5.10 Which of the following is a statement of charge conservation?

A) $\frac{\partial \rho}{\partial t} = -\int \vec{\mathbf{J}} \bullet d\vec{\mathbf{l}}$

B) $\frac{\partial \rho}{\partial t} = -\iint \vec{\mathbf{J}} \bullet d\vec{\mathbf{A}}$

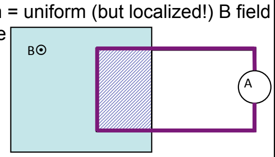
C) $\frac{\partial \rho}{\partial t} = -\iiint (\nabla \bullet \vec{\mathbf{J}})\, d\tau$

D) $\frac{\partial \rho}{\partial t} = -\nabla \bullet \vec{\mathbf{J}}$

E) Not sure/can't remember

Consider two situations:
1) a loop (purple) moves right at velocity v(loop)
2) a magnet (blue region) moves left, at v(mag).
If |v(loop)| = |v(mag)|,
what will the ammeter read in each case?
(Call CW current positive)

Blue region = uniform (but localized!) B field out of page



A) $I_1 > 0$, $I_2 = 0$
B) $I_1 < 0$, $I_2 = 0$
C) $I_1 = I_2$
D) $I_1 = -I_2$
E) $I_1 = 0$, $I_2 = 0$ (special case)

7.2 Maxwell's equations so far...

$$\nabla \bullet \mathbf{E} = \rho / \varepsilon_0 \qquad \nabla \bullet \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

The Curl(E)=0 equation let us define Voltage!
But, it's only true in statics…

7.2 Adding in time dependence modifies the curl(E) equation.

It's now called "Faraday's law".

$$\nabla \bullet \mathbf{E} = \rho / \varepsilon_0 \qquad \nabla \bullet \mathbf{B} = 0$$

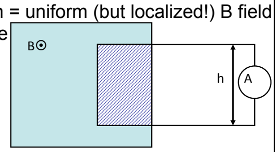
$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Faraday's law resolves the problem we found in the first question today:

Now CHANGING B can *also* induce currents.

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$$

Blue region = uniform (but localized!) B field out of page



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7.2

Maxwell's equations so far...

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Using Stoke's theorem on Faraday's law gives...

A) $\iint \mathbf{E} \cdot d\mathbf{A} = -d\Phi_B / dt$

B) $\iint \mathbf{E} \cdot d\mathbf{A} = -\partial \mathbf{B} / dt$

C) $\oint \mathbf{E} \cdot d\mathbf{l} = -d\Phi_B / dt$

D) $\oint \mathbf{E} \cdot d\mathbf{l} = -\partial \mathbf{B} / dt$

E) NONE of the above is correct!

7.2

Maxwell's equations so far...

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

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A) $\iint \mathbf{E} \cdot d\mathbf{A} = -d\Phi_B / dt$

B) $\iint \mathbf{E} \cdot d\mathbf{A} = -\partial \mathbf{B} / dt$

C) $\oint \mathbf{E} \cdot d\mathbf{l} = -d\Phi_B / dt$

We can do work with *changing* B-fields!

D) $\oint \mathbf{E} \cdot d\mathbf{l} = -\partial \mathbf{B} / dt$

E) NONE of the above is correct!

7.3

Look at our full set of "Maxwell's equations"

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

What is $\nabla \cdot \left(\nabla \times \vec{\mathbf{B}}\right)$?

A) zero
B) non-zero
C) Could be either
D) Could be BOTH at the same time
E) My brain hurts!

Hint: Do you have your textbook? Look in the FRONT flyleaf!

7.3

Look at our full set of "Maxwell's equations"

$$\nabla \bullet \mathbf{E} = \rho / \varepsilon_0 \qquad \nabla \bullet \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

What is $\nabla \bullet \left(\nabla \times \vec{\mathbf{B}}\right)$?

It has to be zero (that's math)!
But that says that

$$0 = \nabla \bullet (\nabla \times \mathbf{B}) = \mu_0 \nabla \bullet \mathbf{J}$$

5.10

Which of the following is a statement of charge conservation?

A) $\frac{\partial \rho}{\partial t} = -\int \vec{\mathbf{J}} \bullet d\vec{\mathbf{l}}$

B) $\frac{\partial \rho}{\partial t} = -\iint \vec{\mathbf{J}} \bullet d\vec{\mathbf{A}}$

C) $\frac{\partial \rho}{\partial t} = -\iiint (\nabla \bullet \vec{\mathbf{J}})\, d\tau$

D) $\frac{\partial \rho}{\partial t} = -\nabla \bullet \vec{\mathbf{J}}$

E) Not sure/can't remember

7.3

Taking div(Ampere's law) gave us:

$$\nabla \bullet \mathbf{J} = 0$$

But current conservation says:

$$\nabla \bullet \vec{\mathbf{J}} = -\frac{\partial \rho}{\partial t}$$

Conclusion: Ampere's law is WRONG!

7.2

$$\nabla \bullet \mathbf{E} = \rho / \varepsilon_0 \qquad \nabla \bullet \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

How can we restore current conservation?
(without losing the "correct" features,
obtained from experiment!)

Not very "symmetric" either…

Maxwell fixed it up!

7.5

Maxwell's equations:

$$\nabla \bullet \mathbf{E} = \rho / \varepsilon_0 \qquad \nabla \bullet \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \partial \mathbf{E}/\partial t$$

This last term saves the day!
NOW when we take div(Maxwell-Ampere), we get an extra term, yielding

$$\nabla \bullet \vec{\mathbf{J}} = -\frac{\partial \rho}{\partial t}$$

You did this on a homework!

7.5

Maxwell's equations:

$$\nabla \bullet \mathbf{E} = \rho / \varepsilon_0 \qquad \nabla \bullet \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \partial \mathbf{E}/\partial t$$

In vacuum…?

7.5

Maxwell's equations:

$$\nabla \bullet \mathbf{E} = 0 \qquad \nabla \bullet \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \partial \mathbf{E}/\partial t$$

In vacuum!

7.5

Maxwell's equations:

$$\nabla \bullet \mathbf{E} = 0 \qquad \nabla \bullet \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \partial \mathbf{E}/\partial t$$

In vacuum, what is $\nabla \times (\nabla \times \mathbf{E})$?

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \bullet \mathbf{E}) - \nabla^2 \mathbf{E}$$

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times (-\partial \mathbf{B}/\partial t) = -\partial(\nabla \times \mathbf{B})/\partial t$$

In vacuum (!!!)....

$$-\nabla^2 \mathbf{E} = -\mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \mathbf{E}$$

$$\nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \mathbf{E}$$

It's a wave equation…. With a solution

$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \bullet \mathbf{r} - \omega t)}$ or, if you prefer, more simply

$\mathbf{E} = E_0 \cos(kz - \omega t)\hat{x}$

With speed $1/\sqrt{\mu_0 \varepsilon_0}$

$\mathbf{B} = B_0 \cos(kz - \omega t)\hat{y}$

7.5

Maxwell's equations:

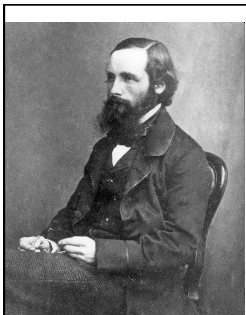
$\nabla \bullet \mathbf{E} = \rho / \varepsilon_0$ $\quad$ $\nabla \bullet \mathbf{B} = 0$

$\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$ $\quad$ $\nabla \times \mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\partial\mathbf{E}/\partial t$

"*It's of no use whatsoever* [...]
this is just an experiment that proves Maestro Maxwell was right - we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there." - Heinrich Hertz, 1888

Asked about the ramifications of his discoveries, Hertz replied, "*Nothing, I guess.*"

Marconi's first wireless radio transmission over large distances (~6 km over water) was in 1897.

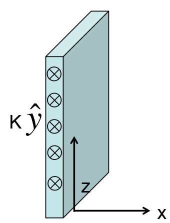


James Clerk Maxwell
Scottish 1831-1879

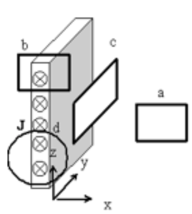
"From a long view of the history of mankind – seen from, say, ten thousand years from now – there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. **The American Civil War will pale into provincial insignificance in comparison with this important scientific event of the same decade."**

– R.P. Feynman

Which Amperian loop would you draw to learn *something* useful about **B** anywhere?



Which Amperian loops are *useful* to learn about B(x,y,z) somewhere?



If $B = B_0$ in the +x direction just RIGHT of the sheet, what can you say about B just LEFT of the sheet?

A) +x direction
B) -x direction
C) +z direction
D) -z direction
E) Something else!