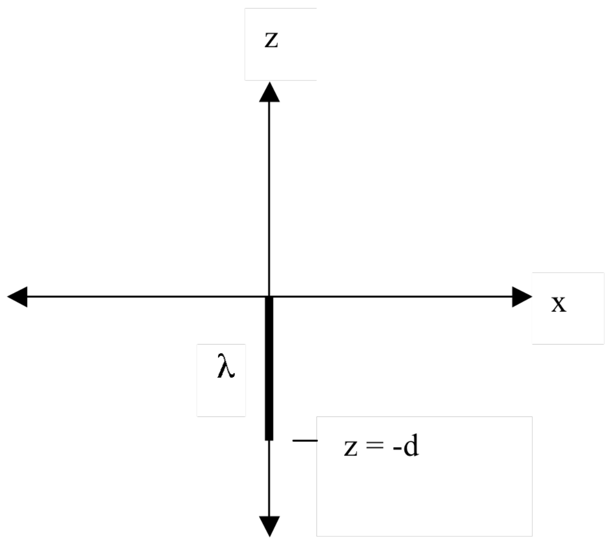
Week 7

Part 1 – Potential from a Line of Charge



A uniform line charge density λ extends from the origin to the point (0,0,-d).

i. Using the script-r technique from earlier in the course, find an expression for the potential anywhere along the z-axis, V(z). Remember:

$$V = \frac{1}{4\pi\varepsilon_o}\int \frac{\rho(\vec{r}\,')\cdot d\tau'}{\vec{r} - \vec{r}\,'}$$

Your answer to part (i.) should have been: $V(z) = \frac{\lambda}{4\pi\varepsilon_o}\ln(1+\frac{d}{z})$. (If you didn't get this, find out where you went wrong.) At large z (z>>d) the potential can be written: $V(z) = \frac{\lambda}{4\pi\varepsilon_o}\ln(1+\varepsilon)$.

ii. Expand the potential, V(z), into a Taylor series. Find the first two non-zero terms.

Taylor expansion about the point $x = x_o$:

$$f(x_o+\varepsilon) = \frac{f(x_o)}{0!}\varepsilon^0 + \frac{f'(x_o)}{1!}\varepsilon^1 + \frac{f''(x_o)}{2!}\varepsilon^2 + \ldots$$

Part 2 – Separation of Variables

i. This problem does not have spherical symmetry. Could the potential have the form: $V(r,\theta) = \sum_l (A_l \cdot r^l + \frac{B_l}{r^{l+1}}) \cdot P_l(\cos\theta)$? If so, in what regions could the solution look like this?

ii. Assuming the potential can have the form $V(r,\theta) = \sum_l (A_l \cdot r^l + \frac{B_l}{r^{l+1}}) \cdot P_l(\cos\theta)$ in the region you specified above, find the two leading non-zero A's and/or B's. Do any terms vanish? Keep in mind that this potential is the same potential you solved by integrating in part 1, so when θ = 0 (the z-axis), the answers must match.

$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$$
$$P_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$$

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Part 3 – Multipole expansion

A potential can be expanded into the form:

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{"monopole"}{r} + \frac{"dipole"}{r^2} + \frac{"quadrapole"}{r3} + \ldots\right)$$

i. For this problem, what are the monopole and dipole moments? Do these answers make sense physically?

ii. Which terms would change if the charge distribution were shifted up by d/2, so that it was centered on the origin?