Phys 3310, HW #9, Due in class Wed Mar 13.

Q1. MULTIPOLES - point charges
You have four point charges. Their location and charges in Cartesian coordinates are:
A positive charge -2q located at (a,0,0), another charge -2q located at (-a,0,0), a 3rd charge -q located at (0,0,b), and finally a fourth charge +5q located at (0,0,-b)

- What is the total charge, and dipole moment, of this distribution of charges? Use the methods of "the multipole expansion" (Griffiths section 3.4.1) to find a simple approximate formula for V(r,θ) (in spherical coordinates!) valid at points far from the origin. ("Simple" means only the *first non-zero term* is needed!)

- Find a simple approximate expression for the electric field valid at points far from the origin. (Again, express your answer in spherical coordinates, so we want
$\vec{E}(r,\theta) = E_r(r,\theta)\hat{\mathbf{r}} + E_\theta(r,\theta)\hat{\theta}$, and you should figure out what E_r and E_θ are.)
Sketch this (approximate) E field. *(Don't worry about what happens near the origin, I just want a sketch of the simple approximation)*

Q2. MULTIPOLES - spherical shell charge distribution
Griffiths derives (end of section 3.3.2) the exact potential V(r,θ) everywhere outside a spherical shell of radius R which has a surface charge distribution $\sigma(R,\theta)= \sigma_0\cos\theta$

a) Calculate the dipole moment of this object, and also the total charge on this object.

b) Use the method of the multipole expansion (Griffiths sections 3.4.1 and 3.4.2) to find an *approximate* form for the potential far from the sphere. Stop with the first leading nonzero term. Compare with Griffiths *exact formula* (from the example, end of section 3.3.2).
- What does this tell you about the quadrupole moment (and higher moments) of this surface charge distribution?

 Q3. SEPARATION OF VARIABLES - DISK
A disk of radius R has a uniform surface charge density σ_0. Way back on Set #2 you found the E-field along the axis of the disk. You can check the following for yourself easily by direct integration, (but you do NOT have to here): I claim that along the z axis, (i.e. θ=0),
$$V(r,\theta = 0) = \frac{\sigma_0}{2\varepsilon_0}\left(\sqrt{r^2 + R^2} - r\right)$$

a) Find the potential *away* from the axis (i.e nonzero θ) , for distances $r > R$, by using the result above and fiddling with the Legendre formula, Griffiths' boxed Equation 3.65. You will in principle need an infinite sum of terms here - but for this problem, just work out explicitly what the first *two* *non-zero* terms are.
(It might help to remember that $P_l(1)$ is always equal to one, and you will have to think mathematically about how the formula above behaves for r>>R.)

b) Look at your answer to part a, and compare it to what Griffiths says it *should* look like (generically) in the boxed equation at the end of 3.4.1, i.e. the multipole expansion.
To discuss: does your answer make some physical sense? In particular, note that there is a "missing term" - why is that?

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Q4. REAL DIPOLE.
a) Griffiths Table 4.1 gives an experimental value for $\alpha/4\pi\varepsilon_0$ for atomic hydrogen. (Read his caption carefully for units!) Here, α is the "atomic polarizibility", defined by $p=\alpha E$. Study Griffiths' Example 4.1, which tells you how to estimate the atomic polarizability, summarize the example in your own words, and then use it (combined with this experimental number) to estimate the atomic radius of hydrogen.
- How well did you do, compared, say, with the Bohr radius? Also, after summarizing his example, can you tell us what physical assumption (simplification!) Griffiths is making about the physical distribution of negative charge inside an atom? Is that realistic?

b) Now suppose you have a single hydrogen atom inside a charged parallel-plate capacitor, with plate spacing 1 mm, and voltage 100 V.
- What is the "separation distance" d (as defined in that same Example 4.1 problem) of the electron cloud and the proton nucleus?
- What fraction of the atomic radius of part a is this? (You should conclude that 100 V across a 1mm gap capacitor is unlikely to ionize a hydrogen atom, do you agree?)
- Use your calculations to <u>roughly</u> <u>estimate</u> what voltage (and thus, what E-field) WOULD ionize this single hydrogen atom. *(I'd say if you can pull the electron cloud one full atomic radius away, it's breaking down!)*

Q5. BOUND CHARGES
Consider a long teflon rod, (a dielectric cylinder), radius a. Imagine that we could somehow set up a permanent polarization $\mathbf{P}(s,\theta,z) = \mathbf{ks}$ $(= k\,s\hat{\mathbf{s}})$, where **s** is the usual cylindrical radial vector from the z-axis, and c is a constant). *Neglect end effects, the cylinder is long.*
a) Calculate the bound charges σ_{bound} (on the outer surface) and ρ_{bound} (in the interior of the rod). What are the units of your constant "k"? *(Do the units work out in <u>all</u> formulas you have used involving "k"?)*
b) Next, use these bound charges (along with Gauss' law, this problem has very high symmetry!) to find the E-field inside and outside the cylinder. (Direction and magnitude)
c) Finally, find the electric displacement field **(D)** inside and outside the cylinder using the fundamental definition (Griffiths boxed Eq 4.21) and verify that Griffiths' Eq 4.23 ("Gauss' law for D-fields") works out. Explain briefly in words why your answers are what they are.

Q6. BOUND CHARGES II
Now let's hollow out that teflon rod, so it has inner radius b, and outer radius is (still) a. Just to make things a little different here, suppose I now set up a <u>different</u> polarization within the teflon material, namely $\mathbf{P}(s,\theta,z) = k\,\hat{\mathbf{s}}$ for b<s<a. (Where k is a given constant)
a) We have vacuum for s<b and s>a. What does that tell you about **P** in those regions? Find the bound charges σ_{bound} (on inner and outer surfaces of the hollow rod) and ρ_{bound} (everywhere else). Use these bound charges, along with Gauss' law, to find the electric field everywhere in space. (Direction and magnitude)
b) Use Griffiths' Eq 4.23 to find **D** everywhere in space. (This should be quick – use symmetry! Are there any *free* charges in this problem?) Use this (simple) result for **D** (along with Griffiths basic relation of **E** to **D**, Eq 4.21) to find **E** everywhere in space. *(This should serve as a check for part a, and shows why sometimes thinking about **D** fields is easier and faster!)*