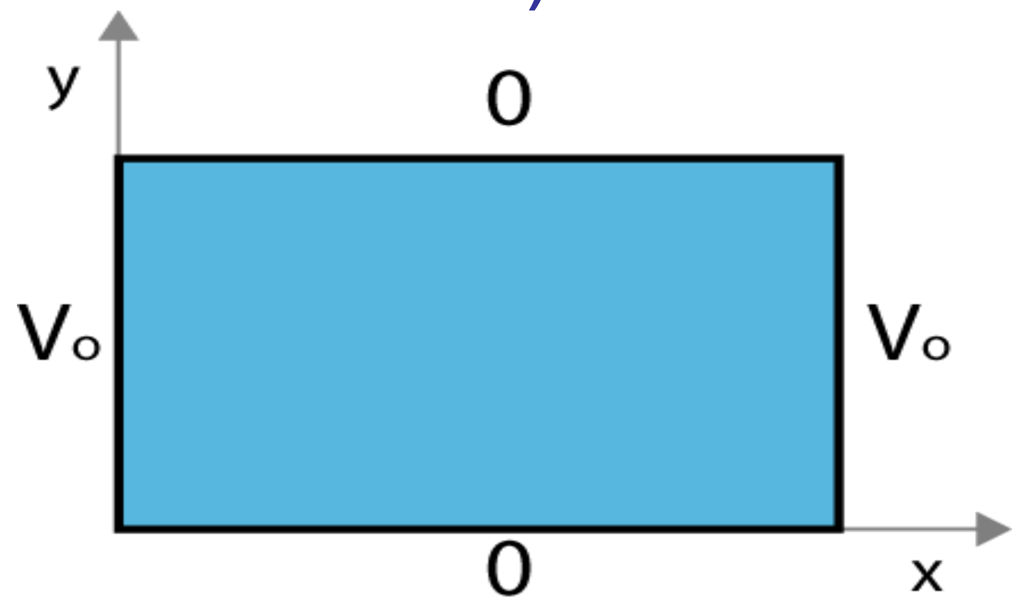
3.11 Given the two diff. eq's :

$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

where $C_1+C_2 = 0$. Given the boundary conditions in the figure, which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?



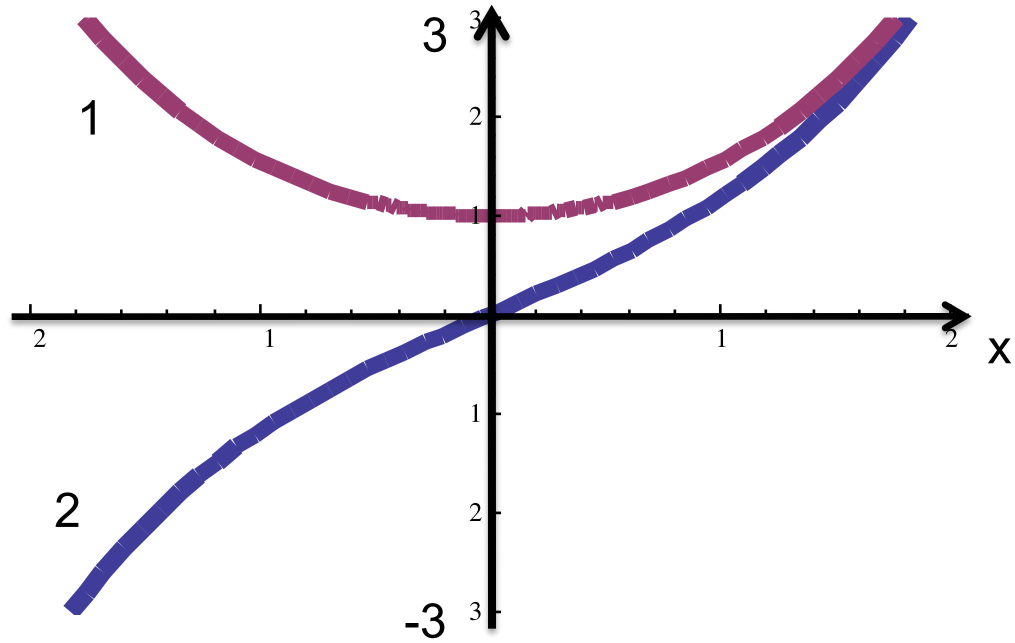

A) x

B) y

C) $C_1 = C_2 = 0$ here

D) It doesn't matter

Two solutions for *positive* C are sinh x and cosh x :




Which is which?

A)Curve 1 is sinh x and curve 2 is cosh x

B)Curve 1 is cosh x and curve 2 is sinh x

3.11 c

Given the two diff. eq's:

$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

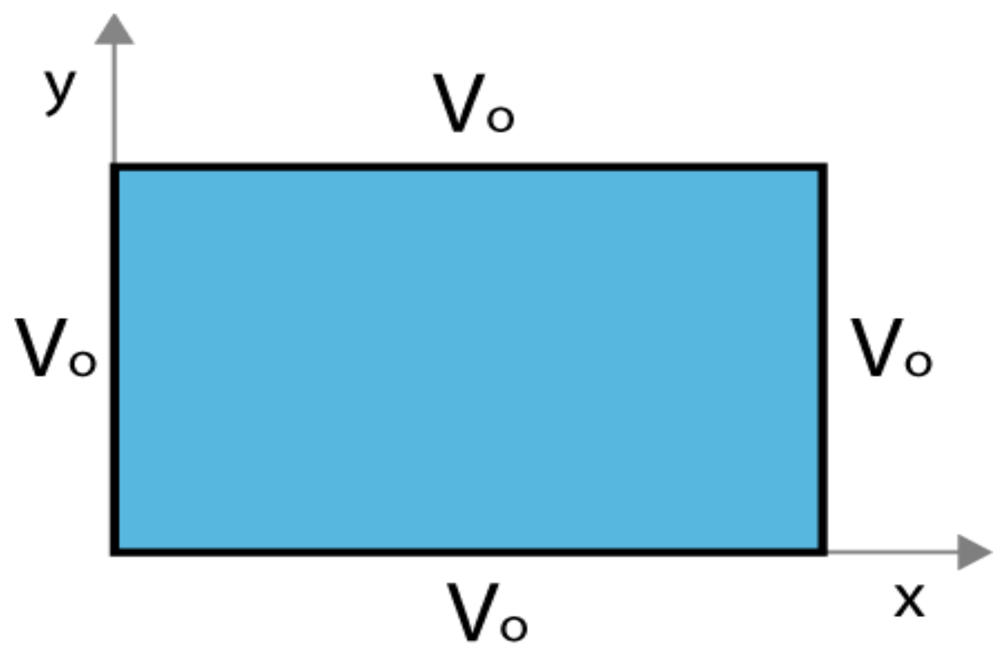
where $C_1+C_2 = 0$. Which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

A) x

B) y

C) $C_1= C_2=0$ here

D) It doesn't matter



3.11 h

The X(x) equation in this problem involves the "positive constant" solutions:
A sinh(kx) + B cosh(kx)

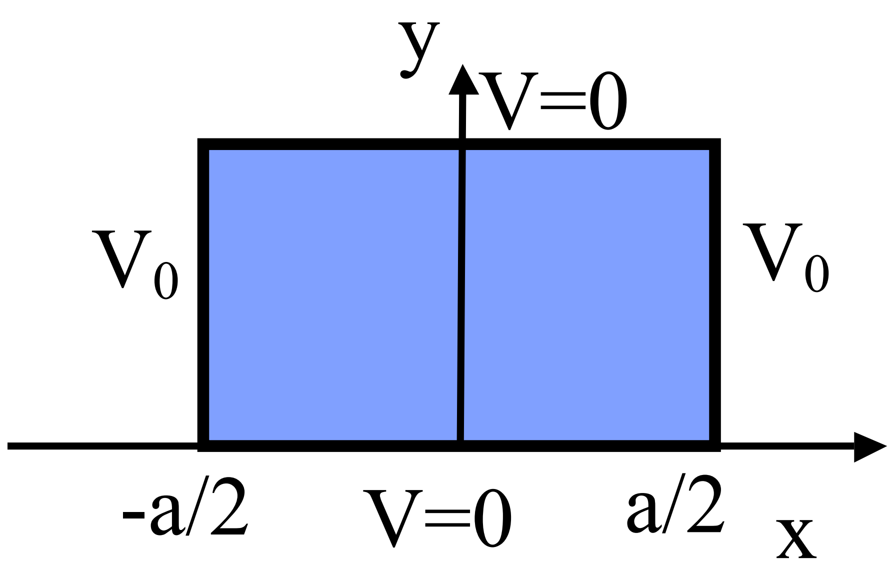
What do the boundary conditions say about the coefficients A and B above?

A) A=0 (pure cosh)

B) B=0 (pure sinh)

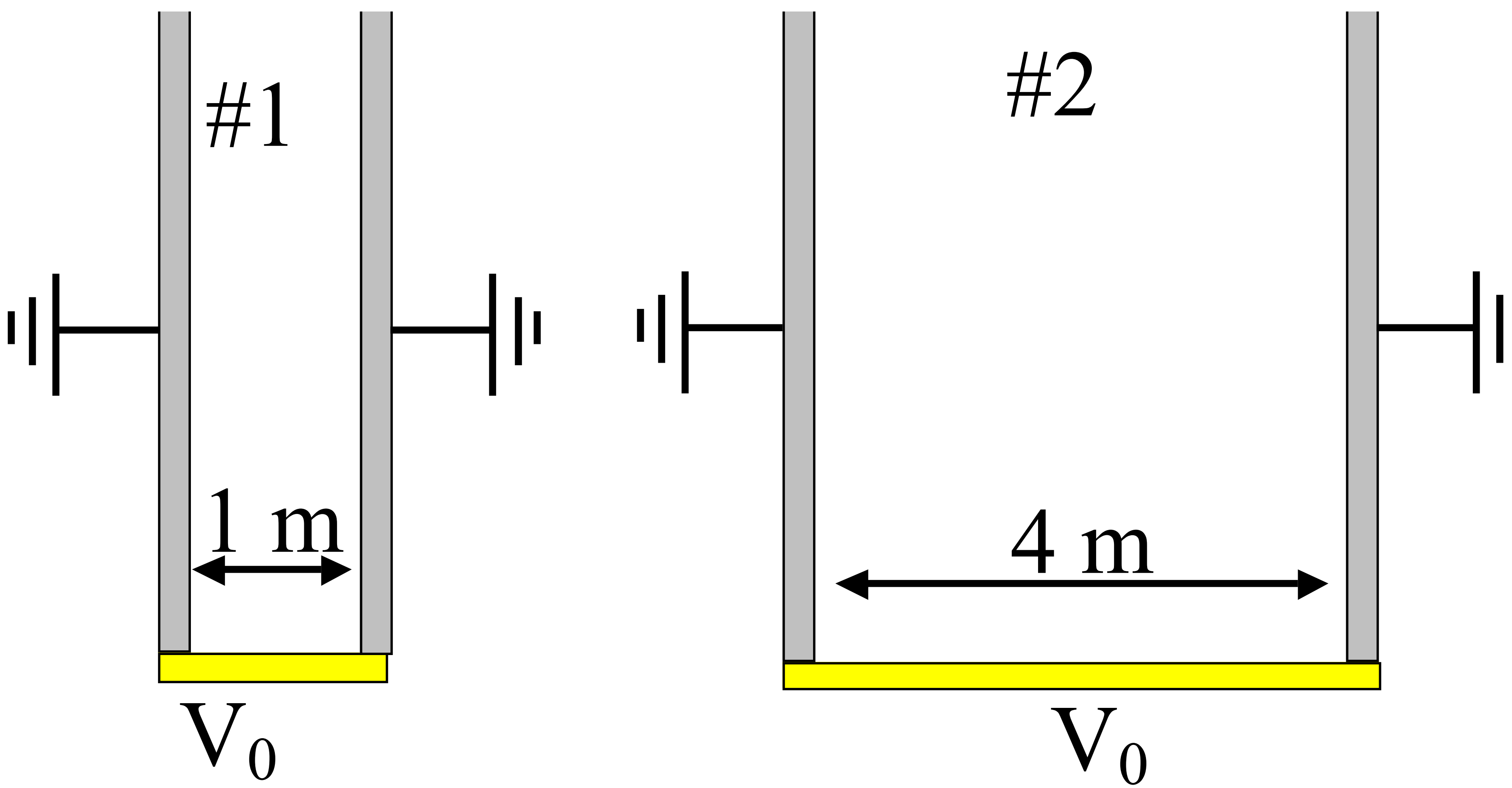
C) Neither: you should rewrite this in terms of $A' e^{kx} + B' e^{-kx}$ !

D) Other/not sure?



3.14

2 troughs (∞ in z, i.e. out of page) have grounded sidewalls.
The base of each is held at V0.



3.14

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5...}^{\infty} \frac{1}{n} \sin(n\pi x / a) e^{-n\pi y/a}$$

How does V(x,y) compare, 4 m above the middle of the base in the two troughs?

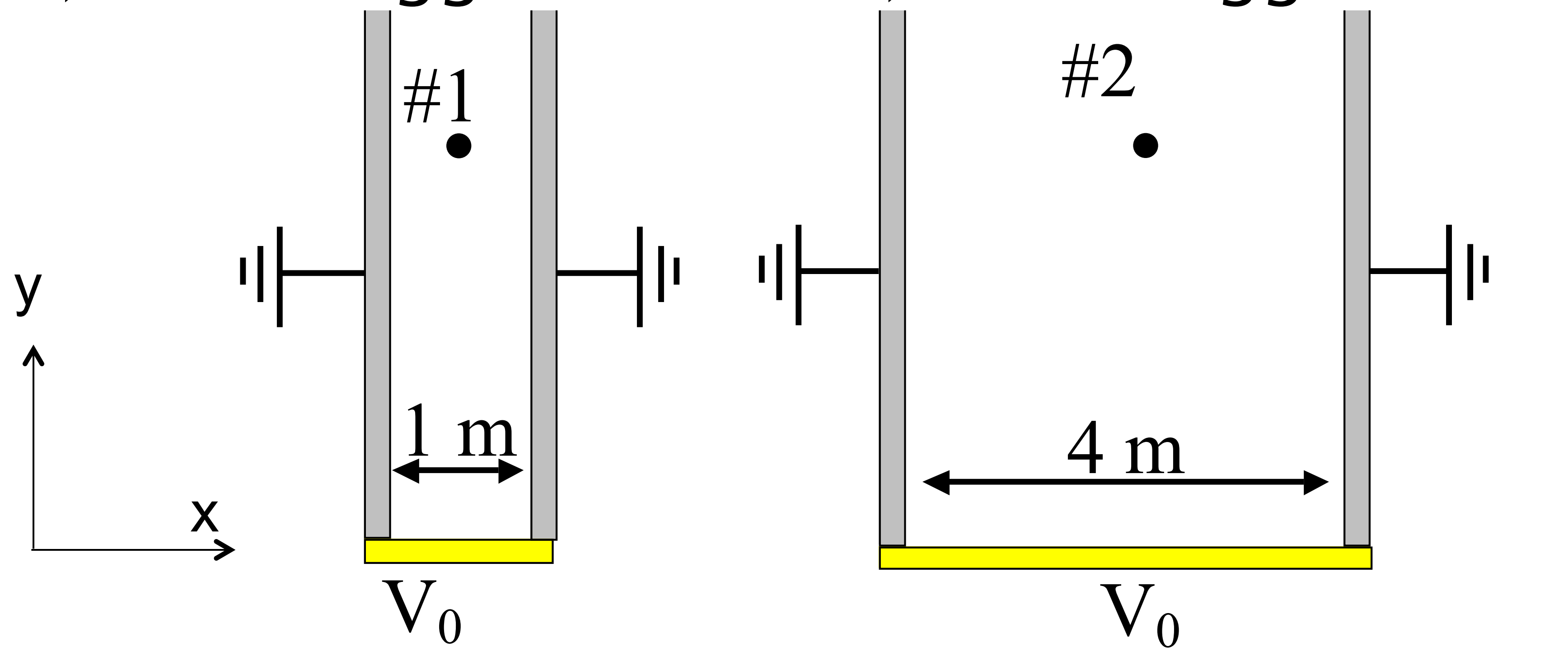
A) Same in each

B) 4x bigger in #1

C) 4x bigger in #2

D) *much* bigger in #1

E) *much* bigger in #2



3.15 Given $\nabla^2 V = 0$ in Cartesian coords, we separated V(x,y,z) = X(x)Y(y)Z(z). Will this approach work in spherical coordinates, i.e. can we separate $V(r,\theta,\varphi) = R(r)P(\theta)F(\varphi)$?

A) Sure.

B) Not quite - the angular components cannot be isolated, e.g. $f(r,\theta,\varphi) = R(r)Y(\theta,\varphi)$

C) It won't work at all because the spherical form of Laplace's Equation has cross terms in it (see the front cover of Griffiths)

3.16 The Rodrigues formula for generating the Legendre Polynomials is

$$P_l(x) = \frac{1}{2^l l!}\left(\frac{dy}{dx}\right)^l (x^2 - 1)^l$$

If the Legendre polynomials are orthogonal, are the leading coefficients $\frac{1}{2^l l!}$ necessary to maintain orthogonality?

A) Yes, $f_m(x)$ must be properly scaled for it to be orthogonal to $f_n(x)$.

B) No, the constants will only rescale the integral

3.17

Given $V(r,\theta) = \sum_l C_l P_l(\cos(\theta))$ we want to get to

the integral: $\int_{-1}^{1} P_l(u) P_m(u) du = \left[ \begin{array}{c} \frac{2}{2l+1}, l = m \\ 0, l \neq m \end{array} \right]$

we can do this by multiplying both sides by:

A) $P_m(\cos\theta)$

B) $P_m(\sin\theta)$

C) $P_m(\cos\theta)\sin\theta$

D) $P_m(\sin\theta)\cos\theta$

E) $P_m(\sin\theta)\sin\theta$

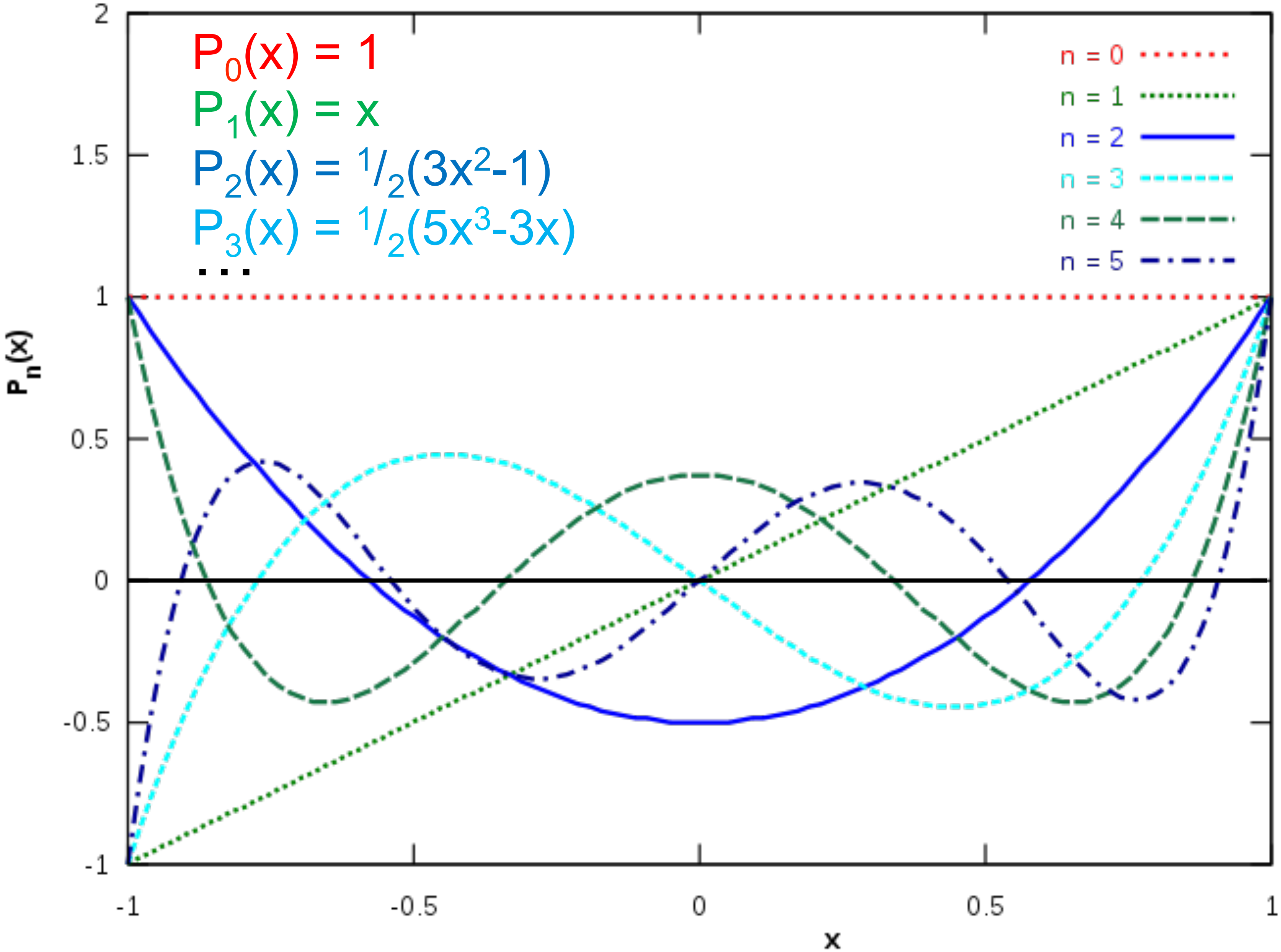
# Orthogonality

$$\int_{-1}^{1} P_l(x) P_m(x)\,dx = \begin{cases} \dfrac{2}{2l+1} & \text{if } l = m \\ 0 & \text{if } l \neq m \end{cases}$$

With: $x = cos\theta$ and: $dx = -\ sin\theta d\theta$ , we get:

$$\int_{0}^{\pi} P_l(\cos\theta) P_m(\cos\theta) \sin\theta\, d\theta = \begin{cases} \dfrac{2}{2l+1} & \text{if } l = m \\ 0 & \text{if } l \neq m \end{cases}$$

# Legendre Polynomials $P_n(x)$



3.18

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is constant, i.e. $V(R, \theta) = V_0$.

Which terms do you expect to appear when finding V(outside) ?

A) Many $A_l$ terms (but no $B_l$'s)

B) Many $B_l$ terms (but no $A_l$'s)

C) Just $A_0$

D) Just $B_0$

E) Something else!!

3.18
b

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is constant, i.e. $V(R, \theta) = V_0$.

Which terms do you expect to appear when finding V(inside) ?

A) Many $A_l$ terms (but no $B_l$'s)

B) Many $B_l$ terms (but no $A_l$'s)

C) Just $A_0$

D) Just $B_0$

E) Something else!

3.19a

$$P_0(\cos\theta) = 1, \qquad P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}, \qquad P_3(\cos\theta) = \frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta$$

Can you write the function $V_0(1+\cos^2\theta)$ as a sum of Legendre Polynomials?

$$V_0(1+\cos^2\theta) \overset{???}{=} \sum_{l=0}^{\infty} C_l P_l(\cos\theta)$$

A) No, it cannot be done
B) It would require an infinite sum of terms
C) It would only involve $P_2$
D) It would involve all three of $P_0$, $P_1$ AND $P_2$
E) Something else/none of the above

3.19

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is

$$V(R,\theta) = V_0(1+\cos^2\theta)$$

Which terms do you expect to appear when finding V(inside) ?

A) Many $A_l$ terms (but no $B_l$'s)
B) Many $B_l$ terms (but no $A_l$'s)
C) Just $A_0$ and $A_2$
D) Just $B_0$ and $B_2$
E) Something else!

3.19
b

$$V(r,\theta)=\sum_{l=0}^{\infty}\left(A_l r^l+\frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is

$$V(R,\theta)=V_0(1+\cos^2\theta)$$

Which terms do you expect to appear when finding V(outside) ?

A) Many $A_l$ terms (but no $B_l$'s)
B) Many $B_l$ terms (but no $A_l$'s)
C) Just $A_0$ and $A_2$
D) Just $B_0$ and $B_2$
E) Something else!

MD11-2

Suppose that applying boundary conditions to Laplace's equation leads to an equation of the form:

$$\nabla^2 V = 0$$

$$\sum_{l=0}^{\infty} C_l P_l(\cos\theta) = 4 + 3\cos\theta$$

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = (3x^2 - 1)/2$$

Can you solve for the coefficients, the $C_l$'s ?

A) No, you need at least one more equation to solve for any the C's.

B) Yes, you have enough info to solve for all of the C's

C) Partially. Can solve for $C_0$ and $C_1$, but cannot solve for the other C's.

D) Partially. Can solve for $C_o$, but cannot solve for the other C's.