Q1. B FIELD FROM CURRENTS *Let's start by practicing writing down current densities:*
A) i) A sphere (radius R, total charge Q uniformly distributed throughout the volume) is
spinning at angular velocity $\omega\hat{\mathbf{z}}$ about its center (which is at the origin) What is the volume
current density $\mathbf{J}(r, \theta, \phi)$ at any point (r, θ, ϕ) in the sphere? (Don't forget direction too!)

ii) A very thin plastic ring (radius R) has a constant linear charge density, and total charge Q.
The ring spins at angular velocity ω about its center (which is the origin). What is the current I,
in terms of given quantities? What is the volume current density **J** in cylindrical coordinates?
(This may be a little tricky, since the ring is "very thin", there will be some δ functions. I
suggest writing down a formula for $\rho(r,\phi,z)$ first. And, remember that **J** should be a vector!)
Now let's take some currents and find the corresponding B field.
B) Compute the magnetic field **B**(0,0,z) along the z-axis for a current loop
(current I, radius R, sitting in x-y plane, centered around the z-axis).
*This is the same situation as part A-ii above, but here just solve in terms of
a given current I (That way, even if you couldn't get part A-ii, this part is
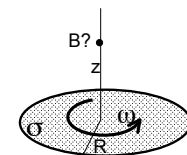still going to be fine! Plus, no δ-functions needed...)*
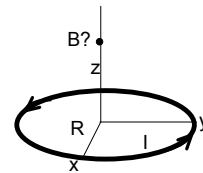
C) Last week we had a CD (radius R) with a fixed, constant, uniform surface electric charge
density σ everywhere on its top surface.
It was spinning at angular velocity $\omega\hat{\mathbf{z}}$ about its center (the origin).
You found the current density **K** at a distance r from the center.
Use that result to find the magnetic field **B**(0,0,z) at any distance z directly
above the origin. (Part B above might help)
Does your answer seem reasonable? Please check it, with units, and some limiting behaviors
(e.g. what do you expect if R→0? ω→0? z→∞? For this last one, don't just say "it goes to
zero". This is a dipole, so B should go to 0 like $1/z^3$. (right?)   <u>Show</u> that it does!

Q2. An infinitely long wire has been bent into a right angle turn, as
shown. The "curvy part" where it bends is a perfect quarter circle, radius
R. Point P is exactly at the center of that quarter circle. A steady current I
flows through this wire. Find **B** at point P (magnitude and direction)
*(No need to derive any formulas "from scratch" if you can get them from
Griffiths' examples!)*
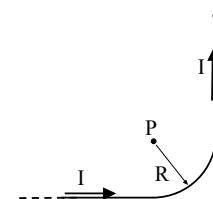
Q3. BIOT-SAVART - SQUARE LOOP
A) Find **B** at the exact center of a square current loop (current I running around a wire bent in
the shape of a square of side a)
(No need to derive any formulas "from scratch" if you can get them from Griffiths' examples!)

B) If I had such a loop in my lab and wanted **B** at the center, I might do the above calculation,
but if I was planning an experiment and just wanted a *rough estimate* of the B-field, I might
"assume a spherical cow": assume the square was really a circle. We've
done that problem   (**B** at the center of a circular loop) it's much simpler
than the square! You don't have to rederive it, but think back to how we
got that result, and why it turned out to be a relatively easy application of
Biot-Savart. But what radius circle would you use, to estimate B? You
might consider finding **B** for the "inscribed" and "circumscribed" circles
and then average. How good an approximation does that turn out to be?
(Can you think of a better way?)

Phys 3310, HW #12, Due in class Wed Apr 10
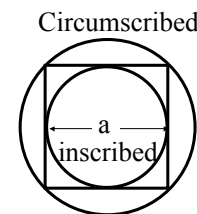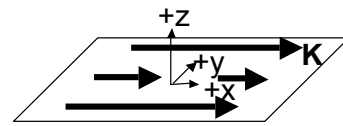
Q4. AMPERES LAW - THEMES AND VARIATIONS
Consider a thin sheet with uniform surface current density $K_0 \, \hat{\mathbf{x}}$
A) Use the Biot-Savart law to find **B**(x,y,z) both above and below the sheet, by integration.
Note: The integral is slightly nasty. Before you turn to Mathematica - simplify as much as possible! Set up the integral, be explicit about what curly R is, what da' is, etc, what your integration limits are, etc. Then, make clear mathematical and/or physical arguments based on symmetry to convince yourself of the *direction* of the **B** field (both above and below the sheet), and to argue how **B**(x,y,z) depends (or doesn't) on x and y. (If you know it doesn't depend on x or y, you could e.g. choose x=y=0 to simplify... But first you must convince us that's legit!)
B) Now solve the above problem using Ampere's law. (Much easier than part a, isn't it?)
Please be explicit about what Amperian loop(s) you are drawing and why. What assumptions (or results from part a) are you making/using?
(Griffiths solves this problem, so don't just copy him, work it out for yourself!)
C) Now let's add a second parallel sheet at z=+a with a current running the other way.
(Formally, this means $\mathbf{J'} = - K_0 \, \delta(z-a)\hat{\mathbf{x}}$. Do you understand this notation?)
Use the superposition principle (do NOT start from scratch or use Ampere's law again, this part should be relatively quick) to find **B** *between* the two sheets, and also *outside* (above or below) both sheets. *Does this remind you of a familiar electrostatics problem at all? How?*
D) Griffiths derives a formula for the B field from a solenoid (Example 5.9) Rewrite his answer (which is in terms of I) so it is expressed in terms of K (See his Fig 5.34 and 5.35 for help with this) Briefly compare with part C, do you see any rough connections?
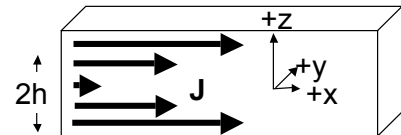
Q5. AMPERE'S LAW-II
Now the *sheet* of current has become a thick SLAB of current.
The slab is *infinite in (both) x and y*, but finite in z.
So we must think about the volume current density **J**, rather than **K**.
The slab has thickness 2h (It runs from z=-h to z=+h)
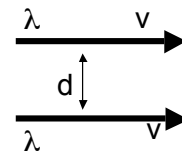Let's assume that the current is still flowing in the +x direction, and

is uniform in the x and y dimensions, but now **J** depends on height linearly, i.e. $\vec{\mathbf{J}} = J_0 \, |z| \, \hat{\mathbf{x}}$
inside the slab (but is 0 above or below the slab). Find the **B** field (magnitude and direction) everywhere in space (above, below, and also, most interesting, *inside* the slab!)

Q6. UNIFICATION
A) Two very long straight line charges (linear charge density λ) are a distance d apart. Both move to the right at constant speed v. How big does v have to be so that the magnetic *attraction* exactly balances the electric *repulsion*? Find the actual number, and comment! *(Note: there are no special relativity complexities or issues here at all: v and λ are measured in the lab frame)*

B) Along with the necessary addition made by Maxwell, Ampere's law in full glory reads
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \dfrac{1}{c^2} \dfrac{\partial \mathbf{E}}{\partial t}$. Take the divergence of this equation, and derive the continuity equation
(Eq 5.29 in Griffiths section 5.1.3) You have just <u>derived</u> the conservation of electric charge!
I think both *these results are pretty cool, and carry very deep messages about the nature and unification of electricity and magnetism,* and *their connection to special relativity!*