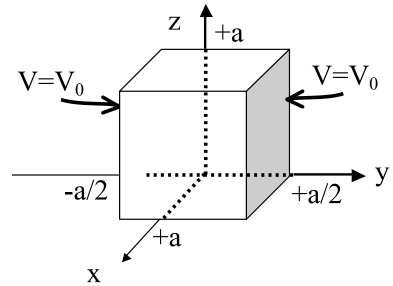
Phys 3310, HW #8, Due in class Wed Mar 6.

Q1. SEPARATION OF VARIABLES - CARTESIAN 3-D

You have a cubical box (sides all of length a) made of 6 metal plates which are insulated from each other.
The left wall is located at y=-a/2,the right wall is at y=+a/2.
Both left and right walls are held at constant potential $V=V_0$.
All four other walls are grounded.
(Note that I've set up the geometry so the cube runs from x=0 to x=a, and from z=0 to z=a, but from y=-a/2 to y=+a/2 This should actually make the math work out a little easier!)



Find the potential V(x,y,z) everywhere inside the box.
Do your formulas give V=0 at the center of this cube? Is E=0 there? (Should they be??)

Q2. SEPARATION OF VARIABLES - SPHERICAL

The potential on the surface of a sphere (radius R) is given by $V=V_0\cos(2\theta)$.
(Assume $V(r=\infty)=0$. Also assume there is no charge in or outside, it's ALL on the surface!)
a) Find the potential everywhere inside and outside this sphere.
(Hint: Can you express cos(2θ) as a simple linear combination of Legendre polynomials?)

b) Find the charge density $\sigma(\theta)$ on the surface of the sphere in part a. Sketch or draw it!

Q3. SEPARATION OF VARIABLES - CONCENTRIC SPHERES

Two concentric spherical surfaces have radii of a and b. If the potential on the outer surface, at r=b, is just a nonzero *constant* (call it V_{out}) and the potential on the inner surface is given by $V(a,\theta) = V_{in}P_1(\cos\theta) \quad (i.e. = V_{in}\cos\theta)$, find the potential in the region *between* the two surfaces (a < r <b).

Q4. SEPARATION OF VARIABLES - SPHERICAL SIGMA

The surface charge density on a sphere of radius *R* is constant, $+\sigma_0$, on the entire northern hemisphere, and $-\sigma_0$ on the entire southern hemisphere. There are no other charges present inside or outside the sphere. Use the method of separation of variables in spherical coordinates to find the electrical potential inside and outside this sphere.
In principle, you will need an infinite sum of terms, but for this problem, just work out explicitly what the first *nonzero* term is, for both *V(r < R)* and *V(r > R)*.

Explain physically why the first "zero term" really should be zero.

Griffiths solves a generic example problem, this is a special case. Work through the details on your own - use Griffiths to guide you if/whenever you need it, but in the end, solve the whole problem yourself and show your work! The charge density in this problem may "look" like a constant, but of course it is not, it is definitely a function of angle!

Q5. DIPOLES.

Let's continue with the situation described in the previous question: Compute the dipole moment of that sphere (with the +*z*-axis up through the pole of the positive, $+\sigma_0$, hemisphere).
(over…)

Phys 3310, HW #6, Due in class Wed Feb 20.

Begin with the definition of a dipole moment, $\vec{p} = \int \rho(\vec{r})\,\vec{r}\,d\tau$, which in this case becomes
$\vec{p} = \int \sigma(\vec{r})\vec{r}\,da = \hat{x}\int da\,x\,\sigma(\vec{r}) + \hat{y}\int da\,y\,\sigma(\vec{r}) + \hat{z}\int da\,z\,\sigma(\vec{r})$
Two of the three components are zero. Working in spherical coordinates, show why those components are zero. (Show this explicitly. Simply stating that they are zero "by symmetry" is not enough.)
Write your final answer for the dipole moment in terms of the charge Q on the upper hemisphere. Does your answer make sense to you?
How does your answer to Question #4 relate to this part?

Extra Credit: (Each is worth half a normal lettered part. They are two unrelated problems, you can do either one, or both…)

i) You have a conducting metal sphere (radius R), with a net charge +Q on it.
It is placed into a pre-existing uniform external field $\mathbf{E}_0$ which points in the z direction. (So, this is *exactly like* Griffiths Example 3.8, except the sphere is not neutral to start with.) Find the potential everywhere inside and outside this sphere. Please explain clearly *where* you are setting the zero of your potential. Do you have any freedom in this matter? Briefly, explain.

ii) In last week's problem 1 (Homework Set 7), you had to solve for the voltage inside a square pipe with the voltage fixed on three side of the pipe and the strange-looking condition $\partial V / \partial y = 0$ on the 4th side. What physical situation would produce such boundary conditions: that is, how would an experimentalist arrange things so as to guarantee that $\partial V / \partial y = 0$ on the 4^{th} side?