

Phys 3310, HW #6, Due in class Wed Feb 20.

Q1. COAX CAPACITORS

A real coaxial cable (found in physics labs) has an inner conducting solid cylinder of radius "a" (i.e. a wire!) and an outer conducting cylindrical shell (inner radius "b"). It is physically easy to set up a fixed, given, potential difference ΔV between the inner and outer conductors, keeping the entire coax neutral.

a) Assuming infinite cylinders, find the capacitance of this system "per unit length".
(Express your answer in terms of only the given quantities: a, b, and constants of nature!)

b) Find the energy stored per unit length inside this capacitor. There are various ways to do this: please use Griffiths eq. 2.45, i.e. integrate the energy density stored in the E field. Express your final answer in terms of only the given quantities: a, b, and ΔV .

Then, explicitly check yourself, by using your answer to part a and Griffiths Eq. 2.55:
stored energy = $\frac{1}{2} C(\Delta V)^2$

c) This is also a physical model for axons, which are long cylindrical cells (basically coax cables!) carrying nerve impulses in your body and brain. Using part a above, estimate the capacitance (in SI metric units, which are Farads) of your sciatic nerve.

Assumptions - the sciatic nerve is the longest in your body, it has a diameter of roughly 1 micron, and a length of perhaps 1 m(!) Axons generally have a value of b which is very close to a (i.e. the gap is extremely tiny, b-a is about 1 nanometer) so you can simplify your expression using $\ln(1+\epsilon) \approx \epsilon$. Think carefully about what ϵ is here, though!

Q2. SPHERICAL CAPACITOR

Griffiths (end of Ch. 2) finds the capacitance of two concentric spherical metal shells. By letting $R(\text{outer shell}) \rightarrow \infty$, you get a finite result, called "the capacitance of the sphere".
(It tells you the Q need on a sphere to get the surface up to voltage V with respect to ∞).

a) Estimate the capacitance of i) an isolated human body and ii) planet earth. (in Farads)
(Here I would "assume a spherical person" for simplicity :-)

These estimates should give you a sense of how BIG a one Farad capacitance is!

Now let's use the results:

b) By shuffling over a carpet on a dry winter day, you can easily charge yourself up to a high voltage. Estimate this voltage by thinking about the length of the spark when your hand comes close to a grounded conductor (like the doorknob) (The basic definition of voltage, Eq 2.21 suggests that over short distances, $\Delta V \approx E\Delta x$, right? And you learned what the breakdown E field of air is last week...) Now use your capacitance (part a) to estimate the energy dissipated when you touch a doorknob (or your little brother/sister!) and spark. (Is it a lot, or a little?)

c) Planet earth has a static E field at sea level of ~ 100 V/m. Use this to estimate the net charge on the earth, and thus the total stored energy our "planetary capacitor" holds. If we could tap this, could it solve our energy problems?

(Just FYI, it turns out that the role of lightning in this story may surprise you, lightning is not DISCHARGING the giant earth capacitor... it's the charging mechanism!)

Q3. UNIQUENESS THEOREM

Griffiths sections 3.1.5 and 3.1.6 discuss and prove “uniqueness theorems”. Read through both sections and make sense of it all! For this homework problem, prove the second uniqueness yourself, using a slightly *different* method than Griffiths (though you will find many common "pieces" are involved!) Do it like this:

Go back to Green's Identity (Problem 1.60c in 3rd ed, or 1.61c in 4th ed, p. 56. You proved it in HW#2!) This identity is true for ANY choice of T and U, so let the functions T and U both be the SAME function: set them both equal to $V_3 = V_1 - V_2$. Then, Green's Identity (along with some arguments about what happens at the boundaries, like Griffiths uses in *his* proof) should let you quickly show that E_3 (which is defined to be the negative gradient of V_3 , as usual) must vanish everywhere throughout the volume. QED.

Understand the game. We are checking if there could be two different potential functions, V_1 and V_2 , each of which satisfies Laplace's equation throughout the region considered. You construct (define) V_3 to be the difference of these, and prove that V_3 (or $E_3 = -\nabla V_3$) must vanish everywhere in the region. Which means there really is only one unique E-field throughout the region after all! This is a pretty "formal manipulation", giving you a chance to practice with the divergence theorem and think about boundary conditions...

Q4. METHOD OF IMAGES – CORNER

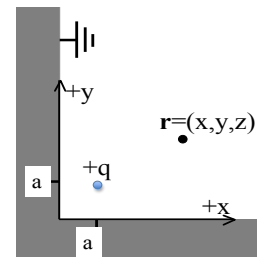
a) Take 4 charges, and put them on the corners of a square. Two are +q, the other two are -q. The like charges should be diagonally opposite one another. Show that there are two simple equipotential surfaces that are planes.

b) Use part a to (rather simply) obtain a formula for the voltage $V(x,y,z)$ arising from a single point charge +q located symmetrically a distance a from the inside corner formed by bending an infinite, grounded metal sheet through a right angle.

- Use the PhET sim “charges and fields”

<http://phet.colorado.edu/en/simulation/charges-and-fields>

to visualize resulting E-field and some relevant equipotential contours. (Take a screen shot and include it, with a few comments to explain what you learned)



Q5. METHOD OF IMAGES - SPHERICAL

Look at Griffiths' Ex. 3.2, and associated Fig 3.12 (either edition) which shows a grounded metal sphere with a charge q outside it. He argues (leading to Eq. 3.17) that there is a simple "method of images" trick available here - you just have to put the right charge (q') at the right spot (b, inside the radius of the sphere).

a) Your first task is to show WHY this particular "image trick" works:

Eq 3.17 of Griffiths (either edition) says that for 2 points charges q and q'

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{\mathcal{R}} + \frac{q'}{\mathcal{R}'} \right)$$

(where all variables are defined in Griffiths' Example 3.2)

Show/convince yourself (and the grader) that you can rewrite this as:

$$V(r, \theta) = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{\sqrt{r^2 + a^2 - 2racos\theta}} - \frac{q}{\sqrt{R^2 + (ra/R)^2 - 2racos\theta}} \right)$$

Phys 3310, HW #6, Due in class Wed Feb 20.

(Here, r and θ are the usual spherical polar coordinates, with the z -axis along the line from the origin to q)

- What is V everywhere on the sphere defined by $r=R$?
- Briefly summarize your understanding of this story: why/how has solving the problem of these 2 particular point charges solved the original problem?

b) Find the induced surface charge density on the surface of the sphere as a function of θ . Integrate this to get the total induced charge. What *should* it be?

Hint: Look again at the end of Griffiths section 2.3.5. to relate surface charge density to a formula for voltage. In particular, what is Griffiths' "n" (the normal direction) if you are on the surface of a sphere?

c) Now let's apply this result to a novel situation:

Imagine a grounded infinite thick conducting plane in the x - y plane, that has a (conducting) hemispherical bump (radius R) in it, centered at the origin, as shown.

A charge q sits a distance " a " above the plane, i.e. at the point $(0,0,a)$

I claim that you can find the potential V anywhere in the plane above the conductor using the method of images, with *three* image charges.

Where should they be? (Explain your reasoning- you need to ensure the boundary condition $V=0$ on the entire conductor.)

Use this to construct a formula for V at any point above the plane.

(Like Griffiths points out, the method of images is often about getting a little creative!)

