LAPLACE'S EQUATION AND UNIQUENESS

Poisson's equation tells us that

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

If the charge density throughout some volume is zero, what else *must* be true throughout that volume:

A) V=0
B) E=0
C) Both V and E must be zero
D) None of the above is necessarily true

2.46

Why is $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{L}}$ =0 in electrostatics?

a) Because $\nabla X \vec{E} = 0$

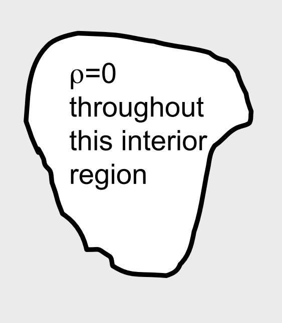
b) Because E is a conservative field

c) Because the potential between two points is independent of the path

d) All of the above

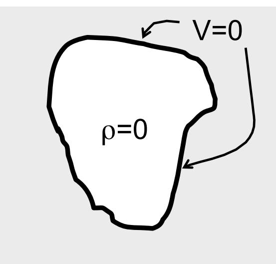
e) NONE of the above - it's not true!

3.1 A region of space contains no charges.
What can I say about V in the interior?



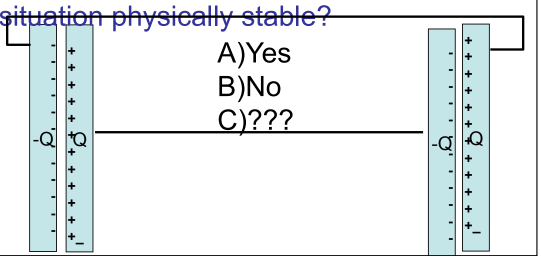
A) Not much, there are lots of possibilities for V(r) in there
B) V(r)=0 everywhere in the interior.
C) V(r)=constant everywhere in the interior

3.2 A region of space contains no charges.
The *boundary* has V=0 everywhere.
What can I say about V in the interior?



A) Not much, there are lots of possibilities for V(r) in there
B) V(r)=0 everywhere in the interior.
C) V(r)=constant everywhere in the interior

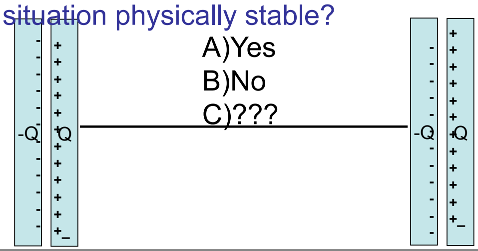
3.3 Two very strong (big C) ideal capacitors are well separated.
If they are connected by 2 thin conducting wires, as shown, is this electrostatic situation physically stable?



A)Yes
B)No
C)???

3.4 Two very strong (big C) ideal capacitors are well separated.

What if they are connected by one thin conducting wire, is this electrostatic situation physically stable?



A)Yes
B)No
C)???

General properties of solutions of ∇^2 V=0

(1) V has no local maxima or minima inside. Maxima and minima are located on surrounding boundary.

(2) V is boring. (I mean "smooth & continuous" everywhere).

(3) V(**r**) = average of V over any surrounding sphere:

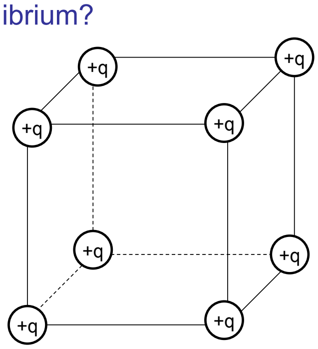
$$V(\vec{r}) = \frac{1}{4\pi R^2} \cdot \oint_{\substack{\text{Sphere with} \\ \text{radius } R \\ \text{around } \vec{r}}} V da$$

(4) V is unique: The solution of to the Laplace eq. is uniquely determined if V is specified on the boundary surface around the volume.

3.5 If you put a + test charge at the *center* of this cube of charges, could it be in stable equilibrium?

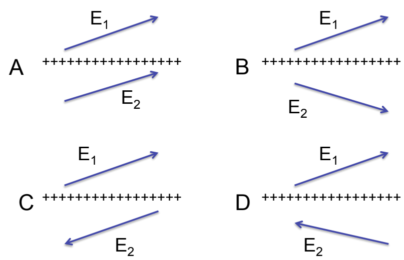
A) Yes
B) No
C) ???

Earnshaw's Theorem



3.X

Which of the following cases could actually occur above and below a sheet of surface charge?



E: None of these

METHOD OF IMAGES

Class Activities: Method of Images

Whiteboards

Method of Images

I set up the "method of images" problem (with +Q above, and -Q below), and had THEM, in pairs, write the formula for V(x,y,z). (They struggled surprisingly with this!) I then had them evaluate V(x,y,0) and V(anything -> infinity). Lastly, I had the faster groups work out Ex, Ey, and/or Ez, and evaluate it on the plane.

Writing

What is Method of Images?

I posted this "On paper (don't forget your name!) in your own words (by yourself):What is the idea behind the method of images? What does it accomplish?What is its relation to the uniqueness theorem?" and collected their answers.

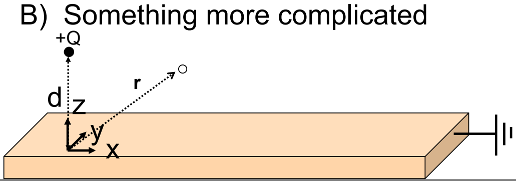
3.7 A point charge +Q sits above a *very large grounded* conducting slab.

What is **E**(**r**) for points *above* the slab?

A) Simple Coulomb's law:

$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0}\frac{\vec{\Re}}{\Re^3} \quad \text{with } \vec{\Re} = (\vec{r} - d\,\hat{z})$$

B) Something more complicated



Whiteboard:

Calculate voltage for 2 point charges a distance "d" above and below the origin. Where is V(r)=0?

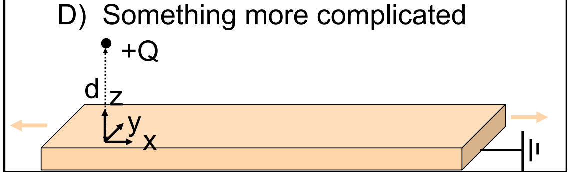
3.8 A point charge +Q sits above a *very large grounded* conducting slab.

What's the electric force on +Q?

A) 0 B) $\frac{Q^2}{4\pi\varepsilon_0(2d)^2}$ downwards

C) $\frac{Q^2}{4\pi\varepsilon_0 d^2}$ downwards

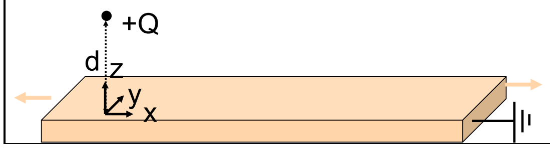
D) Something more complicated



3.8b A point charge +Q sits above a *very large grounded* conducting slab. **What's the energy of this system?**

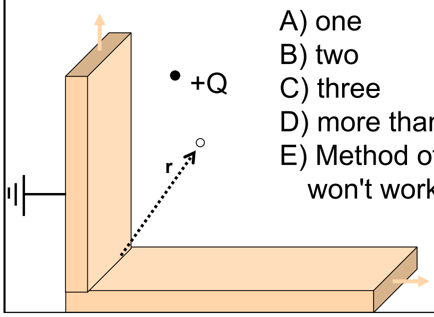
A) $\frac{-Q^2}{4\pi\varepsilon_0(2d)}$

B) Something else.



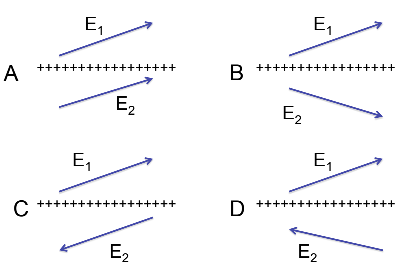
3.9 Two ∞ grounded conducting slabs meet at right angles. How many image charges are needed to solve for V(**r**)?

A) one
B) two
C) three
D) more than three
E) Method of images won't work here



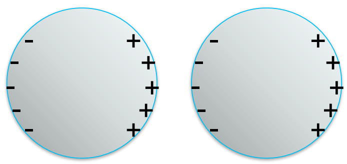
3.X

Which of the following cases could actually occur above and below a sheet of surface charge?



E: None of these

Is this a stable charge distribution for two neutral, conducting spheres?



A) Yes
B) No

C) ???

SEPARATION OF VARIABLES

Class Activities: Sep of Var (1)

DIscussion
Questions for Lecture (from UIUC)
1.) What are the physical reasons/motivation for wanting to solve the Laplace equation or the Poisson equation?
2.) What is the general form of the 1-D solution to Laplaces equation?
3.) The nature of Laplace's equation is such that it tolerates/allows NO local maxima or minima for V(r); all extrema of V(r) must occur at endpoints/boundaries. Why?

Whiteboards
Sinh and Cosh
Draw sinh and cosh, is it even/odd, what's the curvature, behaviour as x=0, infinity, what's cosh(pi), etc...
Show my mathematica solution in powerpoint (I did *not* show the 2D "0-V0-0-V0" boundary value problem, called "alternative_separation_variables.nb")

Whiteboards
Boundary conditions on E
One persistent difficulty that students have is an inability to recreate the mathematical steps to determine the boundary conditions on the parallel and perpendicular components of E. After watching 3 continuous semesters of this course, I strongly recommend having a whiteboard or worksheet activity where students are asked to derive those boundary conditions given a surface charge.

Simulation
Fourier
Show the PhET Fourier sim. Also show the mathematic notebook I made where we can *look* at the Legendre Polynomials. See http://phet.colorado.edu/index.php

Class Activities: Sep of Var (2)

Discussion
Separability
Make up a function f(x) and a function g(y) and come up with a pair that satisfies f(x)+g(y)=0. (You can't do it unless they're constants).

Tutorial
Electric Field Continuity across a Boundary
Oregon State University
Students working in small groups use Maxwell's equations to determine the continuity of the electric field across a charged surface

Discussion
Cartesian separation of variables.
Taking the first several terms in the series will give an approximation of the solution, much like in a Taylor expansion. How many terms should we keep to have a reasonable approximation of the potential? *This discussion question, along with mathematica plots, is contained in the Chapter 3 clicker question document.*

Discussion
Cartesian separation of variables
Rectangular Pipe 2 is twice as wide as rectangular Pipe 1 and the shaded face is at a potential V(x,y). Before applying the boundary conditions how will the solutions to Laplace's Equation be qualitatively different or similar? *This is included in Chapter 3 clicker question document.*

Worksheet
Identifying separable differential equations
Dawn Meredith "Meaning in Mathematics"
http://pubpages.unh.edu/~dawnm/connectm&m.html
This sheet is practice (not guided inquiry) on recognizing separable diff eqs. The second page gives students a chance to see why second order diff eqs are never separable.

Class Activities: Sep of Var (3)

Tutorial
Laplace's Equation
Paul van Kampen – Dublin University (Tutorials 9-16, page 10)
Tutorial on Laplace's equation. Conducting cylinder in E field. First describe what happens to the cylinder when placed in the E field. Do separation of variables in cylindrical coordinates.

Griffiths by Inquiry **(Lab 2): Laplace's Equation and Boundary Value problems**

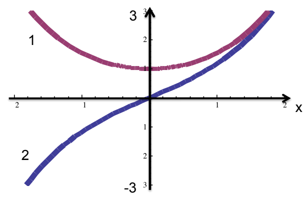
Griffiths by Inquiry **(Lab 3): 2D Boundary Value problems in Cartesian coordinates**

Griffiths by Inquiry **(Lab 4): 2D Boundary Value problems in Cylindrical coordinates**

Griffiths by Inquiry **(Lab 5): 3D Boundary Value problems in electrostatics**

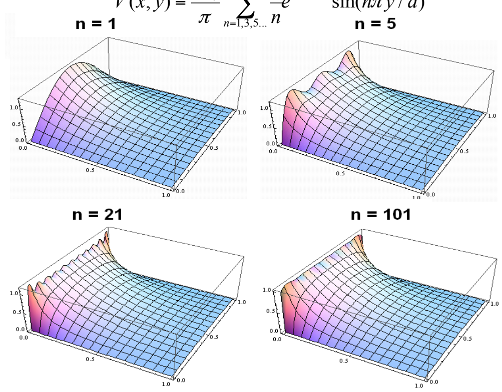
CARTESIAN COORDINATES

Two solutions for *positive C* are sinh x and cosh x :



Which is which?
A)Curve 1 is sinh x and curve 2 is cosh x
B)Curve 1 is cosh x and curve 2 is sinh x

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5...}^{\infty} \frac{1}{n} e^{-n\pi x/a} \sin(n\pi y/a)$$



Second uniqueness Theorem: In a volume surrounded by conductors and containing a specified charge density *ρ(r)*, the electric field is uniquely determined if the total charge on each conductor is given. (The region as a whole can be bounded by another conductor, or else unbounded.)

Griffiths, 3.1.6

Say you have three functions f(x), g(y) and h(z).
f(x) only depends on 'x' but not on 'y' and 'z'.
g(y) only depends on 'y' but not on 'x' and 'z'.
h(z) only depends on 'z' but not on 'z' and 'y'.

If f(x) + g(y) + h(z) = 0 for all x, y, z, then:

A)All three functions are constants (i.e. they do not depend on x, y, z at all.)
B)At least one of these functions has to be zero everywhere.
C)All of these functions have to be zero everywhere

3.10 Suppose $V_1(\mathbf{r})$ and $V_2(\mathbf{r})$ are linearly independent functions which *both* solve Laplace's equation, $\nabla^2 V = 0$

Does $aV_1(\mathbf{r})+bV_2(\mathbf{r})$ also solve it (with a and b constants)?

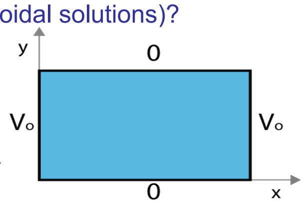
A) Yes. The Laplacian is a linear operator
B) No. The *uniqueness theorem* says this scenario is impossible, there are never two independent solutions!
C) It is a definite yes or no, but the *reasons* given above just aren't right!
D) It depends...

3.11 Given the two diff. eq's:

$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

where $C_1+C_2 = 0$. Given the boundary conditions in the figure, which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

A) x B) y
C) $C_1 = C_2 = 0$ here
D) It doesn't matter



3.11h

The X(x) equation in this problem involves the "positive constant" solutions:
A sinh(kx) + B cosh(kx)

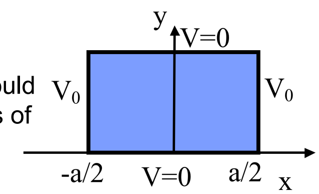
What do the boundary conditions say about A and B?

A) A=0 (pure cosh)

B) B=0 (pure sinh)

C) Neither: you should rewrite this in terms of A e^{kx} + B e^{-kx} !

D) Other/not sure?



3.11c

Given the two diff. eq's:

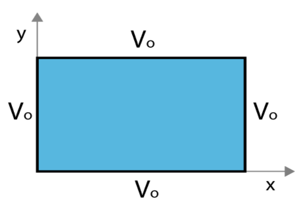
$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

where $C_1+C_2 = 0$. Which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

A) x B) y

C) $C_1 = C_2 = 0$ here

D) It doesn't matter



3.11b

Given the two diff. eq's:

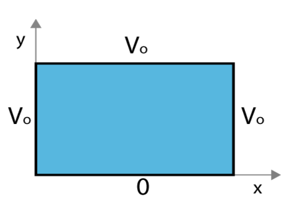
$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

where $C_1+C_2 = 0$. Which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

A) x B) y

C) $C_1 = C_2 = 0$ here

D) It doesn't matter



3.11h The X(x) equation in this problem involves the "positive constant" solutions:
A sinh(kx) + B cosh(kx)

What do the boundary conditions say about A and B?

A) A=0 (pure cosh)

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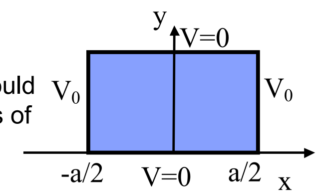
C) Neither: you should rewrite this in terms of A e^{kx} + B e^{-kx} !

D) Other/not sure?



3.12
What is the value of

$$\int_0^{2\pi} \sin(2x)\sin(3x)dx \ ?$$

A) Zero

B) π

C) 2π

D) $\pi/2$

E) Something else/how could I possibly know this?

‘Separation of Variables’ is:

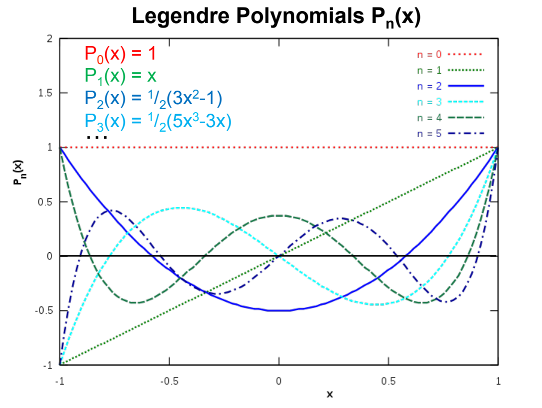
A) … easy – piece of cake!

B) … I’m getting the hang of it. More examples please (maybe in spherical coordinates?).

C) … somewhat confusing.

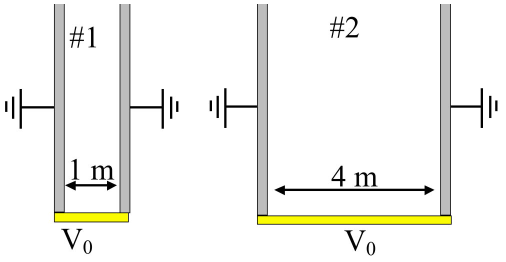
D) … really hard! What’s going on?

E) Fourier’s trick just doesn’t make any sense (or other things, such as: ‘Why did we write V(x,y,z) = X(x)*Y(y)*Z(z) ?’)



3.14

2 troughs (∞ in z, i.e. out of page) have grounded sidewalls. The base of each is held at V0.



3.14 $$V(x,y) = \frac{4V_0}{\pi}\sum_{n=1,3,5...}^{\infty}\frac{1}{n}\sin(n\pi x/a)e^{-n\pi y/a}$$

How does V(x,y) compare, 4 m above the middle of the base in the two troughs?

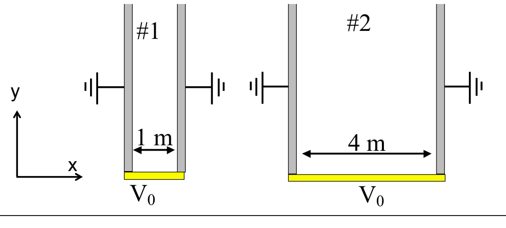
A) Same in each

B) 4x bigger in #1

C) 4x bigger in #2

D) *much* bigger in #1

E) *much* bigger in #2



SEP OF VAR: LEGENDRE POLYNOMIALS

Orthogonality

$$\int_{-1}^{1} P_l(x)P_m(x)dx = \begin{cases} \frac{2}{2l+1} & \text{if } l = m \\ 0 & \text{if } l \neq m \end{cases}$$

With: $x = cos\theta$ and: $dx = - sin\theta d\theta$, we get:

$$\int_{0}^{\pi} P_l(\cos\theta)P_m(\cos\theta)\sin\theta d\theta = \begin{cases} \frac{2}{2l+1} & \text{if } l = m \\ 0 & \text{if } l \neq m \end{cases}$$

3.10

Suppose $V_1(\mathbf{r})$ and $V_2(\mathbf{r})$ are linearly independent functions which *both* solve Laplace's equation, $\nabla^2 V = 0$

Does $aV_1(\mathbf{r})+bV_2(\mathbf{r})$ also solve it (with ‘a’ and ‘b' constants)?

A) Yes. The Laplacean is a linear operator
B) No. The *uniqueness theorem* says this scenario is impossible, there are never two independent solutions!
C) It is a definite yes or no, but the *reasons* given above just aren't right!
D) It depends...

3.15 Given $\nabla^2 V = 0$ in Cartesian coords, we separated V(x,y,z) = X(x)Y(y)Z(z). Will this approach work in spherical coordinates, i.e. can we separate V(r,θ,φ) = R(r)P(θ)F(φ)?

A) Sure.

B) Not quite - the angular components cannot be isolated, e.g. f(r,θ,φ) = R(r)Y(θ,φ)

C) It won't work at all because the spherical form of Laplace's Equation has cross terms in it (see the front cover of Griffiths)

3.16 The Rodrigues formula for generating the Legendre Polynomials is

$$P_l(x) = \frac{1}{2^l l!}\left(\frac{dy}{dx}\right)^l (x^2-1)^l$$

If the Legendre polynomials are orthogonal, are the leading coefficients $\frac{1}{2^l l!}$ necessary to maintain orthogonality?

A) Yes, $f_m(x)$ must be properly scaled for it to be orthogonal to $f_n(x)$.

B) No, the constants will only rescale the integral

3.17 Given $V(\theta) = \sum_{l=0}^{\infty} C_l P_l(\cos\theta)$

(The P_l's are Legendre polynomials.)

If we want to isolate/determine the coefficients

C_l in that series, multiply both sides by:

A) $P_m(\theta)$

B) $P_m(\cos\theta)$

C) $P_m(\theta)\sin\theta$

D) $P_m(\cos\theta)\sin\theta$

E) something entirely different

$$\int_0^\pi P_l(\cos\theta)P_m(\cos\theta)\sin\theta d\theta = \begin{cases} \frac{2}{2l+1} & \text{if } l = m \\ 0 & \text{if } l \neq m \end{cases}$$

$$\int_{-1}^1 P_l(x)P_m(x)dx = \begin{cases} \frac{2}{2l+1} & \text{if } l = m \\ 0 & \text{if } l \neq m \end{cases}$$

3.18

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is constant, i.e. $V(R, \theta) = V_0$.
Which terms do you expect to appear when finding V(outside) ?

A) Many A_l terms (but no B_l's)
B) Many B_l terms (but no A_l's)
C) Just A_0
D) Just B_0
E) Something else!!

3.18 b

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is constant, i.e. $V(R, \theta) = V_0$.
Which terms do you expect to appear when finding V(inside) ?

A) Many A_l terms (but no B_l's)
B) Many B_l terms (but no A_l's)
C) Just A_0
D) Just B_0
E) Something else!

3.19a

$P_0(\cos\theta) = 1,$ $P_1(\cos\theta) = \cos\theta$

$P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2},$ $P_3(\cos\theta) = \frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta$

Can you write the function $V_0(1+\cos^2\theta)$ as a sum of Legendre Polynomials?

$$V_0(1+\cos^2\theta) \overset{???}{=} \sum_{l=0}^{\infty} C_l P_l(\cos\theta)$$

A) No, it cannot be done
B) It would require an infinite sum of terms
C) It would only involve P_2
D) It would involve all three of P_0, P_1 AND P_2
E) Something else/none of the above

3.19

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is

$$V(R,\theta) = V_0(1+\cos^2\theta)$$

Which terms do you expect to appear when finding V(inside) ?
A) Many A_l terms (but no B_l's)
B) Many B_l terms (but no A_l's)
C) Just A_0 and A_2
D) Just B_0 and B_2
E) Something else!

3.19 b

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is

$$V(R,\theta) = V_0(1+\cos^2\theta)$$

Which terms do you expect to appear when finding V(outside) ?
A) Many A_l terms (but no B_l's)
B) Many B_l terms (but no A_l's)
C) Just A_0 and A_2
D) Just B_0 and B_2
E) Something else!

MD11-2

Suppose that applying boundary conditions to Laplace's equation leads to an equation of the form: $\nabla^2 V = 0$

$$\sum_{l=0}^{\infty} C_l P(\cos\theta) = 4 + 3\cos\theta$$

$P_0(x) = 1$
$P_1(x) = x$
$P_2(x) = (3x^2 - 1)/2$

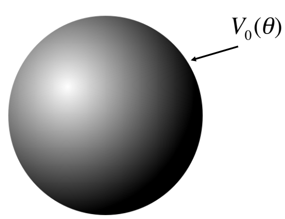
Can you solve for the coefficients, the C_l's ?

A) No, you need at least one more equation to solve for any the C's.
B) Yes, you have enough info to solve for all of the C's
C) Partially. Can solve for C_0 and C_1, but cannot solve for the other C's.
D) Partially. Can solve for C_o, but cannot solve for the other C's.

3.20

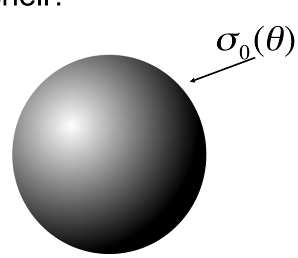
How many boundary conditions (on the potential V) do you use to find V inside the spherical plastic shell?

A) 1
B) 2
C) 3
D) 4
E) It depends on $V_0(\theta)$



3.21

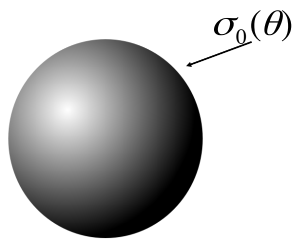
How many boundary conditions (on the potential V) do you use to find V inside the thin plastic spherical shell?



A) 1
B) 2
C) 3
D) 4
E) depends on σ_0

3.21

How many boundary conditions (on the potential V) do you use to find V inside the thin plastic spherical shell?



A) 1
B) 2
C) 3
D) 4
E) depends on σ

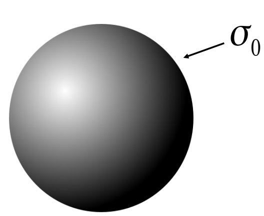
3.21 b

Does the previous answer change at all if you're asked for V *outside* the sphere?

a) yes

b) No

Since the electric field is zero inside this conducting sphere, and $V = -\int \vec{E} \cdot d\vec{l}$, is V=0 inside as well?



a) Yes

b) No

MULTIPOLE EXPANSION

Class Activities: Multipole

Tutorial
Multipole expansion: "Discrete" activity
Oregon State University
Students work in small groups to create power series expansions for the electrostatic potential due to two electric charges separated by a distance D.

Griffiths by Inquiry (Lab 6): Multipole expansions

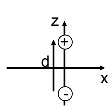
Discussio n
Questions for lecture (from UIUC):
1.) Why should we care about approximate solutions for the scalar potential V(r) and/or E(r)?
2.) What are electric multipole moments of an electric charge distribution?
3.) When is it appropriate to use such approximate solutions for the scalar potential V(r) and/or E(r)?

MD6 - 2

For a dipole at the origin pointing in the z-direction, we have derived

$$\vec{\mathbf{E}}_{dip}(\vec{r}) = \frac{\vec{p}}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\theta}\right)$$

For the dipole **p** = q **d** shown, what does the formula predict for the direction of **E**(**r**=0)?



A) Down B) Up C) some other direction

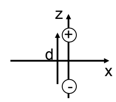
D) The formula doesn't apply.

MD6 - 2

At the end of last class we derived the potential for a dipole at the origin pointing in the z-direction. Using **E** = -∇V we can find the E-field in spherical coordinates:

$$\vec{\mathbf{E}}_{dip}(\vec{r}) = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\theta}\right)$$

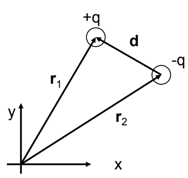
For the dipole **p** = q **d** shown, what does the formula predict for the direction of **E**(**r**=0)?



A)Down B) Up C) some other direction

D) The formula doesn't apply.

MD6.1



$$\sum_i q_i\,\vec{r}_i = ?$$

A) $+q\vec{d}$

B) $+2q\vec{d}$

C) $-2q\vec{d}$

D) zero

E) None of these

MD6 - 3

For a collection of point charges, the dipole moment is defined as $\vec{p} = \sum_i q_i\,\vec{r}_i$

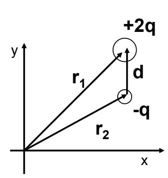
Consider the two charges, +2q and –q, shown. Which statement is true?

A) The dipole moment is independent of the origin.

B) The dipole moment depends on the position of the origin.

C) The dipole moment is zero.

D) The dipole moment is undefined.



3.22a

A small dipole (dipole moment p=qd) points in the z direction.

We have derived $V(\vec{r}) \approx \frac{1}{4\pi\varepsilon_0}\frac{qdz}{r^3}$

Which of the following is correct (and "coordinate free")?

A) $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$

B) $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^3}$

C) $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^2}$

D) $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\times\hat{r}}{r^2}$

E) None of these

3.22 b

An ideal dipole (tiny dipole moment p=qd) points in the z direction.

We have derived $\vec{\mathbf{E}}(\vec{r}) = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\vec{\theta}\right)$

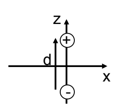
Sketch this E field...
(What would change if the dipole separation d was *not* so tiny?)

MD6 - 2

For a dipole at the origin pointing in the z-direction, we have derived

$$\vec{\mathbf{E}}_{\text{dip}}(\vec{r}) = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\theta}\right)$$

For the dipole **p** = q **d** shown, what does the formula predict for the direction of **E**(**r**=0)?



A) Down B) Up C) some other direction

D) The formula doesn't apply.

3.22
c

You have a physical dipole, +q and -q a finite distance d apart.
When can you use the expression:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

A) This is an exact expression everywhere.

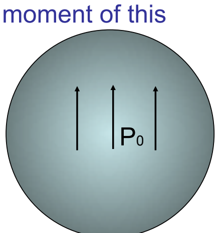
B) It's valid for large r

C) It's valid for small r

D) ?

4.1
alt

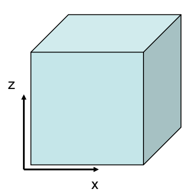
The sphere below (radius a) has uniform polarization $\mathbf{P}_0$ (which points in the z direction.)
What is the total dipole moment of this sphere?



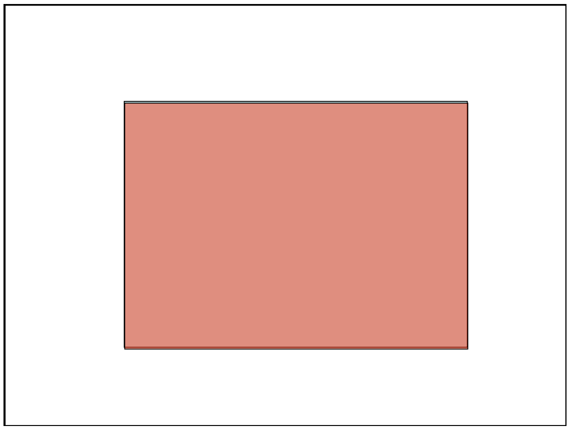
A) zero
B) $P_0 a^3$
C) $4\pi a^3 P_0/3$
D) P_0
E) None of these/must be more complicated

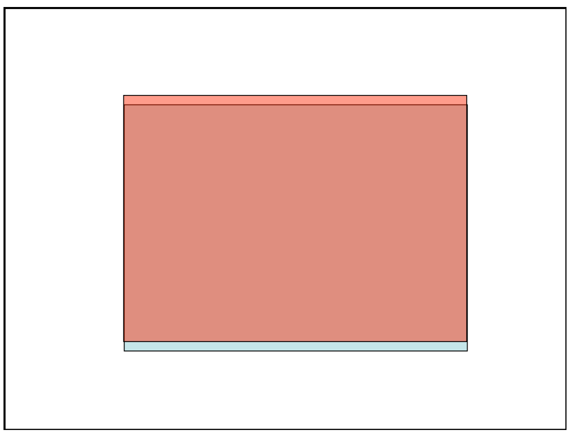
4.1

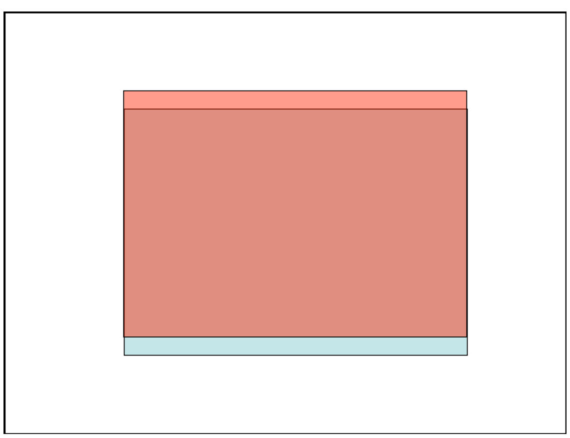
The cube below (side a) has uniform polarization $\mathbf{P}_0$
(which points in the z direction.)
What is the total dipole moment of this cube?

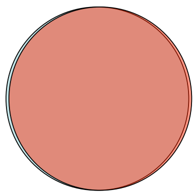


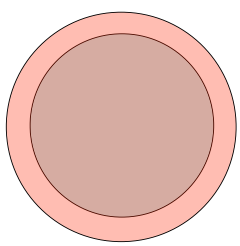
A) zero
B) $a^3 \mathbf{P}_0$
C) $\mathbf{P}_0$
D) $\mathbf{P}_0/a^3$
E) $2\mathbf{P}_0 a^2$











3.22
d

You have a physical dipole, +q and -q, a finite distance d apart.
When can you use the expression

$$V(r) = \frac{1}{4\pi\varepsilon_0}\sum \frac{q_i}{\mathfrak{R}_i}$$

A) This is an exact expression everywhere.

B) It's valid for large r

C) It's valid for small r

D) ?

3.23

Griffiths argues that the force *on* a dipole in an E field is: $\vec{\mathbf{F}} = (\vec{\mathbf{p}} \bullet \vec{\nabla})\vec{\mathbf{E}}$

If the dipole **p** points in the z direction, what direction is the force?

A) Also in the z direction
B) perpendicular to z
C) it could point in any direction

3.23

Griffiths argues that the force on a neutral dipole in an external E field is:

$$\vec{\mathbf{F}} = (\vec{\mathbf{p}} \bullet \vec{\nabla})\vec{\mathbf{E}}_{ext}$$

If the dipole **p** points in the z direction, what direction is the force?

A) Also in the z direction
B) perpendicular to z
C) it could point in any direction
D) the force is zero because the dipole is neutral

3.24

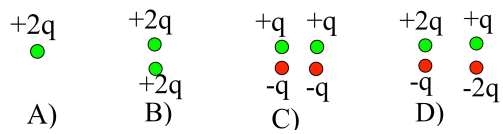
Griffiths argues that the force *on* a dipole in an E field is: $\vec{\mathbf{F}} = (\vec{\mathbf{p}} \bullet \vec{\nabla})\vec{\mathbf{E}}$

If the dipole **p** points in the z direction, what can you say about **E** if I tell you the force is in the x direction?

A) **E** simply points in the x direction
B) Ez must depend on x
C) Ez must depend on z
D) Ex must depend on x
E) Ex must depend on z

3.25

Which charge distributions below produce a potential which looks like C/r^2 when you are far away?

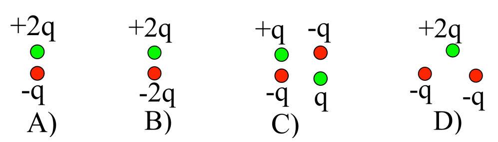


E) None of these, or *more* than one of these!

(Note: for any which you did not select, how DO they behave at large r?)

3.26

Which charge distributions below produce a potential which looks like C/r^2 when you are far away?



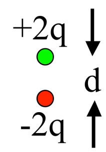
E) None of these, or *more* than one of these!

(Note: for any which you did not select, how DO they behave at large r?)

No flies were harmed in the process

3.27

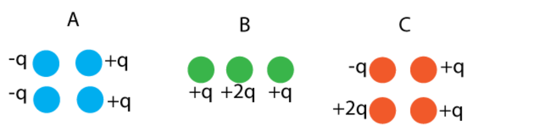
What is the magnitude of the dipole moment of this charge distribution?

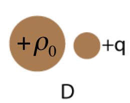


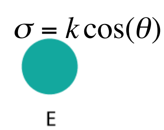
A) qd
B) 2qd
C) 3qd
D) 4qd
E) It's not determined

(To think about: How does V(r) behave as r gets large?)

3.28 In which situation is the dipole term the leading non-zero contribution to the potential?

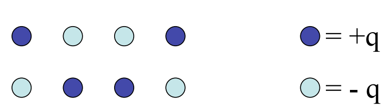






A) A and C
B) B and D
C) only E
D) A and E
E) Some other combo

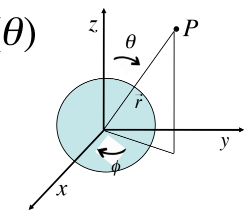
3.29 In terms of the multipole expansion
V(r) = V(mono) + V(dip) + V(quad) + …
the following charge distribution has the form:



A) V(r) = V(mono) + V(dip) + higher order terms
B) V(r) = V(dip) + higher order terms
C) V(r) = V(dip)
D) V(r)=only higher order terms than dipole
E) No higher terms, V(r)=0 for this one.

3.30 What is the direction of the dipole moment of the blue sphere?

$$\sigma = k\sin(\theta)$$



A) $\hat{\theta}$ B) $\hat{r}$

C) $\hat{\phi}$ D) $\hat{z}$

E) dipole moment is zero or ill-defined

On paper (don't forget your name!) in your own words (by yourself):

What is the idea behind the multipole expansion? What does it accomplish?
In what limits/cases is it useful?