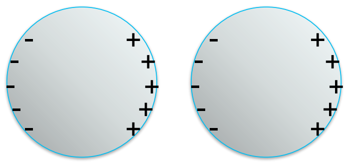
Is this a stable charge distribution for two neutral, conducting spheres? (There are no other charges around)



A) Yes
B) No

C) ???

General properties of solutions of $\nabla^2 V=0$

(1) V has no local maxima or minima inside. Maxima and minima are located on surrounding boundary.

(2) V is boring. (I mean "smooth & continuous" everywhere).

(3) $V(\mathbf{r})$ = average of V over any surrounding sphere:

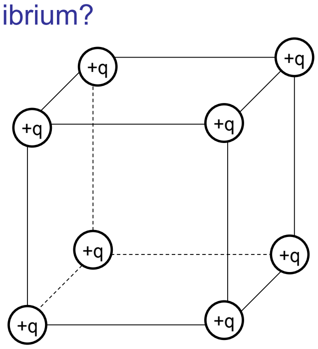
$$V(\vec{r}) = \frac{1}{4\pi R^2} \oint_{\substack{\text{Sphere with} \\ \text{radius } R \\ \text{around } \vec{r}}} V\, da$$

(4) V is unique: The solution of the Laplace eq. is uniquely determined if V is specified on the boundary surface around the volume.

3.5 If you put a + test charge at the *center* of this cube of charges, could it be in stable equilibrium?

A) Yes
B) No
C) ???

Earnshaw's Theorem



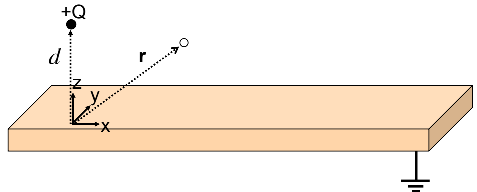
3.7 A point charge +Q sits above a *very large grounded* conducting slab.

What is **E**(**r**) for other points *above* the slab?

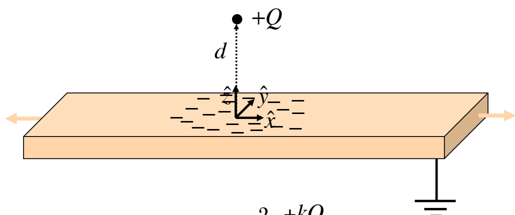
A) Simple Coulomb's law:

$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0}\frac{\vec{\Re}}{\Re^3} \quad \text{with } \vec{\Re} = (\vec{r} - d\,\hat{z})$$

B) Something more complicated



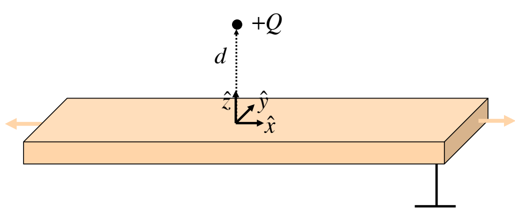
Infinite grounded conducting slab



$$V(z>0) \stackrel{?}{=} \frac{+kQ}{|\mathbf{r}-\mathbf{r}'|}$$

Boundary Conditions: $V(\text{far away}) \rightarrow 0$

$V(z=0) = 0$ (grounded)



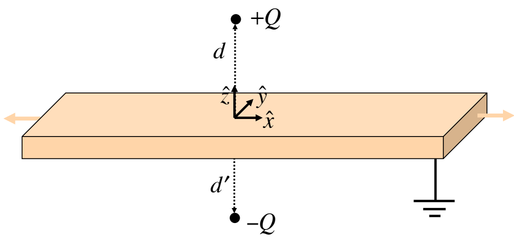
Calculate voltage V(r) (everywhere in space!) for 2 equal/opposite point charges a distance "d" above and below the origin.

Where is V(r)=0?

(If you're done early, figure out Ex, Ey, and Ez from this voltage, and evaluate (simplify) AT the plane z=0)

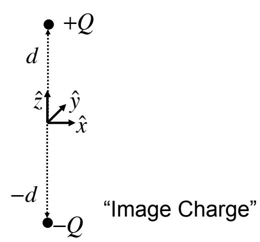
Boundary Conditions: $V(far\ away) \to 0$

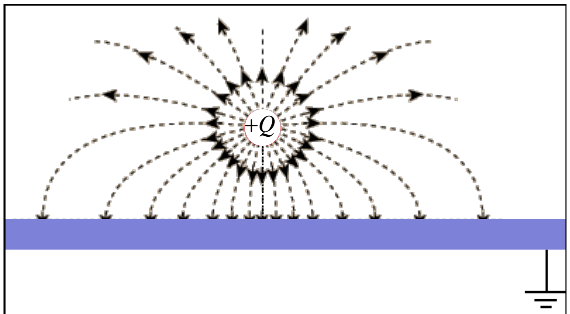
$V(z=0)=0$ (grounded)



Boundary Conditions: $V(far\ away) \to 0$

$V(z=0)=0$ (grounded)





Method of Images

$\nabla^2 V = -\rho/\varepsilon_0$ Poisson's Equation

Last Class: Uniqueness Theorem

If: $\nabla^2 V_A = -\rho/\varepsilon_0$ & $\nabla^2 V_B = -\rho/\varepsilon_0$

& If: $V_A = V_B$ on the boundaries

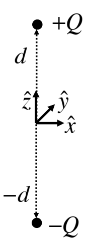
Then: $V_A = V_B$ everywhere

(within the boundaries)

Boundary Conditions: $V(\textit{far away}) \to 0$

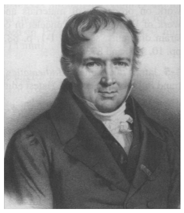
$V(z=0) = 0$ (grounded)

$$V(\mathbf{r})\big|_{z\geq 0} = \sum_i \frac{kQ_i}{|\mathbf{r} - \mathbf{r}_i'|}$$



Uniqueness Theorem

Method of Images



Siméon Denis
Poisson
1781-1840

Method of Images

William Thomson
(Lord Kelvin)
1824-1907

Method of Images

James Clerk
Maxwell
1831-1879

"Treatise on Electricity and Magnetism"
1873, Vol. 1, Ch. XI (p. 245-283, 3rd Ed.)

Boundary Conditions: $V(far\ away) \to 0$

$V(z=0)=0$ (grounded)

$$V(\mathbf{r})\Big|_{z\geq 0} = \left[\frac{kQ}{\left(x^2+y^2+(z-d)^2\right)^{1/2}} - \frac{kQ}{\left(x^2+y^2+(z+d)^2\right)^{1/2}} \right]$$

$$\mathbf{E}(\mathbf{r}) = -\nabla V(\mathbf{r})$$

$$\sigma(x,y) = -\varepsilon_0 \frac{\partial V}{\partial z}\Big|_{z=0}$$

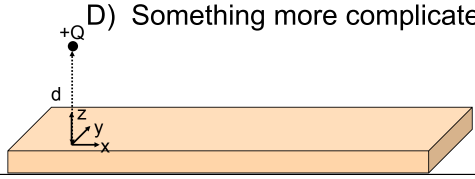
3.8 A point charge +Q sits above a *very large grounded* conducting slab. What is the electric force on this charge?

A) 0

B) $\dfrac{Q^2}{4\pi\varepsilon_0(2d)^2}$ downwards

C) $\dfrac{Q^2}{4\pi\varepsilon_0 d^2}$ downwards

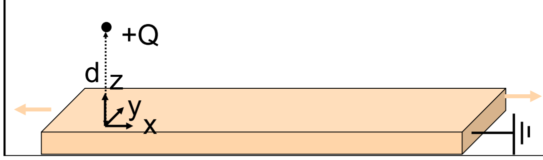
D) Something more complicated

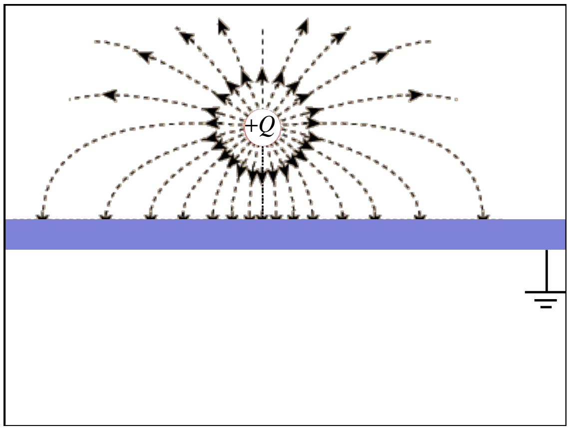


3.8b A point charge +Q sits above a *very large grounded* conducting slab. What's the energy of this system?

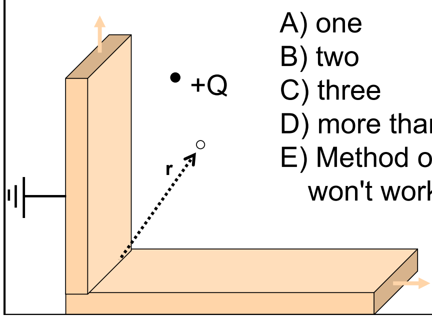
A) $\dfrac{-Q^2}{4\pi\varepsilon_0(2d)}$

B) Something else.





3.9 Two ∞ grounded conducting slabs meet at right angles. How many image charges are needed to solve for V(**r**)?



A) one
B) two
C) three
D) more than three
E) Method of images won't work here