Part 1 – Sketching Vector Potential

One of Maxwell's equations, $\nabla \times \vec{E} = 0$, allowed us to define a scalar potential V, where $\vec{E} = -\nabla V$. Similarly, another one of Maxwell's equations allows us to define the vector potential, **A**.

i. Which Maxwell equation does **A** come from? How does it lead to **A**?

ii. What current density **J** would create the **B**-field in Figure 2 below? Can you write an explicit mathematical formula for it?

iii. Notice that the equations defining **A** are mathematically analogous to Maxwell's equations for **B**:

$$\nabla \bullet \vec{\mathbf{B}} = 0 \quad \Leftrightarrow \quad \nabla \bullet \vec{\mathbf{A}} = 0$$
$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} \quad \Leftrightarrow \quad \nabla \times \vec{\mathbf{A}} = \vec{\mathbf{B}}$$

First, sketch **B** in Figure 1. Then, using the mathematical similarities above, sketch **A** in Figure 2:

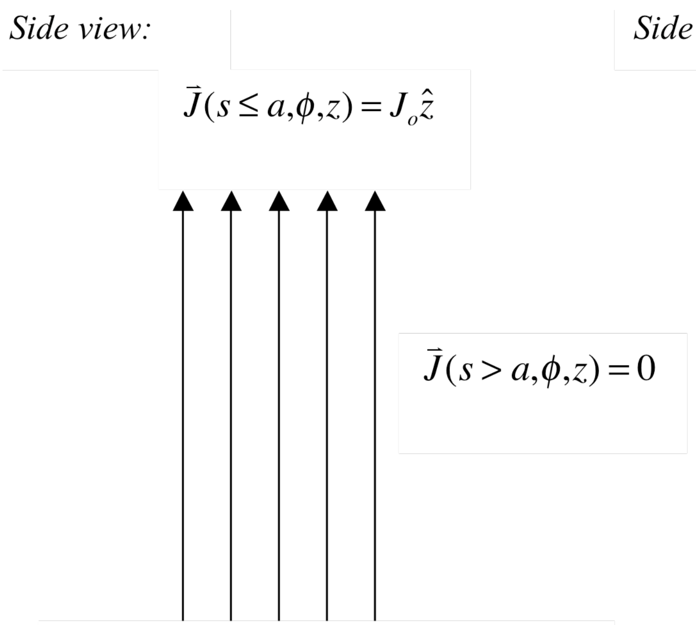


Figure 1: Given **J**, sketch the **B** field.

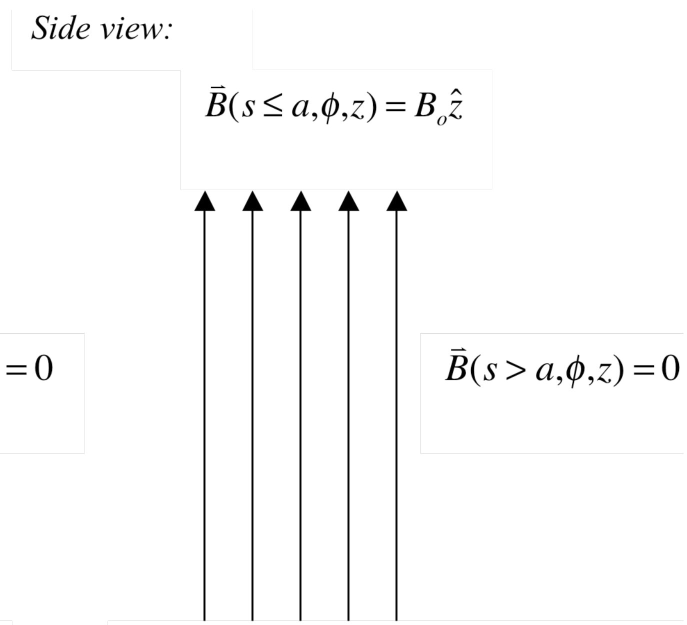


Figure 2: Given **B**, sketch the **A** field.

Written by: Darren Tarshis, Steve Pollock, Stephanie Chasteen
Contact: Steve Pollock (steven.pollock@colorado.edu)

iv. One way to check your previous answer (conceptually) is using an Ampere's Law analogy. Ampere's Law tells you that the **J**-flux (or I_{encl}) is equal to $\oint \vec{B} \bullet d\vec{l}$. What is a similar relationship between the vector potential and magnetic field?

Try using this "Ampere's Law analogy" to (conceptually) check your sketch of **A**.

v. A toroidal inductor looks like a doughnut wrapped with wire. Indicate the direction of **J**, then sketch **B** and **A** for the toroidal inductor.

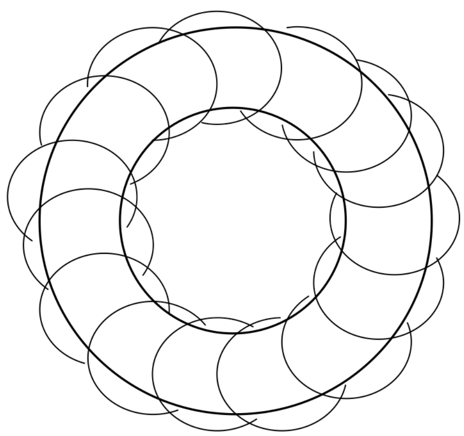
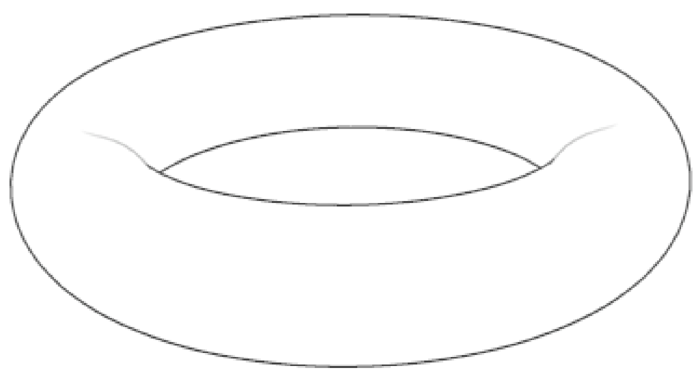


Figure 3



Written by: Darren Tarshis, Steve Pollock, Stephanie Chasteen
Contact: Steve Pollock (steven.pollock@colorado.edu)

Part 2 – Calculating Vector Potential

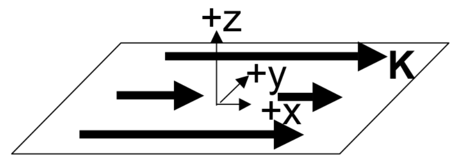


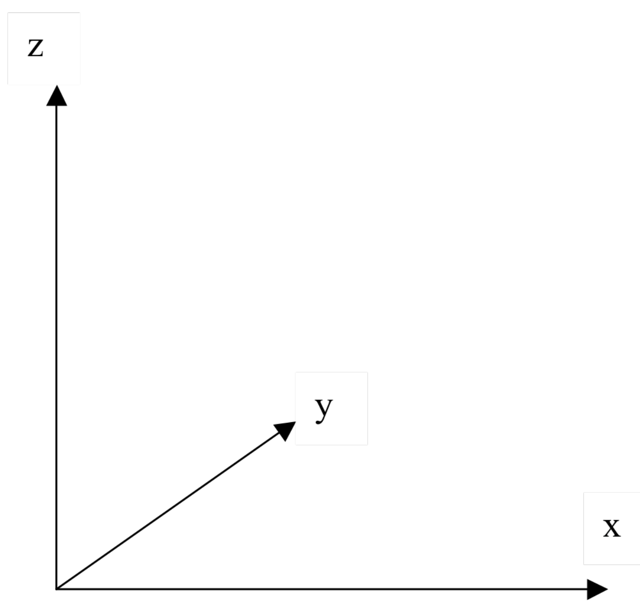
Figure 4

On last week's homework, you calculated the magnetic field produced by a uniform surface current: $K(z=0)=K_o\hat{x}$. The answer you should have calculated is:

$$\vec{B}(z>0)=\frac{-\mu_o K_o}{2}\hat{y} \qquad \vec{B}(z<0)=\frac{+\mu_o K_o}{2}\hat{y}$$

i. Can you think of physical situation(s) that can be modeled by each of the four labeled figures in this Tutorial?

ii. Sketch your best guess of what **A** looks like for the uniform surface current. Which components (x, y, or z) does **A** have (it might help to look at relationship between **A**, **B**, and **J** in the two examples in Part 1)? Which variables (x, y, or z) does **A** depend on?



 4/4/08
Written by: Darren Tarshis, Steve Pollock, Stephanie Chasteen
Contact: Steve Pollock (steven.pollock@colorado.edu)

iii. Using your assumptions for which components **A** has, and which variables **A** depends on, calculate (or guess) what **A** is.

Does your sketch of **A** resemble the answer you calculated (or guessed)?

Written by: Darren Tarshis, Steve Pollock, Stephanie Chasteen
Contact: Steve Pollock (steven.pollock@colorado.edu)