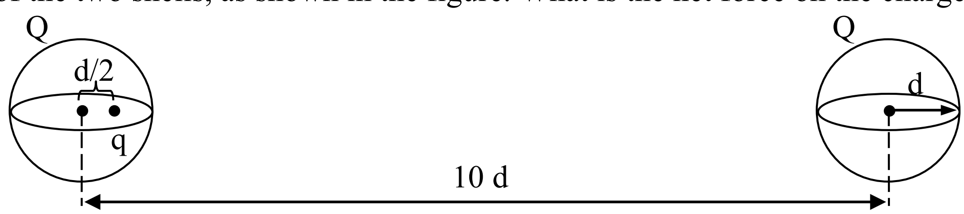
Phys 3310, HW #5, Due in class Wed Feb 13.

Q1. QUICKY.

The following problem is from the 2001 Physics GRE exam. Apparently, students were expected to solve the problem in just a few minutes!

Two spherical, nonconducting, and very thin shells of uniformly distributed positive charge Q and radius d are located a distance 10d apart. A positive point charge q is placed inside one of the shells at a distance d/2 from the center, on the line connecting the centers of the two shells, as shown in the figure. What is the net force on the charge q?



Q2. ESTIMATION

Estimate the maximum charge you can put onto a child's balloon.

Given that, what is the associated "maximum voltage" of the balloon (with respect to ∞)

*Like last week, estimation problems give you a lot of freedom. How big is a balloon? Why should there be a maximum at all? That's key: if the balloon sparks, it can bear no more charge. Can you look up the breakdown electric field of air somewhere? Your answer can be *very* rough, we only want an order of magnitude.*

Q3. ENERGY OF POINT CHARGE DISTRIBUTION

Imagine a small square (side "d") with four point charges -q, one on each corner.

Calculate the total stored energy of this system (i.e. work required to assemble it).

Then, calculate how much work it takes to "neutralize" these charges by bringing in *one* more point charge (+4q) from far away and placing it right at the *center* of this square.

(When studying crystal structures, it is sometimes convenient to model them as rectangular grids of charged ions, this problem forms the starting point for such a model)

Q4. ENERGY OF A CONTINUOUS CHARGE DISTRIBUTION

a) Find the total electrostatic energy stored in a uniformly charge sphere of radius R and charge q. (Note: this is uniform throughout the whole volume, it's not a shell) Express your answer in terms of q, R, and constants of nature. There are many different ways to do this, I want you to use two *different methods* so you can check yourself. The most obvious are:

i) figure out **E**(r) and then use Griffiths Eq. 2.45 (being careful to integrate over *all space*, not just where the charge is! Think about that for yourself, it's quite important)

ii) figure out V(r) and then use Griff. 2.43 (what region do you need to integrate this over?)

There are still other ways if you prefer - e.g. Griffiths Eq 2.44. Or you could build up the charge "shell by shell" and explicitly integrate the work to build each thin shell, to find the total work done building the complete sphere. But two methods is enough for me!

b) According to Einstein, an electron has a rest mass energy $E = mc^2$. It is tempting (though wrong!) to imagine that the electron is NOT a point charge, but instead is a tiny sphere, with uniform charge density, whose total electrostatic energy (found above!) EQUALS this mass energy of the electron. (So, the idea is maybe the rest energy of an electron is purely electromagnetic.) Use your result in part a to deduce the radius an electron would have to have, for this to work out.

You will need to look up the charge and mass of the electron. Don't take your result too literally, we are not taking quantum mechanics into account! This is a "classical radius".

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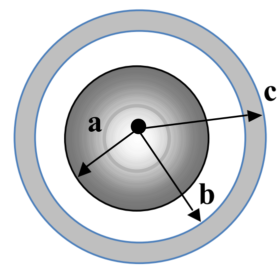
c) Here's another application of part a: compute the electrostatic energy of an atomic nucleus with Z protons and a total of A nucleons (which means protons OR neutrons, so A = # protons + # neutrons), using a (common) approximation for the nuclear radius: $R = (1.2 \times 10^{-15}\text{ m})*A^{1/3}$. *(it's fun but totally not needed here to think about* <u>*why*</u> *that's the 1/3 power!)* Convert your answer (which is in SI units of Joules) to the energy unit of MeV, which is Mega eV. (Give your result in units of MeV times $Z^2/A^{1/3}$) Now use this result to estimate the *change* of electrostatic energy when a uranium nucleus undergoes fission into two roughly equal-sized nuclei. Google the experimental answer - what does this tell you about the source of the energy released in a nuclear explosion? *(You might have thought you would need to know a lot of nuclear physics to estimate this number, but apparently not so much!)*

Q5. GAUSS' LAW AND CONDUCTORS:
A solid metal sphere of radius a, with total charge Q, is surrounded by a thick concentric solid metal shell (inner radius b, outer radius c). This outer shell has NO net charge.



a) Find the surface charge density at radius a, at b, and at c,

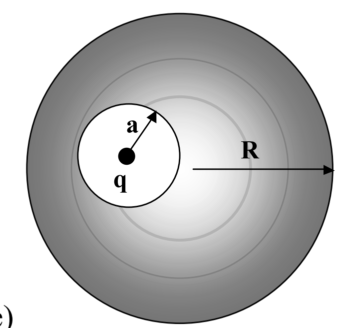
b) Sketch E everywhere, and find the potential at the origin (assuming $V(\infty)=0$)

c) Now ground the outer surface (by connecting a wire between it and ∞). This sets $V(c)=0$ (since c and ∞ are connected by a wire!) How do your answers to the previous 2 questions change?

To

To think about: what happened to the net charge on the outer shell when you grounded it in part b? Is this at all surprising to you? What is going on physically?

Q6. GAUSS' LAW AND CAVITIES:
Dig a spherical cavity (radius a) out of a larger solid (neutral!) conducting sphere of radius R. At the center of the cavity is a point charge q. Please EXPLAIN your reasoning on all parts!



a) Find the surface charge density σ_a and σ_R, and sketch them so we can see how you think they are distributed.

b) Compute and then *sketch* or clearly describe the E field everywhere.(Inside the cavity, in the bulk of the sphere, and outside)

c) If you move q a little off to one side, so it is no longer exactly at the *center* of its little cavity, which of your answers above would change? Please explain. (Detailed calculations are NOT needed, just a qualitative discussion and sketches)

d) Going back to the original problem (as shown), if you bring a new charge Q near the <u>outside</u> of the conductor, which (if any) of your answers (in a and b above) change? (Again, qualitative answers are all I want)

I really like this problem, lots of good physics in it! Think it through carefully, make physical sense of your answers, don't just take someone else's word for it, if you get help!