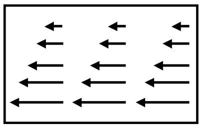
1.7c

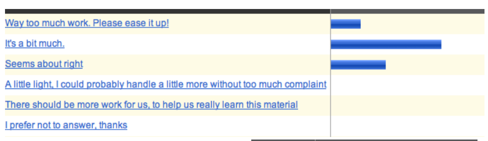
What is the curl of this vector field, in the region shown below?



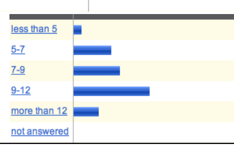
A. non-zero everywhere
B. Non-zero at a limited set of points
C. zero curl everywhere
D. We need a formula to decide for sure

Pace

Load



Hours



Today: Voltage or "Electric Potential"

The 1120 version:

- Voltage V = kq/r from a point charge
- Voltage = potential energy/charge
- ΔV is "path independent"
- E = -dV/dr

Potential and E-field

$$V(\mathbf{r}) \equiv -\int_{\text{origin}}^{\mathbf{r}} \vec{E}(r') \bullet d\vec{r}\,'$$

$$qV(\mathbf{r}) \equiv -\int_{\text{origin}}^{\mathbf{r}} q\vec{E}(r') \bullet d\vec{r}\,'$$

$$= -\int_{\text{origin}}^{\mathbf{r}} \vec{F}_{\text{on q}} \bullet d\vec{r}\,'$$

= external work done on charge to move it

Potential and E-field

$$V(\mathbf{r}) \equiv -\int_{\text{origin}}^{\mathbf{r}} \vec{E}(r') \bullet d\vec{r}\,'$$

$$\mathbf{E}(\mathbf{r}) = -\nabla V(r)$$

The fact that V is *well defined* arises from

$$\oint \vec{\mathbf{E}}(\mathbf{r}) \bullet d\vec{l} = 0$$

Which is exactly the same as (!)

$$\nabla \times \vec{\mathbf{E}}(\mathbf{r}) = 0$$

Potential and E-field

$$V(\mathbf{r}) = -\int_{\text{origin}}^{\text{r}} \vec{E}(r') \bullet d\vec{r}\,'$$

$$\mathbf{E}(\mathbf{r}) = -\nabla V(r)$$

One way to find V(r) (given charges):

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho(\mathbf{r}')}{|\Re|} d\tau'$$

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$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \iiint \frac{dq}{|\Re|}$$

Voltage V = kq/r from a point charge !

So, what does

$$V(\mathbf{r}) \equiv -\int_{\text{origin}}^{\mathbf{r}} \vec{E}(r') \bullet d\vec{r}\,'$$

Have to do with

$$\oint \vec{\mathbf{E}}(\mathbf{r}) \bullet d\vec{l} = 0$$

And *why* is that the same as

$$\nabla \times \vec{\mathbf{E}}(\mathbf{r}) = 0$$

Fundamental theorem:

$$\int_a^b \left(\frac{df}{dx}\right) dx = f(b) - f(a)$$

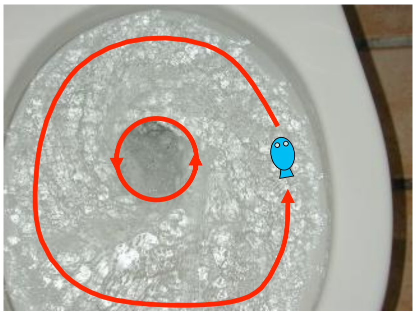
Divergence theorem:

$$\iiint_V (\nabla \bullet \vec{F})\, d\tau = \oiint_S \vec{F} \bullet d\vec{A}$$

Stoke's (or "curl") theorem:

$$\iint_S (\nabla \times \vec{F}) \bullet d\vec{A} = \oint \vec{F} \bullet d\vec{l}$$

Common vortex



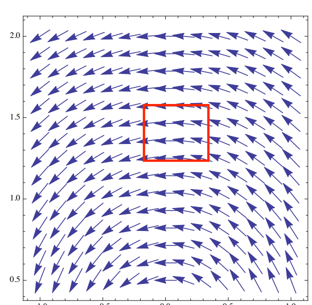
Feline Vortex

1.8

What is the curl of this vector field, **V** in the region shown below?

$$\vec{\mathbf{V}} = c\,\hat{\varphi}$$

A. non-zero everywhere
B. Zero at some points, non-zero others
C. zero curl everywhere shown



1.8

What is the curl of this vector field, **V** in the region shown below?

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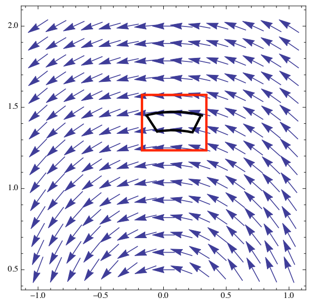

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1.8

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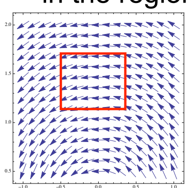


$$\nabla \times \vec{\mathbf{V}} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right]\hat{z}$$

Why is $\nabla \times \vec{\mathbf{E}}(\mathbf{r}) = 0$

1.8

$$\vec{E}(r) = \frac{q}{4\pi\varepsilon_0 r^2}\,\hat{r}$$

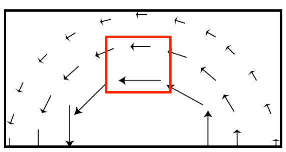
$$\nabla \times \vec{\mathbf{V}} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$

$$+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial(r v_\phi)}{\partial r}\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial(r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial\theta}\right]\hat{\varphi}$$

$\nabla \times \vec{E} =$ 0!

1.8

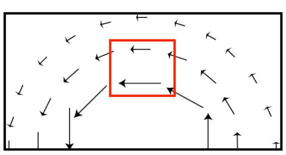
What is the curl of this vector field, in the red region shown below?



A. non-zero everywhere in the box
B. Non-zero at a limited set of points
C. zero curl everywhere shown
D. We need a formula to decide for sure

1.8

What is the curl of this vector field, in the red region shown below?



$$\vec{\mathbf{V}} = \frac{c}{s}\,\hat{\varphi}$$

A. non-zero everywhere in the box
B. Non-zero at a limited set of points
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1.8

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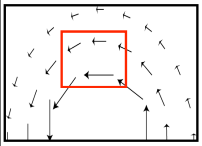
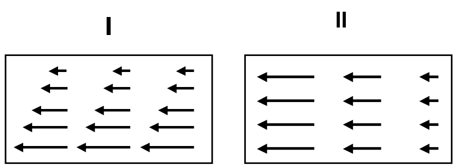


$$\nabla \times \vec{\mathbf{V}} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right]\hat{z}$$

A. non-zero everywhere in the box
B. Non-zero at a limited set of points
C. zero curl everywhere shown

2.43b/1.7b

Which of the following *could* be a static physical E-field in a small region?



A) Only I B) Only II
C) Both D) Neither
E) Cannot answer without further info

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\mathbf{r}')\,\hat{\Re}}{\Re^2}\,d\tau'$$

$$\vec{\Re} = \mathbf{r} - \mathbf{r}' = (x - x', y - y', z - z')$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\mathbf{r}')\,\hat{\mathfrak{R}}}{\mathfrak{R}^2}\,d\tau'$$

(with $\vec{\mathfrak{R}} = \mathbf{r} - \mathbf{r}' = (x - x', y - y', z - z')$)

It is also true that $\frac{\hat{\mathfrak{R}}}{\mathfrak{R}^2} = -\nabla \frac{1}{|\mathfrak{R}|}$

where $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\mathbf{r}')\,\hat{\mathfrak{R}}}{\mathfrak{R}^2}\,d\tau'$$

(with $\vec{\mathfrak{R}} = \mathbf{r} - \mathbf{r}' = (x - x', y - y', z - z')$)

It is also true that $\frac{\hat{\mathfrak{R}}}{\mathfrak{R}^2} = -\nabla \frac{1}{|\mathfrak{R}|}$

where $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \rho(\mathbf{r}')\left(-\nabla \frac{1}{|\mathfrak{R}|}\right)d\tau'$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\mathbf{r}')\,\hat{\mathfrak{R}}}{\mathfrak{R}^2}\,d\tau'$$

(with $\vec{\mathfrak{R}} = \mathbf{r} - \mathbf{r}' = (x - x', y - y', z - z')$)

It is also true that $\frac{\hat{\mathfrak{R}}}{\mathfrak{R}^2} = -\nabla \frac{1}{|\mathfrak{R}|}$

where $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

Question: is the following mathematically ok?

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \rho(\mathbf{r}')\left(-\nabla \frac{1}{|\mathfrak{R}|}\right)d\tau' = -\nabla \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\mathbf{r}')}{|\mathfrak{R}|}\,d\tau'$$

A) Yes B) No C) ???

2.43

Which of the following electrostatic fields could exist in a finite region of space that contains no charges?

A) $Axyz(\hat{i} + \hat{j})$

B) $A(xz\hat{i} - xz\hat{j})$

C) $A(-xy\hat{j} + xz\hat{k})$

D) None of these

$$\mathbf{E}(\mathbf{r}) = -\nabla \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho(\mathbf{r}')}{|\Re|} d\tau'$$

$$\mathbf{E}(\mathbf{r}) = -\nabla V(\vec{r})$$

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho(\mathbf{r}')}{|\Re|} d\tau' \quad + c$$

Summary:

Def of potential: $V(\mathbf{r}) \equiv -\int_{\text{origin}}^{\mathbf{r}} \vec{E}(r') \bullet d\vec{r}\,'$

How do you compute it: $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho(\mathbf{r}')}{|\Re|} d\tau'$

What good is it? $\mathbf{E}(\mathbf{r}) = -\nabla V(\vec{r})$

Where did it come from? $\nabla \times \vec{\mathbf{E}}(\mathbf{r}) = 0$