge density}$$

$$\left(= n_s q \vec{V} \quad \text{with} \quad n_s = \frac{\text{\# of charge carriers}}{m^2} \right)$$

Example @ top, could write $\vec{J} = K\delta(z)\hat{x}$. think about this!

Units of $\vec{K}$ = A/m, it's current passing unit length

+ $\vec{F}_{\text{little piece of ribbon}} = (\vec{K} \times \vec{B})\, da$, also as before...

Conservation of current (+ charge)

Total charge is conserved ← exp'tal fact.

which means, if you pick any volume,

Total inflow of charge = growth of net charge inside

Total outflow of charge = loss of net charge inside.

Since $\vec{J} \cdot d\vec{a} = d\vec{I}$ flowing out through area $d\vec{a}$ (p.10)

Total outflow $= \oiint \vec{J} \cdot d\vec{a}$ (this is $\frac{\text{coulombs}}{\text{sec}}$, it's rate of loss)

Since Q inside $= \iiint \rho \cdot d\tau$,

then rate of loss of charge $= -\frac{d}{dt} \iiint \rho \, d\tau$

so, if Q decreasing, loss is +.

so $\oiint \vec{J} \cdot d\vec{a} = \iiint -\frac{\partial \rho}{\partial t} \, d\tau$ ← Just conservation of charge

Div. theorem

$$\iiint (\vec{\nabla} \cdot \vec{J}) \, d\tau = \iiint -\frac{\partial \rho}{\partial t} \, d\tau$$ ← True for any volume, remember!

so $\boxed{\vec{\nabla} \cdot \vec{J} = -\partial \rho / dt}$ Continuity equation

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Continuity eq'n is basic statement of charge conservation

$$\underbrace{\vec{\nabla}\cdot\vec{J}} + \partial\rho/\partial t = 0$$

"out flow of current" + "increase in local charge" must cancel!

and, to summarize

$\vec{J} = \rho\vec{v}$ = volume current density = Amps passing $a_\perp$

$\vec{K} = \sigma\vec{v}$ = surface current density = Amps passing $\ell_\perp$

$\vec{I} = \lambda\vec{v}$ = line current = Amps passing point

and $\vec{J} = N_{vol}\, q\, \vec{v}$

$\vec{K} = N_{surf}\, q\, \vec{v}$

$\vec{I} = N_{linear}\, q\, \vec{v}$

and, when you need to sum (e.g. finding forces)

$$\iiint \vec{J}\, d\tau \leftrightarrow \iint \vec{K}\, da \leftrightarrow \int \underbrace{\vec{I}\, d\ell}_{\text{or } I\, d\vec{\ell}} \leftrightarrow \underbrace{\sum q_i \vec{v}_i}$$

won't use much for now, 'cause ↓

In magnetostatics, (by definition), charges don't pile up anywhere, $\partial\rho/\partial t = 0$ + $\boxed{\vec{\nabla}\cdot\vec{J} = 0}$ ← statics

[Ohm's Law: $\vec{J} \propto \vec{E}$. In conductors, obeying ohm's law ⇒ magnetostatics!]

Magnetostatics: Steady current flow everywhere

$$\vec{\nabla} \cdot \vec{J} = 0$$

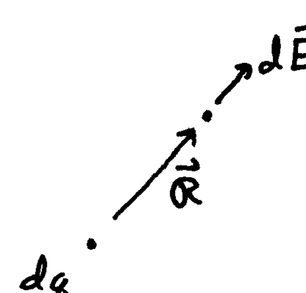
No "individual charges", it's all steady flows, (no $\frac{\partial}{\partial t}$'s to fuss with)

Currents create Magnetic fields:

It's an experimental fact. We have to do exp'ts to determine Direction, magnitude. I cannot derive this!

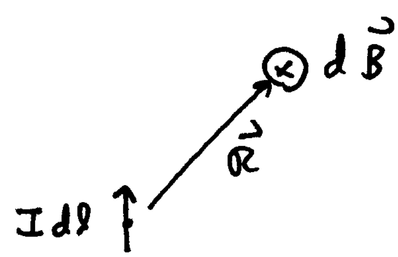
We will see the formula later as two of Maxwell's eq'ns, but historically, Biot & Savart deduced an equivalent result by careful exp'ts (following Oersted's original discovery, during a class lecture demo (!) that current $\Rightarrow \vec{B}$)

Think of Coulomb $\underset{\text{due to } dq}{d\vec{E}} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{R^2} \hat{R}$



a small charge creates a small $\vec{E}$ field

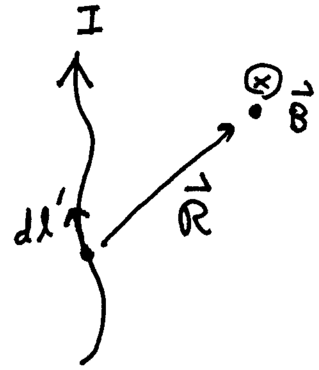
Biot-Savart is similar



$$\underset{\text{due to chunk of current } \vec{I}\,dl}{d\vec{B}} = \frac{\mu_0}{4\pi} \frac{\vec{I}\,dl}{R^2} \times \hat{R}$$

Just like Coulomb! except for the cross product!

Of course, in magnetostatics there are no isolated chunks of current like this, so really must sum over chunks:



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}\, d\ell'}{\mathcal{R}^2} \times \hat{\mathcal{R}}$$

A constant. The magnetic partner of ε_0 = "permittivity of free space"

$$\frac{\mu_0}{4\pi} = 10^{-7}\ \frac{N}{A^2}$$

"permeability of free space"

Units: $\vec{F} = q\vec{v}\times\vec{B}$

says $[B] = \frac{N}{C\cdot m/s} = \frac{N}{A\cdot m}$

1 Tesla $= 1\ N/A\cdot m$

$dB = \frac{\mu_0}{4\pi}\vec{I}\,d\ell/\mathcal{R}^2 \times \hat{\mathcal{R}} \Rightarrow$

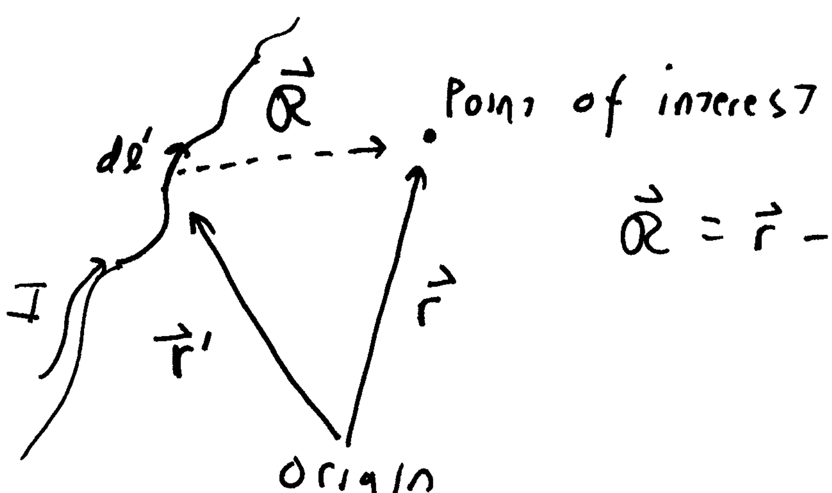
$[\mu_0] = \text{Tesla}\cdot\frac{m}{A} = N/A^2$ ✓

Some authors don't like the vector on $\vec{I}$, + shift it to $d\vec{\ell}$, so I is just the amount of current in $d\vec{\ell}$ direction

so $$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int I\,\frac{d\vec{\ell}'\times\hat{\mathcal{R}}}{\mathcal{R}^2}$$

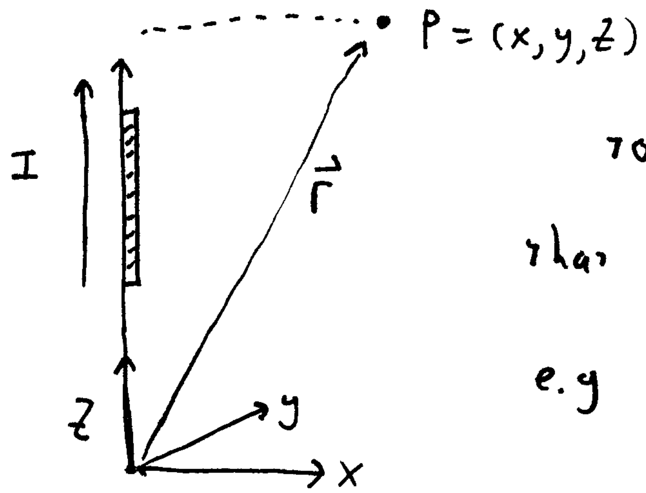
Remember $\vec{\mathcal{R}}$:

(definition) →



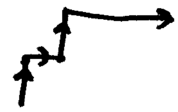
$$\vec{\mathcal{R}} = \vec{r} - \vec{r}'$$

Example 1: Straight current segment $I_0 \hat{z}$

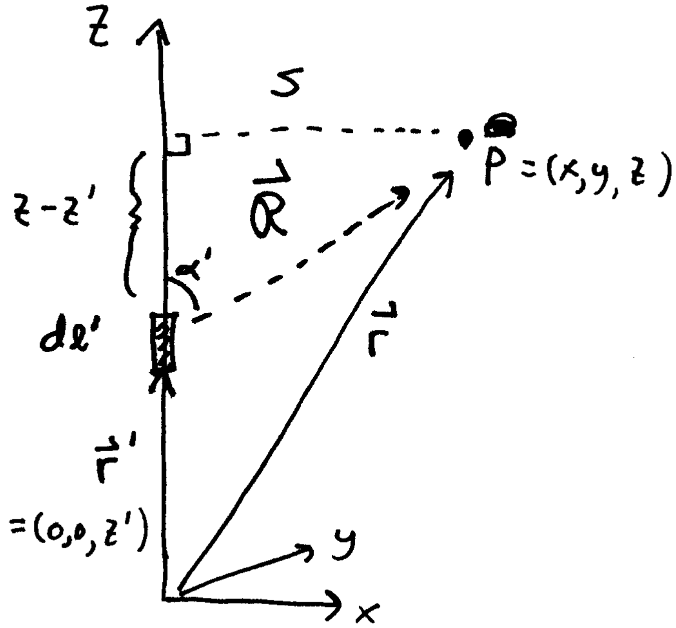


Let's figure out $\vec{B}$ at P due to this chunk (shown hatched)

that way, we can figure out $\vec{B}$ from

e.g I =  by "summing chunk"

This finite chunk is itself a sum (integral) of infinitesimal chunks



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{\ell}' \times \hat{R}}{R^2}$$

Look @ the figure + convince yourself that $d\vec{\ell}' \times \vec{R}$ points in $\hat{y}$ direction by the right hand rule (RHR)

I will use s = distance to P from z-axis (cylindrical radial coordinate)

And I have defined an angle α' in the picture

Here $d\ell' = dz'$, and $|d\vec{\ell}' \times \hat{R}| = d\ell' \cdot 1 \cdot \sin\alpha'$

Also, looking at picture, $R^2 = s^2 + (z - z')^2$; $\sin\alpha' = \frac{s}{R}$

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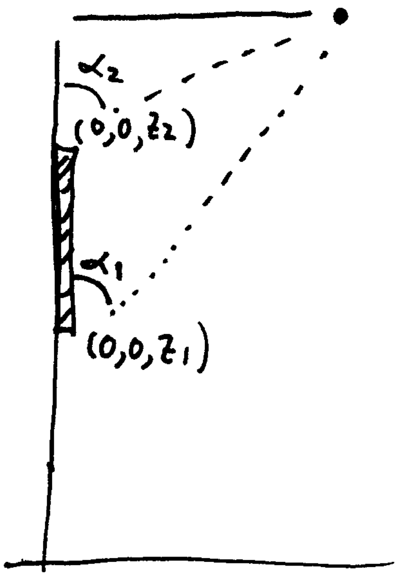
So B at P = $\dfrac{\mu_0}{4\pi} I \displaystyle\int_{z_1}^{z_2} \dfrac{dz' \cdot s}{(s^2+(z-z')^2)^{3/2}}$



s is a constant in this integral, + MMA gives me the result - (Griffiths works it out)

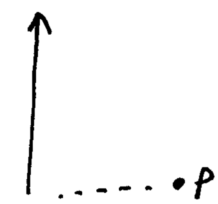
$$B = \frac{\mu_0 I}{4\pi s} \left. \frac{z'-z}{\sqrt{s^2+(z'-z)^2}} \right|_{z'=z_1}^{z'=z_2}$$

I could also observe: $\cos(\alpha_1) = \dfrac{z-z_1}{\sqrt{s^2+(z-z_1)^2}}$
(from this figure)

So $\vec{B} = \dfrac{\mu_0 I}{4\pi s}(\cos\alpha_2 + \cos\alpha_1)$ into the page (RHR!)

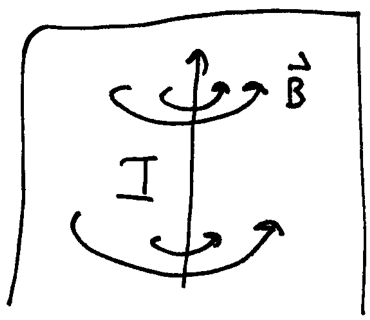
If current is infinite $\alpha_1 = 0$, $\alpha_2 = \pi$, and we get $-1+1=2$
(in parens)

$\vec{B}$ (long wire) = $\dfrac{\mu_0 I}{2\pi s}$ (RHR sense)



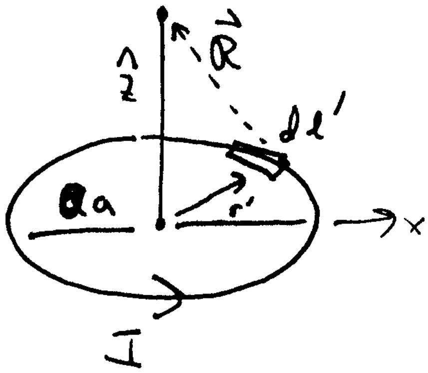
If current is "half infinite, starting across from P
($\alpha_1 = \pi/2$)

$\vec{B} = \dfrac{\mu_0 I}{4\pi s}$



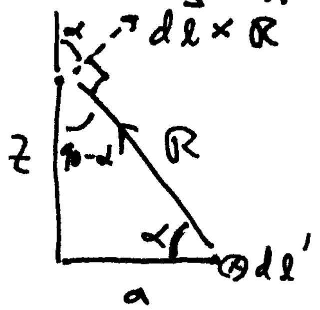
etc.

one more example: Ring of current, B on axis?



Here, $\mathcal{R} = \sqrt{a^2 + z^2}$ = constant, so that's nice & easy!

Unfortunately $d\vec{l} \times \hat{R}$ points at a crazy angle. But if we sum over all dl''s, only the vertical (z) component of those will survive! (Convince yourself!)



$d\vec{l} \times \hat{\mathcal{R}} = dl'$, but the vertical component is

$dl' \cos\alpha$ ~~...~~ $= dl' \dfrac{a}{\sqrt{z^2+a^2}}$
see picture!

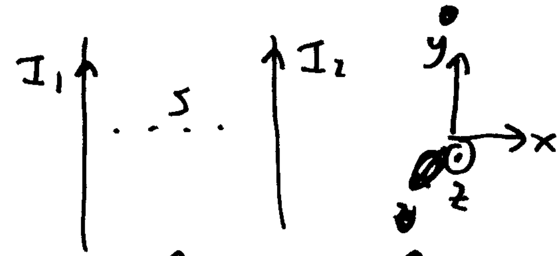
so $$B_z = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}' \times \hat{\mathcal{R}}}{\mathcal{R}^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{1}{a^2+z^2} \cdot \frac{a}{\sqrt{a^2+z^2}} \cdot \underbrace{\int dl'}_{=2\pi a}$$

$$B_z(0,0,z) = \frac{\mu_0 I}{2} \frac{a^2}{(a^2+z^2)^{3/2}}$$

5-20

To wrap up:

- Parallel wires



$B_{\text{due to } I_1} = \frac{\mu_0 I}{2\pi s}$ into page ($-\hat{z}$ direction)

$F_{\text{on } I_2 \text{ due to } B \text{ of } I_1} = \int \underbrace{I_2\, d\vec{l}_2 \times \vec{B}}_{-\hat{x} \text{ direction}} = I_2 \cdot \frac{\mu_0 I_1}{2\pi s} \int dl_2 \,(-\hat{x})$

so $\dfrac{F_{\text{on } I_2}}{\text{unit length}} = \dfrac{\mu_0 I_1 I_2}{2\pi s}$ (towards I_1 if both are parallel, away " antiparallel)

- If current is spread out, Biot-Savart becomes

$$\int d\vec{B} = \iint \frac{\mu_0}{4\pi} \frac{\vec{K} \times \hat{\mathscr{R}}}{\mathscr{R}^2}\, da' \quad \text{for surface currents}$$

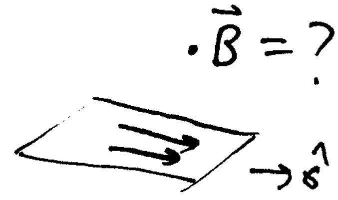
$$= \iiint \frac{\mu_0}{4\pi} \frac{\vec{J} \times \hat{\mathscr{R}}}{\mathscr{R}^2}\, d\tau' \quad \text{" volume currents}$$

- Don't try to use Biot-Savart to find $\vec{B}$ from one single moving charge: that's not magnetostatics, wait till next semester!

- Superposition principle holds for $\vec{B}$ just like $\vec{E}$

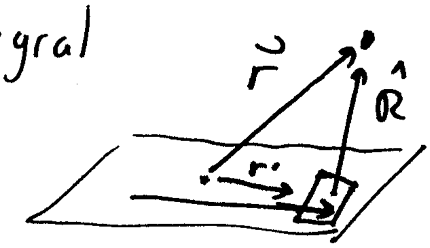
5.21

One more example: <u>sheet</u> of current $\vec{K}$.
(infinite in extent)



this is more common than you think, solenoids
solar wind
...

we could set up the integral
breaking surface into <u>patches</u>
dA with current



$$\frac{\mu_0}{4\pi}\int \frac{(\vec{K}\ \hat{x}) \times \hat{\mathscr{R}}}{\mathscr{R}^2}\, da'$$

But there will be a much easier way, coming soon! So let's hold off.

But by <u>symmetry</u>, convince yourself $\vec{B}$ must point <u>towards</u> you above, + <u>away</u> (below) the sheet in the figure above.