

One of Maxwell's equations,  $\nabla \times E = 0$  made it useful for us to define a scalar potential  $V$ , where  $E = -\nabla V$

Similarly, another one of Maxwell's equations makes it useful for us to define the vector potential,  $\mathbf{A}$ . Which one?

- A)  $\nabla \times E = 0$
- B)  $\nabla \cdot E = \rho / \epsilon_0$
- C)  $\nabla \times B = \mu_0 J$
- D)  $\nabla \cdot B = 0$
- E) something else!

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$$\nabla \times \vec{E} = 0 \rightarrow \vec{E} = -\nabla V$$

Can add a constant 'c' to V without changing E ("Gauge freedom"):  $\nabla c = 0$ ,

$$\nabla \cdot \vec{B} = 0 \rightarrow \vec{B} = \nabla \times \vec{A}$$

Can add any vector function 'a' with  $\nabla \times a = 0$  to  $\vec{A}$  without changing  $\vec{B}$  ("Gauge freedom")  
 $\nabla \times (\vec{A} + \vec{a}) = \nabla \times \vec{A} + \nabla \times \vec{a} = \nabla \times \vec{A} = \vec{B}$

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5.25

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

In Cartesian coordinates, this means:

$$\nabla^2 A_x = -\mu_0 J_x, \text{ etc.}$$

Does it also mean, in spherical coordinates, that  $\nabla^2 A_r = -\mu_0 J_r$

- A) Yes
- B) No

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5.25  
b

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint_{\mathfrak{R}} \frac{\vec{J}(\vec{r}')}{\mathfrak{R}} d\tau'$$

Can you calculate that integral using spherical coordinates?

A) Yes, no problem  
 B) Yes,  $r'$  can be in spherical, but  $\mathbf{J}$  still needs to be in Cartesian components  
 C) No.

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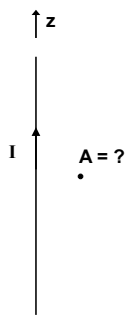
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MD12-3

The vector potential  $\mathbf{A}$  due to a long straight wire with current  $I$  along the  $z$ -axis is in the direction parallel to:



A)  $\hat{z}$   
 B)  $\hat{\phi}$  (azimuthal)  
 C)  $\hat{s}$  (radial)

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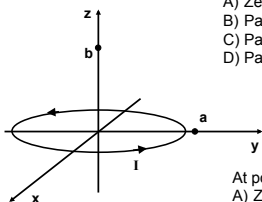
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MD12-4a,b

A circular wire carries current  $I$  in the  $xy$  plane. What can you say about the vector potential  $\mathbf{A}$  at the points shown?



At point a, the vector potential  $\mathbf{A}$  is:  
 A) Zero  
 B) Parallel to x-axis  
 C) Parallel to y-axis  
 D) Parallel to z-axis

At point b, the vector potential  $\mathbf{A}$  is:  
 A) Zero  
 B) Parallel to x-axis  
 C) Parallel to y-axis  
 D) Parallel to z-axis

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AFTER you are done with the front side:

The left figure shows the B field from a long, fat, uniform wire.

What is the physical situation associated with the RIGHT figure?

- A) **A** field from a long, fat wire
- B) **A** field from a long solenoid pointing to the right
- C) **A** field from a long solenoid pointing up the page
- D) **A** field from a torus
- E) Something else/???

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5.27 Suppose **A** is azimuthal, given by

$$\vec{\mathbf{A}} = \frac{c}{s} \hat{\phi}$$

What can you say about  $\text{curl}(\mathbf{A})$ ?

- A)  $\text{curl}(\mathbf{A})=0$  everywhere
- B)  $\text{curl}(\mathbf{A}) = 0$  everywhere except at  $s=0$ .
- C)  $\text{curl}(\mathbf{A})$  is nonzero everywhere
- D) ???

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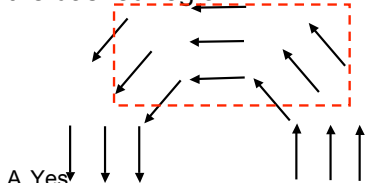
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5.24 If the arrows represent the vector potential **A** (note that  $|\mathbf{A}|$  is the same everywhere), is there a nonzero **B** in the dashed region?



- A. Yes
- B. No
- C. Need more information to decide

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Compare the magnetostatic triangle (p. 240) with the electrostatic triangle (pg. 87). How is the potential similar/different to the vector potential?

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What is  $\oint \vec{A}(\vec{r}) \cdot d\vec{l}$

- A) The current density  $\mathbf{J}$
- B) The magnetic field  $\mathbf{B}$
- C) The magnetic flux  $\Phi_B$
- D) It's none of the above, but is something simple and concrete
- E) It has no particular physical interpretation at all

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