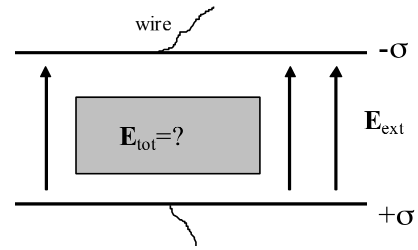
HW 10 HINTS FOR Q6



Q6. MODELING A SOLID DIELECTRIC
The "given constants" here are n, q, k (the spring constant), and $\mathbf{E}_{ext}$ is given, too. However, *note that Δz is not really a "given constant". It will obviously depend on how big an external field you apply! But at first, we'll just give it the name Δz , and later we will eliminate it!*

We want to figure out the dielectric constant of this material.

Below are some fairly detailed steps/suggestions. See if you can use as few of these steps/hints as possible- but if you get stuck, perhaps looking at the next step will help move you along.

Before you begin, review the in-class tutorial, and e.g. figure out the bound surface charge density on the surface of the dielectric, and the polarization in the dielectric, in terms of the givens and *Δz*.

i) FIND THE INDUCED FIELD (IN TERMS OF GIVENS AND UNKNOWN *Δz*)
We have a large (assume infinite) slab, with some bound charge on it. I might start by figuring out the induced electric field, $\mathbf{E}_{ind}$, inside the plastic slab. (Express it in terms of Δz!)

ii) FIND THE TOTAL FIELD (IN TERMS OF GIVENS AND UNKNOWN *Δz*)
So now we know the induced E field. What is then the total electric field inside the plastic slab, $\mathbf{E}_{tot}$? *(First express it very generically in terms of the magnitude of the induced field $\mathbf{E}_{ind}$ and the external field $\mathbf{E}_{ext,.}$ Then, given part i, write it purely in terms of givens, including Δz) Watch out for minus signs!*

iii) USE THE "MODEL" TO FIGURE OUT *Δ*z IN TERMS OF THE GIVENS.
So now you have an expression for the total E field in there. Good – it's $\mathbf{E}_{tot}$ which is "real", that's the actual E field that the little atoms are experiencing! Think physically of this simple model (it's merely charges on springs, with a spring constant k, sitting in a total field $\mathbf{E}_{tot}$)
Can you write a simple expression for Δz in terms of $\mathbf{E}_{tot.}$? Nothing fancy here, what's the force of the spring? What's the electric force? How do they relate in equilibrium?!

iv) ELIMINATE *Δz,* FIND A RELATION BETWEEN TOTAL AND EXTERNAL FIELD.
Now hook it all together: you have Δz in terms of $\mathbf{E}_{tot}$ (from part iii) But in part ii, you also had a rather different expression for $\mathbf{E}_{tot}$ written in terms of Δz ! Ah! Combine these two equations to relate $\mathbf{E}_{tot}$ directly to $\mathbf{E}_{ext}$. This is progress! Δz is now gone from the story, and you have a direction connection between the physical E field in the medium ($\mathbf{E}_{tot}$) to the applied field ($\mathbf{E}_{ext}$)

v) FIGURE OUT THE D FIELD.
Inside the dielectric slab, it should be fairly quick and easy to write a simple formula for the D field in terms of $\mathbf{E}_{ext}$. (Why? Because D arises from FREE charges only, not bound charges! It's a simple Gauss' law situation) But given D in terms of $\mathbf{E}_{ext}$, now we're pretty close to done:
The fundamental definition of the dielectric constant is from the relation $\mathbf{D} = \varepsilon_0\, \varepsilon_r \boldsymbol{E}_{tot}$,
where ε_r is the dielectric constant we're after.
You already found (part iv) the relation between $\mathbf{E}_{tot}$ and $\mathbf{E}_{ext.}$ So I think at this point, you should be able to "read off" the dielectric constant.