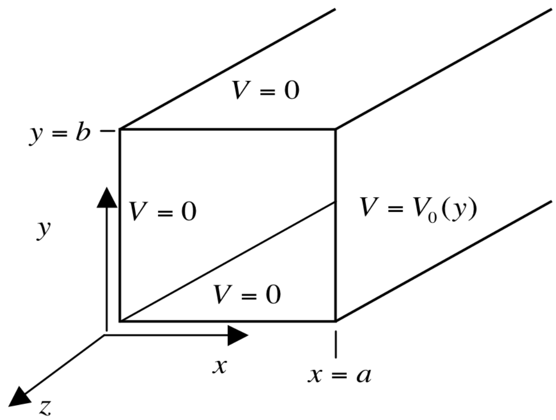
Part 1: Laplace's Equation and Separation of Variables

Within a very long, rectangular, hollow pipe, there are no electric charges. The walls of this pipe are kept at a known voltage (they are known because in a lab, you can control them).

Three of the walls are grounded: $V(x=0,y,z)=0$; $V(x,y=0,z)=0$; $V(x,y=b,z)=0$

The fourth wall maintains a potential that varies with y: $V(x=a,y,z)=V_o(y)$ which will be specified later.



In order to find out the voltage inside the pipe, you will need to solve Laplace's equation:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

i. What does it mean to "separate variables" of $V(x,y,z)$. Can you use that approach here? If so, write down the general form that V must have.

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ii. Plug this general form of V into Laplace's equation. After doing this, you should have several terms.

- Simplify as much as possible.
- Are any of the terms zero?
- What must be true about the remaining terms in order to satisfy Laplace's equation?
- Write down the ordinary differential equations you need to solve to find V.

iii. Solve the differential equations generally. Do *not* apply the boundary conditions yet. Is there any ambiguity? Could this ambiguity be resolved by considering the boundary conditions?

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iv. Now, we'll be more specific about the voltage on the fourth wall (the only wall not grounded). What does each boundary condition below tell you?

1. $V(x, y=0, z)=0$
2. $V(x, y=b, z)=0$
3. $V(x=0, y, z)=0$
4. $V(x=a, y, z)=V_o \sin\frac{17\pi \cdot y}{b}$

Use these boundary conditions to find the voltage everywhere inside the pipe.

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v. Explicitly check that your answer for V(x,y,z) satisfies all boundary conditions. If your solution satisfies the boundary conditions, could there be another (or more than one) solution for V(x,y,z) that would *also* work?

Part 2: Fourier's Trick

i. After applying boundary conditions #1-3 (three grounded walls), what does your answer look like? (write down what you have after the C_n below)

$$V(x,y,z) = \sum_n C_n$$

ii. Suppose the fourth wall now maintains a constant voltage: $V(x = a, y, z) = V_o$

What is the new voltage everywhere inside the pipe?

1. What do you multiply both sides by? (Fourier's trick)
2. What are the limits of integration?
3. What happens to the infinite sum?
4. Find all C_n's.
5. Write down V(x,y,z).

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