In general, inside a dielectric, you may have free charges !, (put there, e.g on a wire in the plastic, or injected, or rubbed on, or outside), and the $\vec{E}$ field from those charges then polarizes the dielectric: (adding bound charges to the mix, which superpose & alter the field).

So $\rho = \rho_b + \rho_f$. This ρ is real, it creates the total, real $\vec{E}$ field.

ρ_b: the "response" of the dielectric

ρ_f: the "placed" ones

so $\vec{\nabla} \cdot \vec{E} = \rho / \varepsilon_0$ ← Gauss' law, no exceptions, law of nature

$$= \frac{\rho_B + \rho_f}{\varepsilon_0} = \frac{-\vec{\nabla} \cdot \vec{P} + \rho_f}{\varepsilon_0}$$

so $\vec{\nabla} \cdot \left(\vec{E} + \frac{\vec{P}}{\varepsilon_0}\right) = \rho_f / \varepsilon_0$

we Define a field $\boxed{\vec{D} \equiv \varepsilon_0 \vec{E} + \vec{P}}$

The "D-field" or "Displacement field", an old word from Maxwell, (no meaning now ?)

and $\vec{\nabla} \cdot \vec{D} = \rho_F$

This looks like Gauss' law. $\Rightarrow \oiint \vec{D} \cdot d\vec{A} = Q_{free, enclosed}$

(Note units, it's C/m^2, not units of $\vec{E}$!)

Why $\vec{D}$? • ρ_F is "externally determined". These are the charges <u>we</u> placed. (The ρ_B is "self determined", it's the <u>response</u>. You don't choose it, + so often don't <u>know</u> it.)

If ρ_F is symm<u>etric</u>, we can use our usual Gauss' law tricks + "read off" $\vec{D}(r)$. <u>If</u> you know $\vec{P}$, <u>then</u> you can infer $\vec{E}$ at this point.

$\vec{D}$ is a mathematical invention, a tool to help us determine $\vec{E}$. (often easier to find $\vec{D}$ first) Engineers use it when dealing with fields in media. (We'll see more tricks soon)

<u>Ex:</u> A <u>small</u> charge q is embedded in a rubber sphere. (radius R) Find $\vec{D}$ everywhere.

The q creates an $\vec{E}$ field which polarizes the rubber, which in turn mo<u>difies</u> the ρ in the sphere, altering $\vec{E}$ in some (as yet) unknown way. So $\vec{E}$ is <u>not</u> easy to find.

But $\vec{D}$ is! $\oint\!\!\oint \vec{D}\cdot d\vec{A} = Q_{\text{free, enclosed}}$

This is q, nothing else!!

3310 4-13

In the rubber, or out, makes no difference (!!)

$\oiint \vec{D} \cdot d\vec{A} = q \Rightarrow D \cdot 4\pi r^2 = q$ ← [Note, no $\frac{1}{\varepsilon_0}$ in the def of $\vec{D}$!]

so $\vec{D}(\vec{r}) = \frac{q}{4\pi r^2} \hat{r}$

→ Outside, there is no P, so $\vec{E} = \frac{\vec{D}}{\varepsilon_0} - \overset{0}{\vec{P}} = \frac{q}{4\pi r^2} \frac{\hat{r}}{\varepsilon_0}$

Just coulomb, the rubber is polarized but neutral, + thus has no effect once you're outside.

→ In the rubber, $\vec{E} = \frac{\vec{D}}{\varepsilon_0} - \vec{P}$. If we don't know $\vec{P}$, we're still stuck.... which will lead us to the next section,

Approximating/Modelling many "Normal" (linear) Dielectrics, to allow us to figure out $\vec{P}$, given $\vec{E}$ (or $\vec{D}$.)

But first, a warning: $\vec{D}$ is not "just like $\vec{E}$ only simpler".

Given ρ_{free}, you can find $\vec{D}$ if there's nice symmetry.

But if you have complicated boundaries, all bets are off.

ρ_{free} does not "determine" $\vec{D}$ like ρ determines $\vec{E}$,

because $\vec{\nabla} \times \vec{D}$ is NOT always zero!

3310 4-14

It's a subtle point, but $\vec{\nabla}\times\vec{D} = \varepsilon_0 \overbrace{\vec{\nabla}\times\vec{E}}^{0} + \vec{\nabla}\times P$

and $\vec{P} = 0$ in vacuum, so you can easily find situations where $\oint \vec{P}\cdot d\vec{l} \neq 0$. Thus, there is no "potential" for $\vec{D}$.

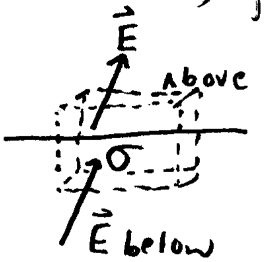
And, no Coulomb's Law either.

So bottom line: $\vec{D}$ is easy to compute (+thus useful) if you have nice symmetry of ρ_f. (Infinite line, spherical, or sheets)

But otherwise... $\vec{D}$ may not help us. For this reason, I don't have great physical intuitions about $\vec{D}$, it's more of a convenient tool to find $\vec{E}$ in certain problems!

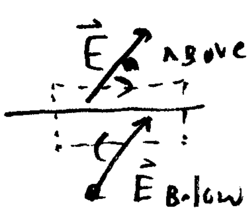
One more side comment before we get to the real useful $\vec{D}$ story (the Linear Dielectric story)

we know, from $\oiint \vec{E}\cdot d\vec{A} = \frac{\rho}{\varepsilon_0}$, that $E^{above}\cdot\hat{n}A + E^{below}\cdot\hat{n}A = \sigma\cdot A/\varepsilon_0$



$$\underline{E_\perp^{above} - E_\perp^{below} = \sigma/\varepsilon_0}$$

and

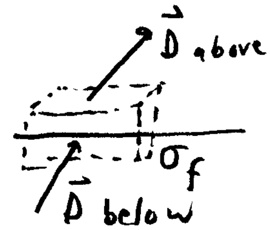


$E_\parallel^{above} - E_\parallel^{below} = 0$ from $\oint \vec{E}\cdot d\vec{l} = 0$ on a "skinny loop", as shown

3310 4-15

Those were our usual boundary conditions (always true).

But, when we have dielectrics, we may not know σ (some of it is $\sigma_f \Rightarrow$ known, but some of it is σ_{bound}, not known until we figure out $\vec{P}$!) So they are true, but we might not be able to use them to figure out $\vec{E}$.



On the other hand, $\oiint \vec{D} \cdot d\vec{A} = \rho_{Free}$

Says $D_\perp^{above} - D_\perp^{below} = \sigma_F$ ← known, most likely

so this might help us deduce D: just like before, in "free space", we sometimes used σ to help us find $\vec{E}$, like near ∞ sheet, or next to a conductor, ~~etc.~~ ...

$$\oint \vec{D} \cdot d\vec{l} = \oint \vec{P} \cdot d\vec{l} \text{ says } P_{\parallel}^{above} - P_{\parallel}^{below} = D_{\parallel}^{above} - D_{\parallel}^{below}$$

This might help too... but again, only if know $\vec{P}$...

So, it's time to finally talk about how to find $\vec{P}$!

Then, we'll be able to relate $\left(\vec{D} = \varepsilon_0 \vec{E} + \vec{P}\right)$ $\vec{D}$, $\vec{E}$, and $\vec{P}$ in general.

(we'll come back + use these Boundary conditions when solving for voltage in problems with Dielectric materials!)

3310 4-16.

A model, an approximate result true for some ordinary substances in ordinary-sized $\vec{E}$ fields:

$\vec{P} \propto \vec{E}$. Seems reasonable! $\vec{E}$ will "stretch" dipoles,

(look back e.g. at Griffiths 4.1, where we saw that atoms polarize, and $\vec{p} = \alpha \vec{E}$, so $\vec{P} = N \frac{\text{atoms}}{m^3} * \vec{p} \frac{\text{polarization}}{\text{atom}}$)

$= N\alpha \vec{E}$.

Didn't have to be linear, though $\longrightarrow$ (AND isn't always.)

Careful! $\vec{E}$ here = total field!

$\vec{E}_{\text{external}}$ will polarize material, which superposes onto E_{ext}, so $\vec{P}$ is not necessarily given directly from E_{ext}, it's proportional to the total resultant $\vec{E}$ field in the material!

But anyway, $\vec{P} = \varepsilon_0 \chi_e \vec{E}$ * For "Linear Dielectrics"* (Homogeneous, Isotropic)

the total $\vec{E}$ field

Polarization of Dielectric

"susceptibility"

(stuck in, to make χ_e unitless, a number.

$\chi_e = 0 \Rightarrow$ material doesn't polarize (e.g vacuum!)

$\chi_e \to \infty \Rightarrow$ very polarizable (like, conductor-like!)

(This is sort of like "Ohm's Law" in 1120. Practical, approximate, not a deep law of nature but a handy rule, accurate in many situations.)

3310 4-17

For such linear materials,

$\vec{P} = \varepsilon_0 \chi_\varepsilon \vec{E}$, so $\vec{D} \equiv \varepsilon_0 \vec{E} + \vec{P}$

$= \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E}$

$= \varepsilon_0 (1+\chi_e) \vec{E}$

which means

$\vec{D} = \varepsilon \vec{E}$ with $\varepsilon \equiv \varepsilon_0 (1+\chi_e)$

$\equiv \varepsilon_0 \, \varepsilon_r$

[Some people, like in 1120, call this $\underline{\underline{K}}$!]

"permittivity of the dielectric"

"Dielectric constant"

[Sorry for <u>all</u> the (equivalent!) notations, χ_e, ε, and ε_r!]

Unitless! Typically [1-2 in solids, 1.000... in gases.]

[Easy to measure, <u>known</u> for most materials]

Realize the usefulness now:

$\oiint \vec{D} \cdot d\vec{A} = Q_{free,enc}$ ← so, often, you can <u>easily</u> find $\vec{D}$,

but then $\vec{E} = \dfrac{\vec{D}}{\varepsilon}$ ← a simple constant,

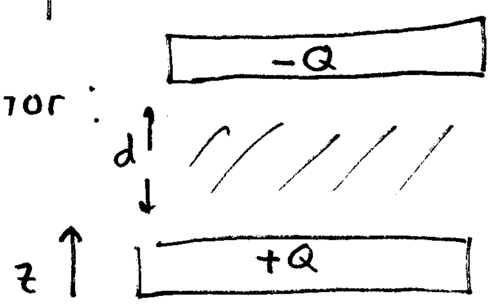
So if you known <u>any</u> <u>one</u> ($\vec{D}$, $\vec{E}$, or $\vec{P}$) the other two are trivial to find if you know the dielectric of the medium

$$\left.\begin{aligned} \vec{D} &= \varepsilon \vec{E} \quad (= \varepsilon_0 \varepsilon_r \vec{E})(= \varepsilon_0 (1+\chi_e) \vec{E}) \\ \vec{P} &= \varepsilon_0 \chi_e \vec{E} = \frac{\chi_e}{\varepsilon_r} \vec{D} = \left(\frac{\chi_e}{1+\chi_e} \vec{D}\right) \end{aligned}\right\}$$

3310 4-18

Examples of uses:

① Capacitor:



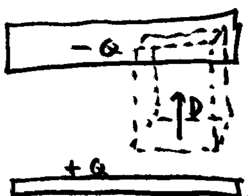
← Free charges on metal

← Dielectric, which polarizes

In the Dielectric, what's $\vec{E}_{tot}$? It's due to the capacitor (free) charge (which gives $\vec{E}_{ext} = \frac{\sigma \hat{z}}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}\hat{z}$ here) but also it polarizes (!) adding in some bound charges which in turn add to this (superpose on this) field!

Note that $\vec{P} \neq \varepsilon_0 \chi_e \vec{E}_{ext}$, it's $\vec{P} = \varepsilon_0 \chi_e \vec{E}_{tot}$, and we don't know $\vec{E}_{tot}$ yet!

But check this out:



← No $\vec{D}$ in the conductor! ($\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$, no $\vec{E}$ in there, no $\vec{P}$ " ")

$\iint \vec{D} \cdot d\vec{A} = Q_{\substack{free \\ enclosed}} \Rightarrow D \cdot A = \sigma \cdot A \Rightarrow D = \sigma$ in there!

So $\vec{D} = Q/A \, \hat{z}$ throughout the Dielectric.

Like I said, if you know Q_{free}, + have symmetry, $\vec{D}$ is quick-n-easy with Gauss' law.

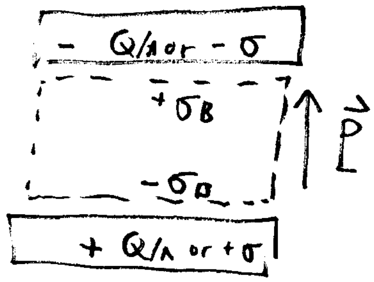
~~But Now~~

But now $\vec{E}_{tot} = \vec{D}/\varepsilon_0 \varepsilon_r$ (always true in Dielectrics)

So $\vec{E}_{tot} = \dfrac{Q}{A\varepsilon_0 \varepsilon_r}\hat{z}$. (It's $\dfrac{E_{ext}}{\varepsilon_r}$ in this case!)

The dielectric <u>weakened</u> the $\vec{E}$ field, by $\dfrac{1}{\varepsilon_r}$. This is common!

why? It polarized!



$$\vec{P} = \varepsilon_0 \chi_e \vec{E}_{tot}$$
$$= \varepsilon_0 \chi_e \frac{\sigma}{\varepsilon_0 \varepsilon_r}\hat{z}$$
$$= \frac{\chi_e}{1+\chi_e}\sigma\,\hat{z}$$

$\sigma_B = \vec{P}\cdot\hat{n} = \pm \dfrac{\chi_e}{1+\chi_e}\sigma$ at top + bottom.

this partly cancels the $\mp\sigma$ on capac!

<u>Effectively</u>, $\sigma_{tot} = \sigma - \sigma_B = \sigma\left(1 - \dfrac{\chi_e}{1+\chi_e}\right) = \dfrac{\sigma}{1+\chi_e} = \dfrac{\sigma}{\varepsilon_r}$!

So $\vec{E}_{tot}$ resulting from σ_{tot} is suppressed by $\dfrac{1}{\varepsilon_r}$.

<u>Note</u> $|\Delta V| = \left|\int \vec{E}\cdot d\vec{l}\right| = \dfrac{\sigma}{\varepsilon_0 \varepsilon_r}\cdot d$

This is $\dfrac{1}{\varepsilon_r}\times$ what we get w/o Dielectric

So $C = Q/\Delta V$ is $\varepsilon_r \times$ <u>bigger</u>. ← Faraday first studied this!

$\Rightarrow$ Capacitors always have dielectrics in them...

$\rightarrow$ Bigger capacitance

$\rightarrow$ weaker $\vec{E}$ for given $V \Rightarrow$ less likely to break down

$\rightarrow$ Can get $\varepsilon_r = 10{,}000$ for some materials! (Barium Titanate)

$\rightarrow$ Stored energy $= \frac{1}{2} C \Delta V^2$ is thus also bigger by $\varepsilon_r \Rightarrow$ (nice!)

Going back to my example on p. 13, q in rubber ball.

we found $\vec{D} = \frac{q}{4\pi r^2}\hat{r}$ in rubber. If rubber is "linear",

then $\vec{E}_{\substack{\text{in}\\\text{rubber}}} = \frac{\vec{D}}{\varepsilon} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}\cdot\left(\frac{1}{\varepsilon_r}\right)$ ["screening" but not "shielding"]

again, just like normal, but down by factor $1/\varepsilon_r$. (Like conductors)

Here again, we had nice symmetry $\Rightarrow$ could find $\vec{D}$.

(Remember our warning: without symmetry, don't know $\vec{D}$,

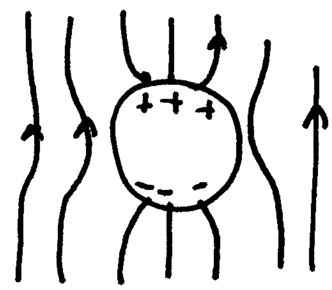
and can't assume $\vec{E}$ = "Normal $\vec{E}$" / ε_r ...)

What about finding V (+ thus $\vec{E}$) the old Ch. 3 way?

In linear dielectric, $\rho_B = -\vec{\nabla}\cdot\vec{P} \propto \vec{\nabla}\cdot\vec{D} = \rho_{free}$

so, if don't have ρ_F (no "embedded Q's"), then

$\rho_B = \rho_f = 0$, $\nabla^2 V = 0$, + we can use old tricks!



Example: Remember the "classic" conductor in $\vec{E}$ field.

we solved this in ch. 3 by following logic:

① → $V(\text{outside})$ solves $\nabla^2 V = 0 \Rightarrow V = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$

② → $V(\text{far away}) = -E_0 z = -E_0 r\cos\theta$

③ → $V(\text{on surface}) = 0$

③ told us $A_l R^l = -B_l / R^{l+1}$

② told us all $l \neq 1$ terms vanish, and $A_1 = -E_0$

The E_{ext} was $\uparrow\uparrow\uparrow\uparrow$, and the sphere polarized, cancelling out $\vec{E}$ entirely inside.

In end, we found $\left.\frac{\partial V}{\partial r}\right|_{out} - \left.\frac{\partial V}{\partial r}\right|_{in}^{\nearrow 0} = -\frac{\sigma(\theta)}{\varepsilon_0}$

gave us $\sigma = 3\varepsilon_0 E_0 \cos\theta$ ← "perfect polarization"

this is just the σ you need to create (by itself) ($\vec{E} = -E_0 \hat{z}$ inside, i.e. cancelling E_{ext})

Let's do the same thing with dielectric sphere!

Put it into $E_{ext} = \uparrow\uparrow\uparrow\uparrow$.

It will polarize... but not perfectly, it's dielectric, not conductor!

Here again: $V_{out} = \sum_{\ell=0}^{\infty} \left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right) P_\ell(\cos\theta)$

But $V_{out}(r\to\infty) = -E_0 r\cos\theta$ tells us

all $A_{\ell\neq 1}$ must vanish, and $A_1 = -E_0$

so $V_{out} = -E_0 r\cos\theta + \sum_{\ell=0}^{\infty} \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$

Inside, <u>cannot</u> assume $\vec{E} = 0$!!

$V_{in}(r,\theta) = \sum_{\ell=0}^{\infty} \tilde{A}_\ell r^\ell P_\ell(\cos\theta)$

- ↳ No $\tilde{B}_\ell$ terms to keep $V(0)$ finite
- $\tilde{A}_\ell$'s are <u>different</u> than A_ℓ's!

We need <u>Boundary Conditions</u>.

<u>As always</u> $V(in)\big|_R = V(out)\big|_R$

(But the other B.C. is new, it comes from p. 15 ideas)

4-22

The "New" B.C. when Dielectrics are present:

$$D_\perp^{above} - D_\perp^{below} = \sigma_F \quad \leftarrow \text{from } \oint \vec{D}\cdot d\vec{A} = Q_{free,\,enc.}$$

For linear Dielectrics, this means

$$\varepsilon_{above} E_\perp^{above} - \varepsilon_{below} E_\perp^{below} = \sigma_F$$

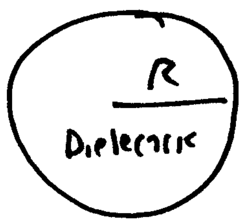
or

$$\boxed{\varepsilon_{above} \frac{\partial V}{\partial n}\Big|_{above} - \varepsilon_{below} \frac{\partial V}{\partial n}\Big|_{below} = -\sigma_F}$$

(Because $\vec{E} = -\vec{\nabla} V$ still, as always!)

Very much like Ch. 3, just bear in mind ε might change at a boundary, if dielectric material ends!

In our problem



vacuum outside $\Rightarrow \varepsilon = \varepsilon_0$

$$\varepsilon_0 \frac{\partial V^{out}}{\partial r}\Big|_{at\ R} - \varepsilon \frac{\partial V^{in}}{\partial r}\Big|_R = -\sigma_F,$$

but we have no free charges in this problem!

So use this... $\varepsilon_0 \dfrac{\partial V^{out}}{\partial r} = \varepsilon \dfrac{\partial V^{in}}{\partial r}$ in this problem

4-23

$$\varepsilon_0\left(-E_0\cos\theta\right) + \varepsilon_0 \sum_{\ell=0}^{\infty} \frac{B_\ell}{R^{\ell+2}}(-)(\ell+1)P_\ell(\cos\theta)$$

$$= \varepsilon \sum_{\ell=0}^{\infty} \ell \tilde{A}_\ell R^{\ell-1} P_\ell(\cos\theta)$$

and $V_{in}|_R = V_{out}|_R$

$$\Rightarrow \sum_{\ell=0}^{\infty} \tilde{A}_\ell R^\ell P_\ell = -E_0 R\cos\theta + \sum_{\ell=0}^{\infty} \frac{B_\ell}{R^{\ell+1}} P_\ell$$

this tells me $\sum\left(\tilde{A}_\ell R^\ell - \frac{B_\ell}{R^{\ell+1}}\right)P_\ell = -E_0 R P_1$

→ must vanish for all $\ell \neq 1$!

so $\tilde{A}_\ell = \frac{B_\ell}{R^{2\ell+1}}$ for <u>all</u> $\ell \neq 1$ (i)

and $\tilde{A}_1 R - \frac{B_1}{R^2} = -E_0 R$ (ii)

The <u>upper</u> eq'n tells me

$$\sum_{\ell=0}^{\infty}\left(\ell\varepsilon\tilde{A}_\ell R^{\ell-1} + \frac{(\ell+1)\varepsilon_0 B_\ell}{R^{\ell+2}}\right)P_\ell = -\varepsilon_0 E_0 \overbrace{\cos\theta}^{P_1}$$

so here,

$\ell\varepsilon\tilde{A}_\ell R^{\ell-1} + \frac{(\ell+1)\varepsilon_0}{R^{\ell+2}} B_\ell = 0$ for <u>all</u> $\ell \neq 1$ (iii)

<u>and</u> $\varepsilon\tilde{A}_1 + \frac{2\varepsilon_0}{R^3} B_1 = -\varepsilon_0 E_0$ (iv)

(c) and (iii) tell me $\tilde{A}_\ell = B_\ell = 0$ for $\underline{\underline{\text{all}}}$ $\ell \neq 1$!

ii + iv tell me

$$\frac{2\varepsilon_0}{R}\left(\tilde{A}_1 R - \frac{B_1}{R^2}\right) = \frac{2\varepsilon_0}{R}\left(E_0 R\right)$$

$$\varepsilon \tilde{A}_1 + \frac{2\varepsilon_0}{R^3} B_1 = -\varepsilon_0 E_0$$

$$\tilde{A}_1 (2\varepsilon_0 + \varepsilon) = -\varepsilon_0 E_0 (2+1)$$

or $\tilde{A}_1 = -3\varepsilon_0 E_0 / (\varepsilon + 2\varepsilon_0) = \dfrac{-3E_0}{(2+\varepsilon_r)}$

and $B_1 = (\tilde{A}_1 + E_0) R^3 = R^3 E_0 \left(1 - \dfrac{3}{2+\varepsilon_r}\right) = E_0 R^3 \left(\dfrac{\cancel{-1}+\varepsilon_r}{2+\varepsilon_r}\right)$

so $V_{in} = \tilde{A}_1 r P_1 = -\dfrac{3E_0}{2+\varepsilon_r} r\cos\theta = \dfrac{-3E_0}{2+\varepsilon_r} z$

so $\vec{E}_{in} = -\dfrac{\partial V_{in}}{\partial z}\hat{z} = +\dfrac{3E_0}{2+\varepsilon_r}\hat{z}$

This is uniform, and $\dfrac{3}{2+\varepsilon_r}$ x's E_{ext}.

$\varepsilon_r = 1 \Rightarrow$ Nothing there at all, vacuum everywhere, $E_{in} = E_{ext}$

$\varepsilon_r \to \infty \Rightarrow E_{in} = 0$, perfect conductor ($\infty$ Dielectric)

E_{in} is damped but not zero.

4-23
24] alt. (quicker)

Look, seems clear (with $\cos\theta$ term) we're going to only get

$$V_{in} = -C_1 r \cos\theta$$

$$V_{out} = -E_0 r \cos\theta + \frac{C_2 R^3}{r^2} \cos\theta \qquad !$$

Continuity $\Rightarrow$ $C_1 = E_0 - C_2$

$$-\varepsilon \left.\frac{\partial V_{in}}{\partial r}\right|_R = -\varepsilon_0 \left.\frac{\partial V_{out}}{\partial r}\right|_R \Rightarrow \varepsilon C_1 = \varepsilon_0 E_0 + 2\varepsilon_0 C_2$$

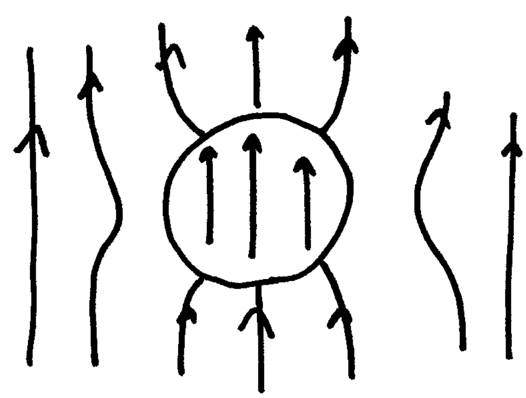
or $\varepsilon_r C_1 = E_0 + 2C_2$

so $C_1(1-\varepsilon_r) = C_2(-3)$

and thus $C_1 = E_0 + C_1 \frac{(1-\varepsilon_r)}{3} \Rightarrow C_1 \left(\frac{3-1+\varepsilon_r}{3}\right) = E_0$

$$C_1 = \frac{3}{2+\varepsilon_r} E_0$$

$$C_2 = \frac{\varepsilon_r - 1}{2+\varepsilon_r} E_0$$



4-25

$$\vec{P}_{in} = \varepsilon_0 \chi_e \vec{E}_{in} = \frac{\varepsilon_0 (\varepsilon_r - 1)\cdot 3}{(\varepsilon_r + 2)} E_0 \hat{z}$$

(always $= \varepsilon_r - 1$)

so $\sigma_{BOUND} = \vec{P}\cdot\hat{r} = 3\varepsilon_0 \left(\frac{\varepsilon_r - 1}{2+\varepsilon_r}\right) E_0 \cos\theta$.

So this is again $\propto \cos\theta$, it's "dipole",

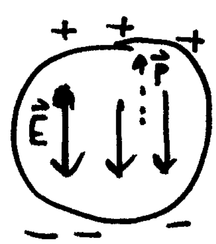
Vacuum $\Rightarrow \varepsilon_r \to 1 \Rightarrow \sigma_B = 0$ ✓

conductor $\Rightarrow \varepsilon_r \to \infty \Rightarrow \sigma_B = 3\varepsilon_0 E_0$ ✓ (see p. 21a!)

OUTSIDE, it's also just a dipole field (+ orig),

but dipole is "weaker" than the ideal conductor would give.

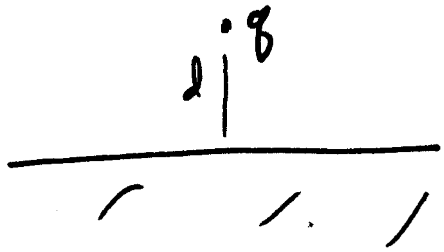
A pure polarized sphere makes internal $\vec{E}$ that partly, but not totally, "fights" the $\vec{E}_{ext}$ that polarized you in the first place



4-26

<u>One</u> <u>last</u> <u>example</u>:

Like "image charge", but w. Dielectric. →



Dielectric

Can we <u>use</u> method of images?

- No <u>free</u> charges on surface, so $D_z|_{above} = D_z|_{below}$

$$\oint \vec{D} \cdot d\vec{A} = Q_{free,enc}$$

Thus $\varepsilon_0 E_z|_{above} = \varepsilon E_z|_{Below}$

or $\varepsilon_0 \dfrac{\partial V_{above}}{\partial z}\Big|_{z=0} = \varepsilon \dfrac{\partial V_{below}}{\partial z}\Big|_{z=0}$

<u>Could</u> we use an image Q' at $z=-d$? well, let's try!

<u>Above</u>, $V^{above}(x,y,z) = \dfrac{kQ}{\sqrt{x^2+y^2+(z-d)^2}} + \dfrac{kQ'}{\sqrt{x^2+y^2+(z+d)^2}}$

<u>below</u> ... well, there <u>really</u> isn't a Q' below, it's all σ_B on surface. <u>Seen</u> from below, <u>that</u> surface charge looks like a Q' <u>above</u>!

$$V^{below}(x,y,z) = \frac{kQ}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{kQ'}{\sqrt{x^2+y^2+(z-d)^2}}$$

!!

[N.B. if $Q'=-Q$, $V_{below}=0$ as it was for conductor] so $$\varepsilon_0 \frac{\partial V_{above}}{\partial z}\bigg|_{z=0} = \varepsilon_0 K \left[\frac{-(z-d)\,Q}{(x^2+y^2+(z-d)^2)^{3/2}} + \frac{-(z+d)\,Q'}{(x^2+y^2+(z+d)^2)^{3/2}}\right]_{z=0}$$

$$= \frac{\varepsilon_0 K}{(x^2+y^2+d^2)^{3/2}}\left[dQ - dQ'\right]$$

$$\varepsilon \frac{\partial V_{below}}{\partial z}\bigg|_{z=0} = \varepsilon K \left[\frac{-(z-d)(Q+Q')}{(x^2+y^2+(z-d)^2)^{3/2}}\right]_{z=0}$$

$$= \frac{\varepsilon K d (Q+Q')}{(x^2+y^2+d^2)^{3/2}}$$

so B.C. requires $\varepsilon_0 (Q-Q') = \varepsilon(Q+Q')$

[Continuity of $V \Rightarrow Q+Q' = Q+Q'$ doesn't help, it's automatic.]

or $Q(\varepsilon_0 - \varepsilon) = Q'(\varepsilon + \varepsilon_0)$

or $$Q' = Q\left(\frac{1-\varepsilon_r}{1+\varepsilon_r}\right)$$

[Note: conductor $\Rightarrow \varepsilon_r \to \infty$
$Q' = -Q$ ✓]

[Note: $\varepsilon_r \to 0 \Rightarrow$ No image needed!]

- Now we know V everywhere.
  - → Vanishes at ∞
  - → Continuous, +right B.C.s at $z=0$

Could find σ_b from $\vec{P}\cdot\hat{z} = \varepsilon_0 \chi_e E_z^{below} = \varepsilon_0 \chi_e \left(-\frac{\partial V^{below}}{\partial z}\right)\Big|_{z=0}$.

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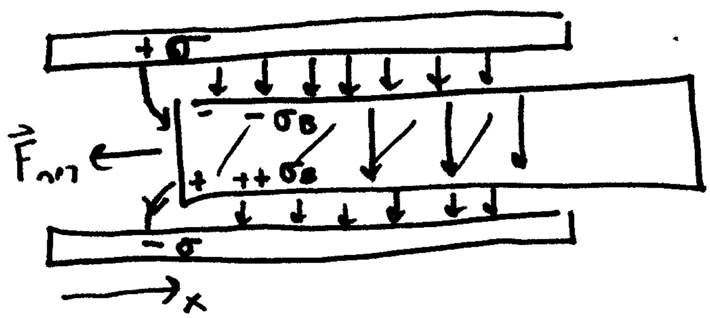

Fringe fields: pull dielectric in!

Very complicated fields. Could we possibly hope to figure out $\vec{E}$'s, then $\vec{F}$'s? Not directly!

Use energy argument:

$$\vec{F}_{ext} = \frac{dW}{dx} = -\vec{F}_{field}$$

and $W = \frac{1}{2} C V^2 = \frac{1}{2} Q^2/C$ ← Better, then no battery to worry about! Keep Q fixed!

Can find $C(x)$ ← exercise!

+ thus deduce $\vec{F}$.

As Dielectric enters, $E \downarrow$, $C \uparrow$, $W \downarrow$.

shielding, $\varepsilon_r > 1$, $\frac{Q^2}{C}$ or $\int E^2$

so, gets "sucked in" (by fringe field!)