3.18 b

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

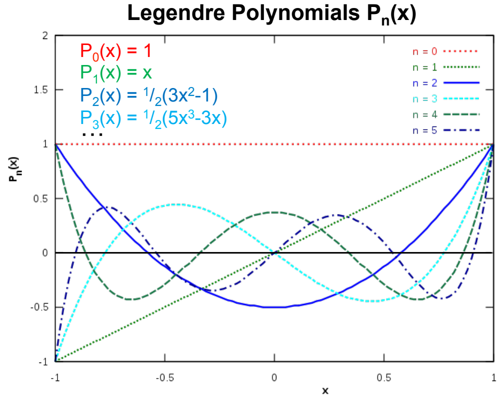
V everywhere on a spherical shell is a given constant, i.e. $V(R, \theta) = V_0$.
There are no charges inside the sphere.
Which terms do you expect to appear when finding V(inside) ?

A) Many A_l terms (but no B_l's)
B) Many B_l terms (but no A_l's)
C) Just A_0
D) Just B_0
E) Something else!

Hint:

V must be finite everywhere.

Legendre Polynomials $P_n(x)$



Orthogonality

$$\int_{-1}^{1} P_l(x)P_m(x)dx = \begin{cases} \frac{2}{2l+1} & \text{if } l = m \\ 0 & \text{if } l \neq m \end{cases}$$

With: $x = cos\theta$ and: $dx = - sin\theta d\theta$, we get:

$$\int_{0}^{\pi} P_l(\cos\theta)P_m(\cos\theta)\sin\theta d\theta = \begin{cases} \frac{2}{2l+1} & \text{if } l = m \\ 0 & \text{if } l \neq m \end{cases}$$

3.17

Given $V_0(\theta) = \sum_l C_l P_l(\cos(\theta))$ we want to get to

the integral: $\int_{-1}^{1} P_l(u) P_m(u) du = \begin{bmatrix} \frac{2}{2l+1}, l = m \\ 0, l \neq m \end{bmatrix}$

we can do this by multiplying both sides by:

A) $P_m(\cos\theta)$
B) $P_m(\sin\theta)$
C) $P_m(\cos\theta) \sin\theta$
D) $P_m(\sin\theta) \cos\theta$
E) $P_m(\sin\theta) \sin\theta$

3.18 b

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Suppose V on a spherical shell is constant, i.e. $V(R, \theta) = V_0$.
Which terms do you expect to appear when finding V(inside) ?

A) Many A_l terms (but no B_l's)
B) Many B_l terms (but no A_l's)
C) Just A_0
D) Just B_0
E) Something else!

Hints:

V must be finite everywhere.

$P_0(\cos\theta) = 1$
$P_1(\cos\theta) = \cos\theta$
$P_2(\cos\theta) = 3/2 \cos^2\theta - ½$
Etc.
- Legendre Polynomials are orthogonal

3.18

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Suppose V on a spherical shell is constant, i.e. $V(R, \theta) = V_0$.
Which terms do you expect to appear when finding V(outside) ?

A) Many A_l terms (but no B_l's)
B) Many B_l terms (but no A_l's)
C) Just A_0
D) Just B_0
E) Something else!!

3.19a

$P_0(\cos\theta) = 1,\qquad P_1(\cos\theta) = \cos\theta$

$P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2},\qquad P_3(\cos\theta) = \frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta$

Can you write the function $V_0(1+\cos^2\theta)$ as a sum of Legendre Polynomials?

$$V_0(1+\cos^2\theta) \overset{???}{=} \sum_{l=0}^{\infty} C_l P_l(\cos\theta)$$

A) No, it cannot be done
B) It would require an infinite sum of terms
C) It would only involve P_2
D) It would involve all three of P_0, P_1 AND P_2
E) Something else/none of the above

3.19

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is

$$V(R,\theta) = V_0(1+\cos^2\theta)$$

Which terms do you expect to appear when finding V(inside) ?
A) Many A_l terms (but no B_l's)
B) Many B_l terms (but no A_l's)
C) Just A_0 and A_2
D) Just B_0 and B_2
E) Something else!

3.19 b

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is

$$V(R,\theta) = V_0(1+\cos^2\theta)$$

Which terms do you expect to appear when finding V(outside) ?
A) Many A_l terms (but no B_l's)
B) Many B_l terms (but no A_l's)
C) Just A_0 and A_2
D) Just B_0 and B_2
E) Something else!

MD11-2

$\nabla^2 V = 0$

Suppose that applying boundary conditions to Laplace's equation leads to an equation of the form:

$$\sum_{l=0}^{\infty} C_l P_l(\cos\theta) = 4 + 3\cos\theta$$

$(x \equiv \cos\theta)$

$P_0(x) = 1$

$P_1(x) = x$

$P_2(x) = (3x^2 - 1)/2$

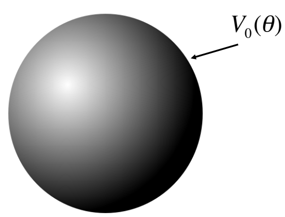
Can you solve for the coefficients, the C_l's ?

A) No, you need at least one more equation to solve for any the C's.
B) Yes, you have enough info to solve for all of the C's
C) Partially. Can solve for C_0 and C_1, but cannot solve for the other C's.
D) Partially. Can solve for C_o, but cannot solve for the other C's.

3.20

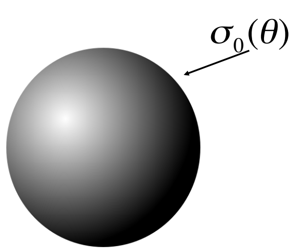
How many boundary conditions (on the potential V) do you use to find V inside the spherical plastic shell?

A) 1
B) 2
C) 3
D) 4
E) It depends on $V_0(\theta)$



3.21

How many boundary conditions (on the potential V) do you use to find V inside the thin plastic spherical shell?



A) 1
B) 2
C) 3
D) 4
E) depends on σ_0

3.21
b

Does the previous answer change at all if you're asked for V *outside* the sphere?

a) yes

b) No

Since the electric field is zero inside this conducting sphere, and $V = -\int \vec{E} \cdot d\vec{l}$, is V=0 inside as well?

a) Yes

b) No