Phys 3310, HW #13, Due in class Wed Apr 17

Q1. AMPERE'S LAW: (A quicky!)
Suppose B in a region of space centered on the origin has cylindrical symmetry and is given by $\vec{B} = B_0\,\hat{\varphi}$ where B_0 is a constant, and $\hat{\varphi}$ is the azimuthal direction in cylindrical coordinates.
i) What is the current density in this region of space?
ii) Suppose the current density that you found extends out to a radius R and is zero for r > R. What is the magnetic field for r > R?

Q2. FORMAL MANIPULATIONS, VECTOR CALCULUS!
A) Griffiths (section 5.3.2) shows that, *given* Biot-Savart, we can arrive at Ampere's law. Go through that derivation and try to recreate it/make sense of it. Don't just copy it down - do all the steps yourself. There are a few "gaps" in his derivation that you should be explicit about - e.g., Eq 5.50 (.52 in the 4th ed) is missing terms, what happened to them? Are you convinced of the minus sign shenanigans leading to 5.52 (.54 in 4th ed)? Convince us you understand them! Do you understand the ending: why did the contribution in Eq. 5.53 (.55 in 4th ed) "go away"?

B) Use similar math gymnastics to start from the Biot-Savart law and end with $\vec{\mathbf{B}} = \vec{\nabla}\times\vec{\mathbf{A}}$, where $\vec{\mathbf{A}} = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}')}{\mathfrak{R}}\,d\tau'$. (this is Griffiths Eq 5.63, or .65 in 4th edition)
You'll do this in a different way than Griffiths does (*though I suggest you convince yourself you can see it his way too, which is section 5.4.1!*). (Note: *Part B is easier than part A, really!!)*
- Start with the Biot-Savart law (Eq 5.45 , or 5.47 in the 4th ed), and make use of the handy identity we've seen several times this term: $\vec{\nabla}\frac{1}{\mathfrak{R}} = -\frac{1}{\mathfrak{R}^2}\hat{\mathfrak{R}}$ (Do you know where this relation comes from, can you show it?) Then use Griffiths' product rule #7 (front flyleaf) to manipulate your expression until you get to $\vec{\mathbf{B}} = \vec{\nabla}\times(\text{something})$. That "something" should be precisely the formula we're after! *At some point you will need to pull the curl past an integral sign - be sure to justify why this is a perfectly legitimate thing to do.*

Q3. FIELDS AND STRENGTHS
A) Estimate the density ρ of mobile charges in a piece of gold-wire speaker wire, assuming each atom contributes one free electron. (Look up any necessary physical constants!) Then, think about the definition of current, and *estimate* the average electron speed in a gold speaker wire of ordinary household size carrying an ordinary household current. *Your answer will come out quite slow - it might surprise you.* If you flip on the stereo, and the speakers are, say, 2 meters away, would there be a noticeable "time lag" before you hear the speaker come on?

B) If you cut open this wire, you'll see it is really two wires, each insulated, and wrapped close together in a single plastic cylinder (since you need a complete circuit, current has to flow TO and FROM the speaker, right?). Make reasonable guesses for the dimensions involved in a real, ordinary speaker wire, to estimate the TOTAL magnetic force between the "outgoing" and "return" wires. Is it attractive or repulsive? Now - if you could somehow remove the stationary positive ions in the metallic conductor (which play no role in the *flow of current,* right?), make a rough estimate for the total *electrical* force of repulsion between the two wires. How does it compare with the magnetic force you just found?

Q4. VECTOR POTENTIAL I
A) A long (infinite) wire (cylindrical conductor, radius R, whose axis coincides with the z axis) carries a uniformly distributed current I_0 in the +z direction. Assuming $\nabla\bullet\vec{\mathbf{A}} = 0$ (the "Coulomb

gauge"), and choosing A=0 at the edge of the wire, show that the vector potential *inside the wire* could be given by A= c $I_0(1-s^2/R^2)$. Find the constant c (including units.) Things to explicitly find/discuss: What is the vector direction of **A**? (Does it "make sense" in any way to you?) Is your answer unique, or is there any remaining "ambiguity" in **A**?
*(Note that I'm not asking you to derive **A** from scratch, just to see that this choice of **A** "works")*

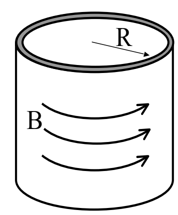
B) What is the vector potential *outside* that wire? (Make sure that it still satisfies $\nabla \bullet \vec{\mathbf{A}} = 0$, and make sure that **A** is continuous at the edge of the wire, consistent with part a)
Here again, is your answer unique, or is there any remaining "ambiguity" in **A**(outside)?

Q5. VECTOR POTENTIAL II
A) Griffiths Fig 5.48 is a handy "triangle" summarizing the mathematical connections between **J**, **A**, and **B** (like Fig. 2.35) But there's a missing link, he has nothing for the left arrow from **B** to **A**. Note the *equations* defining **A** are very analogous to the basic Maxwell's equations for **B**:

$$\nabla \bullet \vec{\mathbf{B}} = 0 \qquad \Leftrightarrow \qquad \nabla \bullet \vec{\mathbf{A}} = 0$$

$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} \qquad \Leftrightarrow \qquad \nabla \times \vec{\mathbf{A}} = \vec{\mathbf{B}}$$

So **A** depends on **B** in the same way (mathematically) the **B** depends on **J**. (Think, Biot-Savart!) Use this idea to just *write down* a formula for **A** in terms of **B** to finish off that triangle.



B) In lecture notes (and/or Griffiths Ex. 5.9) we found the B field everywhere inside (and outside) an infinite solenoid (which you can think of as a cylinder with uniform surface current flowing azimuthally around it. See Griff. Fig 5.35. Use the basic idea from the previous part of *this* question to, therefore, quickly and easily just write down the vector potential **A** in a situation where **B** looks analogous to that, i.e. $\vec{\mathbf{B}} = C\delta(s-R)\,\hat{\varphi}$, with C constant. (Sketch this **A** for us, please) *(You should be able to just "see" the answer, no nasty integral needed!)*
For you to discuss: What physical situation creates such a B field? (This is tricky - think about it!) Also, if I gave you some B field and asked for **A**, can you now think of an "analogue method", i.e. an experiment where you could let nature do this for you, instead of computing it?
*It's cool - think about what's going on here. You have a previously solved problem, where a given **J** led us to some **B**. Now we immediately know what the vector potential is in a* very different *physical situation, one where **B** happens to look like **J** did in that previous problem.*

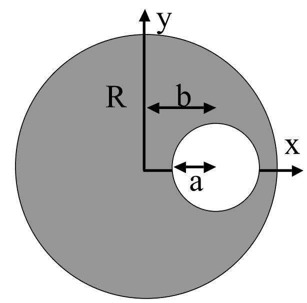
Q6. SQUARE LOOP - FAR AWAY
Last week we considered a square current loop (current I running around a wire bent in the shape of a square of side a) sitting flat in the x-y plane, centered at the origin. (You found B at the center). It is a straightforward extension to find **B**(0,0,z) at a point along the central axis (there's just a little trig to do, because the B from each of the 4 legs "tilts", and you need to add only the z components). I claim the result is: (check if you want! But you don't have to)

$$\mathbf{B}(0,0,z) = \frac{\mu_0 I}{2\pi} \frac{a^2}{(z^2 + a^2/4)} \frac{1}{\sqrt{z^2 + a^2/2}} \hat{z}$$

Take the limit z>>a to find an approximate simple formula for B_z(0,0, large z). Now do the same for a circular current loop of radius a, and compare. (The exact expression is derived in Griffiths, Ex. 5.6, you don't have to rederive that, just consider the large-z limit) Check both answers by comparing with Griffiths dipole approximation (last equation in section 5.4.3), looking only along the +z-axis of course.
Note how simple everything gets far away, and how your expressions for different shapes differ only by a constant out front, which goes like $m = I \times$(loop area) *(The magnetic dipole moment!)*

Phys 3310, HW #13, Due in class Wed Apr 17

Extra Credit: I really like this one! AMPERE AND SUPERPOSITION



<u>A)</u> A long (infinite) wire (cylindrical conductor, radius R, whose axis coincides with the z axis) carries a uniformly distributed current I_0 in the +z direction. (Basically, like question #1). But now, a long (infinite) cylindrical hole is drilled out of the conductor, parallel to the z axis, (see figure for geometry). The center of the hole is at x = b, and the radius is a. Determine the magnetic field *inside the hole region*.

You should find that the B field in the hole is uniform - that was just a little surprising to me! The "superposition" in the title is the trick: finding the B field from a single "uniform" current in a long wire is easy. This problem is two uniform currents superposed (can you see how?) The only issue is that the "center" of the two currents here do not coincide, so you need to be just a little careful about adding vectors.
Draw a careful picture, use a big piece of paper! Here's a little trick that makes the math quick and elegant:
In cylindrical coordinates, B fields from uniform current in wires centered around an origin point in the $\hat{\varphi}$ *direction, yes? But cylindrical unit-vectors form a right-handed system, i.e.*
$\hat{\varphi} = -\hat{s} \times \hat{z}$ *That helps here, because although* $\hat{s}$ *is different if you shift origins,* $\hat{z}$ *is not!*
Just draw the picture…

<u>B)</u> If this is an ordinary wire carrying ordinary household currents, and the drilled hole has dimensions roughly shown to scale in the figure above, make an order of magnitude estimate for the strength of the B field in that region. How does it compare to the earth's field?
The "comparing with B(earth)" question is always interesting to me. I'd say that generally sets the scale for B-fields that surely can't be worth worrying much about in normal settings!